PRACTICAL ARITHMETIC

S. W. BAIRD

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GRAMMAR GRADES

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PRACTICAL

ARITHMETIC

BY

S. W. BAIRD

PRINCIPAL FRANKLIN GRAMMAR SCHOOL, WILKESBARRE, PA.

GRAMMAR GRADES

NEW YORK ·:· CINCINNATI ·:· CHICAGO

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BAIRD'S PRACT. ARITH.
W. P. 4

PREFACE

This book completes a well-graded and progressive series of arithmetics, and furnishes to grammar-school pupils a text-book carefully planned to strengthen their power of mathematical reasoning, presenting a range of topics sufficiently comprehensive to familiarize the students at the same time with the important practical applications of the science of numbers.

The subjects have advisedly been presented in an order differing from that usually set forth in books of the kind. Percentage and all applications thereof not involving the time element most naturally follow decimals, and the computing of simple interest as treated on page 121 is but one step in advance. The preliminary treatment of percentage and interest at this stage makes it possible for pupils who leave school before the course is completed to gain some knowledge of these important subjects.

The book contains no meaningless 'rules,' and only such definitions and explanations as are indispensable for the intelligent pursuit of the study. In their place are found an unusually large number of practical problems in every department of the subject.

The *Statement Plan* is a device to indicate the solution of a problem, just as the diagram shows the analysis of the sentence. It saves as much time and labor for the pupil and for the teacher in arithmetic as the diagram

does in grammar. It gives new life and interest to the study of arithmetic; it trains the pupil to observe carefully the conditions of the problem, and to consider in logical order the various steps necessary to its solution; it breaks up the tendency on the part of the pupil to follow the rule, or to experiment to obtain the answer; and it leads to accurate observation and logical thinking.

The growing demand for the introduction of some elementary algebra into the arithmetic classes of the higher grammar grades has led the author to devote one chapter to this subject. No attempt has been made to treat it in a formal way, the chief aim being to familiarize pupils with the use of letters in the solution of arithmetical problems and with the use of the equation.

The author tenders his grateful acknowledgments to those able and efficient teachers who have given him encouragement, advice, and assistance in his efforts to make the book a valuable instrument of education.

S. W. BAIRD.

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ARITHMETIC

DEFINITIONS

1. *Arithmetic* treats of numbers and the art of using them.

2. The fundamental idea of arithmetic is the unit.

3. A *Unit* is any single thing; as *one, one apple, one pen, one cent,* etc.

4. A *Number* is a unit or collection of units; as *one, four, seven, two cents, nine men,* etc.

Note. — Strictly speaking, number cannot be defined. In arithmetic it has reference to that kind of quantity which can be expressed by figures.

5. Numbers not united with any particular kind of quantity, as *six, nine, twelve,* etc., are called *Abstract* numbers. Numbers prefixed to some particular kind of quantity, as *six trees, nine pictures, twelve pounds,* etc., are called *Concrete* numbers.

6. *Notation, Numeration, Addition, Subtraction, Multiplication,* and *Division* are called the *Fundamental Operations* of arithmetic, because all other operations depend upon them.

NOTATION AND NUMERATION

7. *Notation* treats of the writing of numbers.

8. *Numeration* treats of the reading of numbers.

9. There are three ways of writing numbers:

1. By words, as *one, two, three, four*, etc.
2. By figures, called the *Arabic Method.*
3. By letters, called the *Roman Method.*

ARABIC NOTATION

10. In the *Arabic,* or *Decimal* system of notation ten characters called figures are employed to represent numbers.

FIGURES	NAMES	FIGURES	NAMES
1	one	6	six
2	two	7	seven
3	three	8	eight
4	four	9	nine
5	five	0	naught, zero, or cipher

The first nine of the above figures are sometimes called *significant* figures, because each represents a definite number of units when standing alone. They are also called *digits* (Latin *digitus,* a finger), from an ancient custom of using the fingers in counting.

11. The character which is called *naught, zero,* or *cipher,* has no value in itself. It is employed to show the *absence* of significant figures, and to give *place value* to them.

12. The first *nine* numbers are developed as follows:

One	unit	is represented by	1, one
One	unit and one more	are represented by	2, two
Two	units and one more	are represented by	3, three
Three	units and one more	are represented by	4, four
Four	units and one more	are represented by	5, five
Five	units and one more	are represented by	6, six
Six	units and one more	are represented by	7, seven
Seven	units and one more	are represented by	8, eight
Eight	units and one more	are represented by	9, nine

13. In order to represent more than nine units, two or more figures are combined.

14. When a figure stands alone, neither to the right nor to the left of another figure, it is said to occupy the units' place, or to represent units of the *first* order. A figure placed to the left of units represents units of the *second* order, or tens. A figure placed to the left of tens represents units of the *third* order, or hundreds.

15. In column 1, below, the first nine figures represent units of the *first* order; in 2, units of the *second* order, or tens; and in 3, units of the *third* order, or hundreds.

(1)		(2)		(3)		
1 (unit)		10, ten	(units)	100, one	hundred	(units)
2 (units)		20, twenty	"	200, two	"	"
3	"	30, thirty	"	300, three	"	"
4	"	40, forty	"	400, four	"	"
5	"	50, fifty	"	500, five	"	"
6	"	60, sixty	"	600, six	"	"
7	"	70, seventy	"	700, seven	"	"
8	"	80, eighty	"	800, eight	"	"
9	"	90, ninety	"	900, nine	"	"

16. The value of a figure is increased tenfold every time it is moved one place to the left, and decreased tenfold every time it is moved one place to the right.

Thus, the relative values of 1, 2, 3, 4, 5, 6, 7, 8, and 9, are ten times as great in column 2, Art. 15, as in column 1; and in column 3 they are ten times as great as in column 2, and one hundred times as great as in column 1.

Hence,

One unit			1
Ten units	make	1 ten,	10
Ten tens	"	1 hundred,	100
Ten hundreds	"	1 thousand,	1000
Ten thousands	"	1 ten thousand,	10000
Ten ten thousands	"	1 hundred thousand,	100000
Ten hundred thousands	"	1 million,	1000000

17. Numbers from 9 to 30.

9 and 1 make 10, ten

10	and	1	make	11, eleven	20	and	1	make	21, twenty-one
11	"	1	"	12, twelve	21	"	1	"	22, twenty-two
12	"	1	"	13, thirteen	22	"	1	"	23, twenty-three
13	"	1	"	14, fourteen	23	"	1	"	24, twenty-four
14	"	1	"	15, fifteen	24	"	1	"	25, twenty-five
15	"	1	"	16, sixteen	25	"	1	"	26, twenty-six
16	"	1	"	17, seventeen	26	"	1	"	27, twenty-seven
17	"	1	"	18, eighteen	27	"	1	"	28, twenty-eight
18	"	1	"	19, nineteen	28	"	1	"	29, twenty-nine
19	"	1	"	20, twenty	29	"	1	"	30, thirty

18. For convenience in reading, when numbers are expressed by more than three figures, they are sometimes separated by commas into periods, or groups, of three figures each, beginning at units and counting toward the left, as shown in the following table:

NUMERATION TABLE (FRENCH)

HUNDRED-QUADRILLIONS	TEN-QUADRILLIONS	QUADRILLIONS	HUNDRED-TRILLIONS	TEN-TRILLIONS	TRILLIONS	HUNDRED-BILLIONS	TEN-BILLIONS	BILLIONS	HUNDRED-MILLIONS	TEN-MILLIONS	MILLIONS	HUNDRED-THOUSANDS	TEN-THOUSANDS	THOUSANDS	HUNDREDS	TENS	UNITS
	6	9,	0	0	9,	7	0	0,	9	0	8,	6	1	0,	4	9	8
	6. QUADRILLIONS			5. TRILLIONS			4. BILLIONS			3. MILLIONS			2. THOUSANDS			1. UNITS	

19. Three figures constitute each period, except the left-hand period, which may be composed of one, two, or three figures.

20. Units, Thousands, Millions, Billions, Trillions, etc., are the names of the periods.

21. *One* of each period, as 1 unit, 1 thousand, 1 million, 1 billion, 1 trillion, etc., is called the unit of the period.

22. To read numbers begin at the left-hand period and read, to the right, each period separately, in the order of succession, giving the name or unit of each period except the last.

23. The number in the preceding table is read: 69 quadrillion, 9 trillion, 700 billion, 908 million, 610 thousand, 498 (units).

24. Periods and parts of periods not occupied by significant figures should be filled with ciphers.

Note. — A thorough knowledge of the names of the first four or five periods in their regular order from left to right and from right to left, is indispensable to pupils, in writing and reading numbers.

25. A few of the periods following *Quadrillions* are, *Quintillions, Sextillions, Septillions, Octillions, Nonillions,* and *Decillions.*

EXERCISES IN READING NUMBERS

26. 1. Point off and read 8406721.

Model. — By pointing off the numerical expression into periods of three figures, we find it to contain *three periods*, thus, 8,406,721. Therefore the number is 8 million, 406 thousand, 721.

Point off and read the following numerical expressions:

2.	4837	11.	16129438
3.	64832	12.	40972963
4.	71050	13.	243902684
5.	923846	14.	693284728
6.	903862	15.	800921639
7.	1428396	16.	1243840932
8.	1004043	17.	7263389601
9.	1490392	18.	9007238246
10.	2960004	19.	73080893682

27. Express by figures:

1. Eighty-four thousand seven hundred fifty-four.
2. Six hundred seven thousand five hundred one.
3. Two million four thousand eighty-nine.
4. Ninety-six million seventy-eight thousand six.
5. Seventeen billion seventeen thousand seventeen.
6. Twenty-seven trillion twenty-seven million twenty-seven.
7. Seven hundred forty-five trillion twenty-nine billion three hundred thousand sixty-nine.

28. Read the following:

1.	8612489	10.	1187206748
2.	40297601	11.	10108117010
3.	101009017	12.	405127006909
4.	700056839	13.	2456297410200
5.	927000002	14.	16123006972097
6.	909701100	15.	606505444333222
7.	100100001	16.	810720387796100
8.	999809009	17.	884369000007002
9.	900009019	18.	900000700605001

29. Write in words:

1.	47603	6.	1483697483
2.	743102	7.	18000104020
3.	4600007	8.	109362000600
4.	64027182	9.	1000796384761
5.	783202100	10.	72001100000483

30. Write:

1.	3894601	6.	8809643729
2.	6009031	7.	10140127683
3.	70101001	8.	80000001001
4.	96300700	9.	146308401800
5.	804729384	10.	800008800080

Note.—A short drill in notation and numeration should precede the regular exercise in arithmetic daily, till the pupils become proficient in these operations.

31. There are two methods of numeration in use, known as the *French Method* and the *English Method.*

According to the French Method each period consists of *three* figures; according to the English Method each period consists of *six* figures. The French Method is the one used in the United States. (See Art. 18.)

NUMERATION TABLE (ENGLISH)

HUNDREDS OF THOUSANDS OF TRILLIONS	TENS OF THOUSANDS OF TRILLIONS	THOUSANDS OF TRILLIONS	HUNDREDS OF TRILLIONS	TENS OF TRILLIONS	TRILLIONS	HUNDREDS OF THOUSANDS OF BILLIONS	TENS OF THOUSANDS OF BILLIONS	THOUSANDS OF BILLIONS	HUNDREDS OF BILLIONS	TENS OF BILLIONS	BILLIONS	HUNDREDS OF THOUSANDS OF MILLIONS	TENS OF THOUSANDS OF MILLIONS	THOUSANDS OF MILLIONS	HUNDREDS OF MILLIONS	TENS OF MILLIONS	MILLIONS	HUNDREDS OF THOUSANDS	TENS OF THOUSANDS	THOUSANDS	HUNDREDS	TENS	UNITS
1	8	6	4	2	7,	2	4	8	0	0	9,	7	0	0	9	0	8,	6	1	0	4	9	8
			4. TRILLIONS						3. BILLIONS						2. MILLIONS						1. UNITS		

32. The number in the preceding table is read: One hundred eighty-six thousand four hundred twenty-seven trillion, two hundred forty-eight thousand nine billion, seven hundred thousand nine hundred eight million, six hundred ten thousand four hundred ninety-eight (units).

ROMAN NOTATION

33. According to the Roman Notation seven letters of the Roman alphabet are employed to represent numbers.

I	V	X	L	C	D	M,	Letters employed
1	5	10	50	100	500	1000,	Values

34. Other numbers are represented by repeating or combining these letters according to certain principles.

PRINCIPLES

1. *Every time a letter is repeated, the number represented by it is repeated.*

Thus, I represents 1; II represents 2; III represents 3; X represents 10; XX represents 20; etc.

2. *By writing a letter before one of greater value, the difference of their values is represented.*

Thus, IV represents 4; IX, 9; XL, 40.

3. *By writing a letter after one of greater value, the sum of their values is represented.*

Thus, VI represents 6; XV, 15.

4. *When a letter is written between two letters of greater value, it is combined with the one following it, according to principle 2.*

Thus, XIV represents 14; CIX, 109.

5. *The value of an expression is increased a thousand fold by placing a dash over it.*

Thus, $\overline{\text{V}}$ represents 5000; $\overline{\text{XV}}$, 15000; $\overline{\text{D}}$, 500000.

35. The above principles are further illustrated in the following table:

TABLE

ROMAN NOTATION

I	1	XI	11	C	100
II	2	XIII	13	CC	200
III	3	XIV	14	D	500
IV	4	XIX	19	DCC	700
V	5	XXI	21	M	1000
VI	6	XXIII	23	MDC	1600
VII	7	XXIX	29	MDCCCXCIII	1893
VIII	8	XL	40	$\overline{\text{IX}}$	9000
IX	9	XLIX	49	$\overline{\text{XIX}}$	19000
X	10	XCIX	99	$\overline{\text{XL}}$	40000

NOTE.—This method of notation takes its name from the people who invented and used it, the ancient Romans. It is chiefly used now to number volumes and divisions of books, dial faces, etc.

36. Write by the Roman Method:

1. 77
2. 92
3. 104
4. 749
5. 1642
6. 5475
7. 8063
8. 9005
9. 10946
10. 18407
11. 85001
12. 20031

37. Read, or express by figures:

1. CX
2. CCIX
3. CCCXC
4. DXXIX
5. DCCXLIX
6. MC
7. MDXIV
8. MDCCCLXXVI
9. MDCCCXCI
10. MMXLIX
11. $\overline{\mathrm{XI}}$
12. $\overline{\mathrm{XL}}$
13. $\overline{\mathrm{CC}}$
14. $\overline{\mathrm{M}}$
15. $\overline{\mathrm{DC}}$

38. Read and express by Arabic notation the numbers represented by Roman letters:

1. Columbus discovered Cuba in MCCCCXCII.
2. Ponce de Leon discovered Florida in MDXII.
3. William Penn arrived in Pennsylvania in MDCLXXXII.
4. The population of the United States in MDCCC was 5305482.
5. The National Constitution was adopted in the year MDCCLXXXVII.
6. Abraham Lincoln was assassinated in MDCCCLXV.
7. Philadelphia was made the "seat of government" in MDCCXC.
8. The Declaration of Independence was adopted in MDCCLXXVI.
9. The Stamp Act was passed in MDCCLXV.
10. Washington was inaugurated first President of the United States in MDCCLXXXIX.

UNITED STATES MONEY

39. United States Money is the legal currency of the United States. It is sometimes called *Federal Money.*

40. The unit of value is the dollar.

The sign for dollars is $. It is commonly known as the "dollar mark." Hence two dollars, three dollars, four dollars, etc., in business are generally written $ 2, $ 3, $ 4.

41. For convenience in business, half-dollars, quarter-dollars, dimes, nickels, and cents are coined.

42. Dollars are written and read like simple whole numbers. Thus, $ 3,849,674 is read 3 million 849 thousand 674 dollars.

43. Table of United States Money

10 mills (m.)	= 1 cent (¢)
10 cents	= 1 dime (d.)
10 dimes	= 1 dollar ($)
10 dollars	= 1 eagle (E.)

Note. — The *mill* is not coined.

44. In writing United States money, dollars are separated from cents by a small dot (.), called the decimal point. All figures to the left of the decimal point denote dollars; all figures to the right, denote parts of the dollar. The first two places to the right of the decimal point represent cents; the third, mills; and the fourth, tenths of a mill.

Thus, $ 29.2453 is read, twenty-nine dollars, twenty-four cents, five mills, and three tenths of a mill.

45. The beginner will be aided very much in writing United States money by studying carefully the following:

One cent	$ .01	Seventy-five cents	$ .75
Nine cents . . .	$ .09	One dollar and ten cents . . .	$ 1.10
Ten cents . . .	$ .10	Five dollars and seventy-five cents	$ 5.75
Eleven cents . .	$ .11	Ten dollars and fifty cents . .	$10.50
Nineteen cents . .	$ .19	Thirty-seven cents and five mills	$ .375
Twenty cents . .	$ .20	A quarter dollar	$ .25
Fifty cents . . .	$ .50	A half dollar	$ .50

46. Read the following:

1. $8.10	6. $49.06	11. $405.906
2. $10.50	7. $50.50	12. $3847.62
3. $14.57	8. $72.04	13. $80796.314
4. $20.73	9. $96.043	14. $789600.032
5. $27.27	10. $100.104	15. $7001306.302

47. Express by figures:

1. Four dollars and seventy-five cents.

2. Twenty-eight dollars and thirty-seven cents.

3. Two hundred forty-nine dollars and fourteen cents.

4. Nine hundred sixty dollars, eighty-five cents and four mills.

5. Two thousand forty-nine dollars and seventy-two cents.

6. Seventy-eight thousand four hundred dollars and ten cents.

7. One hundred fifty-three thousand four hundred sixty-three dollars and twenty-nine cents.

8. Eight hundred thirty-seven thousand nine hundred twenty-four dollars, thirty-nine cents and seven mills.

ADDITION

DEFINITIONS AND PRINCIPLES

48. *Addition* is an operation by which two or more numbers are combined into one number, called their *Sum* or *Amount*.

49. In the expression "6 and 3 are 9," instead of the words "and" and "are," we use the two signs "+" and "=". The first is the sign of addition, and is read *plus*. When it is placed between two numbers, it indicates that their sum is to be found. The second is the sign of equality, and is read *equals*. When it is placed between two numbers, it denotes that they are equal in value.

The expression 6 and 3 are 9, when expressed mathematically becomes $6 + 3 = 9$, and is read 6 plus 3 equal 9.

Principles

50. 1. *Only similar numbers can be added. Units must be added to units, tens to tens, dollars to dollars, pounds to pounds, etc.*

2. *The sum is always of the same kind of unit as the numbers added.*

51. Such mathematical expressions as $6 + 4 = 10$, and $9 + 5 = 14$, are called *Equations*. The part of the equation to the left of the sign "=" is the *first* member of the equation; the part to the right the *second* member. Thus, in the equation $8 + 5 = 13$, the first member is $8 + 5$, and the second 13.

52. 1. Find the sum of 384, 928, 452.

Beginning at the right and adding, we find the first column to contain 14 units. Since 10 units make 1 ten, 14 units equal 1 ten and 4 units. Writing the 4 units under the column of units, and carrying the 1 ten to the column of tens and adding, we get 16 tens. Since 10 tens make 1 hundred, 16 tens equal 1 hundred and 6 tens. Writing the 6 tens under the column of tens, and carrying the 1 hundred to the column of hundreds and adding, we get 17 hundreds. Since 10 hundreds make 1 thousand, 17 hundreds equal 1 thousand and 7 hundreds. We write the 7 hundreds under the column of hundreds and the 1 thousand in the thousands' place. Hence the sum of 384, 928, 452 is 1 thousand, 7 hundreds, 6 tens, and 4 units, or 1764.

OPERATION

$$\begin{array}{r} 384 \\ 928 \\ 452 \\ \hline 1764 \end{array}$$

2. Find the sum of 638, 479, 608, 359, 867.

3. Find the sum of 9683, 9842, 7063, 5460, 7293.

4. Find the sum of 80,609, 47,630, 79,206, 47,063.

53. 1. What is the sum of \$386.40, \$960.213, \$84.635, \$806.31, \$9386.40, and \$46?

We write units of like order in the same column, separating dollars from cents by the decimal point (.), and add as in simple numbers.

OPERATION

$$\begin{array}{r} \$386.40 \\ 960.213 \\ 84.635 \\ 806.31 \\ 9386.40 \\ 46.00 \\ \hline \$11669.958 \end{array}$$

2. Find the sum of \$9672.32, \$8064.50, \$9604.315, and \$10906.327.

3. Find the sum of \$4063.34, \$3827.415, \$90001.84, and \$306.74.

4. What is the sum of \$86043.72, \$42.361, \$96.84, \$200.021, and \$9380.47?

PROBLEMS

54. 1. In January, 1897, there were 31 days, in February 28, in March 31, in April 30, in May 31, in June 30, in July 31, in August 31, in September 30, in October 31, in November 30, and in December 31. How many days were there in 1897?

2. If a thermometer registered 12 degrees below zero in March and 96 degrees above zero in July, what was the difference of these temperatures?

3. A man owns 5 tracts of land. The first contains 1824 acres, the second 864 acres, the third 972 acres, the fourth 536 acres, and the fifth 1654 acres. How many acres does he own?

4. McKinley received 603,514 more votes for President in 1896 than Bryan. If Bryan received 6,502,685 votes, how many did McKinley receive?

5. A farmer sold 1200 bushels of wheat for $1195, 1640 bushels of corn for $1192, and 2060 bushels of oats for $972. How many bushels of grain did he sell, and how much did he receive for all?

6. Commodore Dewey's fleet, which gained the victory at Manila, consisted of the Olympia, 5800 tons; Baltimore, 4600 tons; Boston, 3189 tons; Raleigh, 3182 tons; Concord, 1700 tons; Petrel, 890 tons. What was the total tonnage of the fleet?

7. It is estimated that 111,100,000 people in the world speak English, 51,200,000 French, 75,200,000 German, 33,400,000 Italian, 42,800,000 Spanish, 13,000,000 Portuguese, and 75,000,000 Russian. How many people all together speak these languages?

8. A society for the relief of sick babies issued 25,500 tickets for fresh-air excursions in 1895, 30,150 in 1896, 44,978 in 1897, and 50,000 in 1898. How many babies were helped in the four years?

SUBTRACTION

DEFINITIONS AND PRINCIPLES

55. *Subtraction* is an operation by which the difference between two numbers is found.

56. The terms used in subtraction are Minuend, Subtrahend, and Difference or Remainder.

57. The *Minuend* is the number from which another number is taken, or subtracted.

58. The *Subtrahend* is the number subtracted, or taken from, the minuend.

59. The *Difference* or *Remainder* is the number left after the subtrahend is taken from the minuend. Thus, in the expression "9 less 4 equals 5," the *minuend* is 9, the *subtrahend* 4, and the *difference* 5.

60. The sign of subtraction is a short horizontal line, –, called *minus*. When it is placed between two numbers it indicates that their difference is to be found. Thus, the expression "9 – 5" signifies that 5 is to be taken from 9, and is read, 9 minus 5.

61. In arithmetic the minuend must always equal or exceed the subtrahend.

PRINCIPLES

62. 1. *Only like numbers can be subtracted.*

Thus, units must be taken from units, tens from tens, hundreds from hundreds, dollars from dollars, etc.

2. *The difference is of the same kind of unit as the minuend and subtrahend.*

3. *The difference of two numbers plus the smaller number equals the larger number.*

4. *If equal numbers are added to both minuend and subtrahend, or equal numbers are subtracted from both, the difference will remain unchanged.*

63. The *Parenthesis*, ( ), is used to show that all the numbers inclosed by it are to be considered together, as one number, thus, $10 - (3 + 4)$ means that the sum of 3 and 4 is to be subtracted from 10. The *Vinculum*, $\overline{\quad}$, is often used instead of the parenthesis. Thus, $\overline{8 + 9} - (3 + 6)$ means that from the sum of 8 and 9, the sum of 3 and 6 is to be subtracted.

THE BORROWING METHOD

64. 1. From 735 take 456.

OPERATION

735	Minuend
456	Subtrahend
279	Difference

We write the subtrahend under the minuend, and begin at units to subtract. Since 6 units cannot be taken from 5 units, we borrow, or take 1 ten, 10 units, from the 3 tens and add it to the 5 units, thus making 15 units. We take the 6 units from the 15 units, and write the difference, or 9 units, under the column of units. As 1 ten has been taken from the 3 tens, only 2 tens remain. Since 5 tens cannot be taken from 2 tens, we borrow, or take 1 hundred, 10 tens, from the 7 hundreds and add it to the 2 tens, thus making 12 tens. We take the 5 tens from the 12 tens and write the difference, or 7 tens, under the column of tens. As 1 hundred has been taken from 7 hundreds, only 6 hundreds remain. Subtracting the 4 hundreds from 6 hundreds, we get 2 hundreds, which we write under the column of hundreds. Therefore the difference between 735 and 456 is 279.

The Carrying Method

65. 1. From 735 take 456.

OPERATION

735	Minuend
456	Subtrahend
279	Difference

Since 6 units cannot be taken from 5 units, we add 10 units to the 5 units, thus making 15 units. Subtracting 6 units from 15 units, we have 9 units remaining. Since 10 units have been added to the minuend, the difference will be 10 units too large. This is corrected by adding the same amount, 1 ten, to the subtrahend (Art. 62, 4), thus making in the subtrahend 6 tens. Since 6 tens cannot be taken from 3 tens, we add 10 tens to the 3 tens, thus making 13 tens. Subtracting 6 tens from 13 tens, we have 7 tens remaining. Since 10 tens have been added to the minuend, an equal amount, 1 hundred, is added to the subtrahend, thus making 5 hundreds. Subtracting 5 hundreds from 7 hundreds, we have 2 hundreds remaining.

Proofs

66. 1. The subtrahend, plus the difference, equals the minuend.

2. The minuend, minus the difference, equals the subtrahend.

	1.	2.	3.	4.	5.
67. Subtract:	4521	9785	80907	$930.616	$600.01
	1376	6598	7859	46.307	163.22

6. From 30,021 rd. take 27,500 rd.

7. From 86,020 bu. take 24,031 bu.

8. $(\$700.07 - \$275.24) + \overline{\$6628 - \$645} = ?$

9. From 13 million seven hundred eleven thousand three hundred forty-nine, subtract 4 million 649 thousand three hundred sixty-nine.

PROBLEMS

68. 1. The difference is 16,904, and the subtrahend is 56,726. What is the minuend?

2. The subtrahend is 5390, which is 412 less than the difference. Find the minuend.

3. A farmer raised 932 bushels of wheat and 1600 bushels of corn. He sold 726 bushels of wheat and 742 bushels of corn. How many bushels of each had he left?

4. The land surface of California is 155,980 square miles; of Maine, 29,895 square miles; of New York, 47,620 square miles; of New Hampshire, 9005 square miles; of Vermont, 9135 square miles; of Massachusetts, 8040 square miles; of Rhode Island, 1085 square miles; of Connecticut, 4845 square miles; of Pennsylvania, 44,985 square miles. How much greater is the land surface of California than that of all the others combined?

5. The imports from the Philippine Islands to the United States in 1896 were valued at $ 4,982,857 and those of 1897 at $ 4,383,740. By how much did the 1896 imports exceed those of 1897?

6. In 1890 there were 254 cotton mills in the South, with 1,712,000 spindles and 39,000 looms; in 1898 there were 490 mills, 4,100,000 spindles, and 115,000 looms. What was the increase in mills, in spindles, and in looms during the eight years?

7. By how much did the tonnage of the American fleet at Manila (see § 54, Ex. 6) exceed that of the Spanish fleet, which consisted of the Reina Cristina, 3520 tons; Castilla, 3342 tons; Velasco, 1152 tons; Don Antonia de Ulloa, 1130 tons; Don Juan de Austria, 1130 tons, and several smaller vessels of 2054 tons in all?

8. The polar diameter of the earth is 41,707,620 feet, the equatorial diameter 41,847,426 feet. What is their difference?

9. The following are the areas in square miles of the Great Lakes: Lake Superior, 31,200; Lake Huron, 23,800; Lake Michigan, 22,450; Lake Erie, 9960; Lake Ontario, 7240. By how much do the first two exceed the last three?

MULTIPLICATION

DEFINITIONS AND PRINCIPLES

69. *Multiplication* is a short operation of finding the sum of as many expressions of one number as there are units in another number.

OPERATION

If we wish to find the sum of five 8's by addition we express 8 five times, as in the operation at the right, and add the column, finding the sum 40.

But in multiplication, such facts as 5 times 8 are 40 are committed to memory, by means of a *multiplication table*, and employed in finding the result with larger numbers.

$$\begin{array}{r} 8 \\ 8 \\ 8 \\ 8 \\ 8 \\ \hline 40 \end{array}$$

70. The terms used in multiplication are Multiplicand, Multiplier, and Product.

In the expression 5 times 8 = 40, or $\begin{array}{r} 8 \\ 5 \\ \hline 40 \end{array}$, 8 is the *multiplicand*, 5 is the *multiplier*, and 40 the *product*.

It will readily be seen that 40 is the result obtained by taking the multiplicand, 8, as many times as there are units in the multiplier 5; that is, it is the result obtained by taking "8" *five* times.

71. The *Multiplicand* is the number which is to be taken a certain number of times.

72. The *Multiplier* is the number which shows how many times the multiplicand is to be taken.

73. The *Product* is the result obtained by taking the multiplicand as many times as there are units in the multiplier.

74. The sign of multiplication is an inclined cross, $\times$, and is read *times* or *multiplied by*. Thus, 3×8 is read 3 *times* 8, or 3 multiplied by 8.

75. The multiplicand and multiplier are factors of the product. Thus, 3 and 5 are factors of their product, 15.

PRINCIPLES

76. 1. *The product is always of the same kind of unit as the multiplicand.*

If the multiplicand is dollars, the product is dollars; if it is pencils, the product is pencils; if it is apples, the product is apples, etc.

2. *The multiplier is always an abstract number.*

Although the multiplier may be a concrete number in the problem, yet it must be considered as abstract in the operation, because it simply denotes the number of times the multiplicand is to be taken. In the problem, "Find the cost of 5 oranges at 3 cents apiece," 5 oranges is a concrete number, but in the operation it becomes abstract. Thus, at 3 cents apiece, 5 oranges will cost 5 times 3 cents (5×3 cents), or 15 cents.

3. *The product of abstract numbers will be the same in whatever order the factors are written.*

Thus, 7 times 9 = 9 times 7, or $7 \times 9 = 9 \times 7$.

NOTE.—The number unlike the required product is always the multiplier, whether larger or smaller than the other, and should be so designated in analyzing problems; but in practice, the smaller number, even though concrete, may be used as multiplier to shorten the operation.

EXAMPLE.—What is the cost of 13 pounds of sugar at 7 cents a pound?

By examining the above problem, it will be readily seen that the product is to be *cents*. Therefore, the number unlike the product (13) must be the multiplier.

Solution. — At 7 cents a pound, 13 pounds of sugar will cost 13 times 7 cents (13 × 7 cents), or 91 cents.

In this case, as in many other cases, the larger number is the multiplier.

77. 1. Multiply 538 by 7.

To multiply 538 by 7 simply means to find the sum of 538 used 7 times, as shown in Operation I (Art. 69).

Instead of writing 538 *seven* times, we write it but *once*, with the multiplier under it, as shown in Operation II.

Beginning at the right and multiplying, 7 times 8 units are 56 units, which equal 5 tens and 6 units. The 6 units we write in the units' place, and the 5 tens we add to the product of tens. 7 times 3 tens are 21 tens, to which we add the 5 tens, thus making 26 tens, which equal 2 hundreds and 6 tens. We write the 6 tens in the tens' place, and add the 2 hundreds to the product of hundreds. 7 times 5 hundreds are 35 hundreds, to which we add the 2 hundreds, thus making 37 hundreds, which equal 3 thousands and 7 hundreds. We write the 7 hundreds in the hundreds' place, and the 3 thousands in the thousands' place. Therefore we find the entire product to be 3 thousands, 7 hundreds, 6 tens, and 6 units, or 3766.

OPERATION I

$$\begin{array}{r} 538 \\ 538 \\ 538 \\ 538 \\ 538 \\ 538 \\ 538 \\ \hline 3766 \end{array}$$

OPERATION II

$$\begin{array}{r} 538 \\ 7 \\ \hline 3766 \end{array}$$

Multiply:

2. 5386 by 7	4. $84,069 by 8	6. $3840.38 by 9
3. 4069 by 8	5. $30,604 by 9	7. $6093.084 by 7

78. Multiply:

1.	2.	3.	4.	5.	6.
324	726	820	970	7860	8910
20	40	30	300	70	800
6480					

7. Multiply $846 by 800	11. Multiply $960.30 by 200
8. Multiply $7321 by 1000	12. Multiply $309.60 by 700
9. Multiply $6341 by 2000	13. Multiply $563.40 by 500
10. Multiply $9451 by 4000	14. Multiply $741.89 by 5000

79. 1. Multiply 578 by 748.

To multiply 578 by 748 means that 578 is to be taken 8 times + 40 times + 700 times.

OPERATION I

$$\begin{aligned} 8 \times 578 &= 4624 = \text{1st partial product} \\ 40 \times 578 &= 23120 = \text{2d} \quad " \quad " \\ 700 \times 578 &= 404600 = \text{3d} \quad " \quad " \\ \hline 748 \times 578 &= 432344 = \text{Entire product} \end{aligned}$$

In practice, however, the numbers are usually arranged, and the multiplication performed as shown in the following operation:

OPERATION II

```
  578 Multiplicand
  748 Multiplier
 ----
 4624
2312
4046
------
432344 Product
```

It will be noticed that the ciphers at the right of the 2d and 3d partial products, in Operation I, do not appear in Operation II.

As ciphers do not affect the result in addition, they are omitted in practice.

Multiply:

2. 9684 by 316	5. 9382 by 504	8. 93,268 by 872
3. 8932 by 329	6. 7293 by 729	9. 60,606 by 909
4. 7605 by 456	7. 9081 by 860	10. 800,608 by 989

80. 1. What is the product of 89,638 pounds by 89?

2. What is the product of 928,443 tons by 802?

3. What is the product of $ 96,384.62 by 819?

4. What is the product of $ 799,980 by 879?

5. What is the product of 869,384 bushels by 7063?

PROBLEMS

81. 1. The multiplicand is 563,920 and the multiplier is 3752. Find the product.

2. How much will 32 horses cost at $195 apiece?

3. Sixty seconds make a minute, and 60 minutes make an hour. How many seconds are there in 48 hours?

4. If it is 5698 miles from San Francisco to Jeddo, in Japan, how many miles would a ship sail in making 28 complete trips?

5. If it requires 91,378 bricks to build a house, how many bricks will be required to build 68 houses of the same size?

6. Parrish & Co., at one time, bought 248 acres of coal land, at $7575 an acre; at another time, 468 acres, at $8792 an acre. Find the cost of both purchases.

7. Two railroad trains leave the same station at the same time and travel in the same direction. The first travels at the rate of 38 miles an hour, and the second at the rate of 30 miles an hour. How far will they be apart at the end of two days, or 48 hours?

8. A certain brigade consisted of 3 regiments, each regiment of 3 battalions, each battalion of 4 companies, and each company of 100 men. How many men were in the brigade?

9. A miller bought 3046 bushels of wheat, 2 times as much corn, and 3 times as much buckwheat. He paid 80 cents a bushel for the wheat, 48 cents for the corn, and 52 cents for the buckwheat. How much did he pay for all?

10. If each car load of coal weighs 42,360 pounds, what will be the entire weight of 64 car loads?

DIVISION

DEFINITIONS AND PRINCIPLES

82. *Division* is the operation of finding how many times one number is contained in another, or of dividing a number into equal parts.

83. The terms used in division are Divisor, Dividend, and Quotient. Thus, in the expression $12 \div 3 = 4$, the dividend is 12, the divisor 3, and the quotient 4.

84. The *Dividend* is the number to be divided.

85. The *Divisor* is the number by which we divide. It is the standard by which the dividend is to be measured, or it shows into how many equal parts the dividend is to be divided.

86. The *Quotient* shows how many times the dividend contains the divisor, or the value of one of the equal parts into which the dividend has been divided. Thus, in the expression $\$12 \div \$3 = 4$, the quotient shows how many times the dividend contains the divisor; and in $\$12 \div 3 = \$4$, the quotient shows the value of *one* of the three equal parts of the dividend.

87. The sign of division is $\div$, and is read *divided by.* It indicates that the number placed before it is to be divided by the one placed after it. The division of one number by another is indicated in *three* ways. Thus,

$$18 \div 6; \qquad \frac{18}{6}; \qquad 6\overline{)18}.$$

88. When the dividend does not contain the divisor an exact number of times, there remains an undivided part of the dividend called the remainder. Thus, 8 is contained in 17 two times, and a remainder of 1.

PRINCIPLES

89. 1. *When the dividend and divisor are similar numbers, the quotient is an abstract number.*

2. *When the dividend and divisor are dissimilar concrete numbers, in the operation the divisor is abstract, and the quotient concrete.*

3. *The remainder is similar to the dividend.*

When there is no remainder,

4. *The Dividend = Divisor × Quotient.*

5. *The Divisor = Dividend ÷ Quotient.*

When there is a remainder,

6. *The Dividend = (Divisor × Quotient) + Remainder.*

7. *The Divisor = (Dividend − Remainder) ÷ Quotient.*

8. *Multiplying the dividend or dividing the divisor multiplies the value of the quotient.*

9. *Dividing the dividend or multiplying the divisor divides the value of the quotient.*

10. *Multiplying or dividing both dividend and divisor by the same number does not change the value of the quotient.*

NOTE.—In multiplication, *two* factors are given to find the product. In division but *one* factor (divisor) and the product (dividend) are given to find the other factor (quotient). Hence, division might be considered as arising from multiplication. It is claimed by many authors that division is an outgrowth of subtraction. Subtraction may be regarded as the reverse operation of addition; division as the reverse operation of multiplication; multiplication as a concise method of addition; and division as a concise method of subtraction.

90. 1. Divide 31,260 by 4.

OPERATION

$$4\overline{)31260}$$
$$7815 \quad \textit{Ans.}$$

Since 4 is not contained in 3 ten-thousands any ten thousand times, we reduce 3 ten-thousands to the next lower order. 3 ten-thousands = 30 thousands, which, added to the 1 thousand, make 31 thousands. 4 is contained in 31 thousands 7 thousand times, with 3 thousands remainder. 3 thousands = 30 hundreds, which, added to the 2 hundreds, make 32 hundreds. 4 is contained in 32 hundreds 8 hundred times. 4 is contained in 6 tens 1 ten times, with 2 tens remainder. 2 tens = 20 units. 4 is contained in 20 units 5 units times.

Divide:

2.	3.	4.	5.
7)356288 50898 2 Rem.	6)480387	8)725042	9)8643287
6.	**7.**	**8.**	**9.**
5)965384	4)687538	5)964802	9)379891

91. When the dividend and divisor are similar numbers, the quotient is an abstract number (Art. 89, 1).

1. \$ 6)\$ 356288
   59381, and \$ 2 Rem.

2. 7 pounds)48087 pounds
   6869, and 4 pounds Rem.

3. 9 cents)864327 cents

4. 7 quarts)725042 quarts

92. When the dividend and divisor are dissimilar concrete numbers, in the operation the divisor is abstract and the dividend concrete. (Art. 89, 2.)

1. If 9 mowing machines cost \$3375, how much will 1 cost?

OPERATION

9)\$ 3375
\$ 375 *Ans.*

1 mowing machine will cost $\frac{1}{9}$ of \$3375, or \$3375 ÷ 9 = \$375.

2.	3.	4.
6)9354 seconds	7)8176 minutes	8)9237 hours
seconds	minutes	hours
5.	6.	7.
9)5682 days	7)4239 weeks	5)8369 months

PROBLEMS

93. 1. There are 7 days in a week. How many weeks are there in 728 days?

2. There are 8 quarts in a peck. How many pecks are there in 640 quarts?

3. There are 9 square feet in a square yard. How many square yards are there in 1224 square feet?

4. If 8 horses eat 216 pecks of oats in a certain time, how many quarts will 1 horse eat in the same time?

5. If 5 barrels of flour cost $21.25, how much will 1 barrel cost?

94. Dollars may be changed to cents by omitting the symbol "$," adding two ciphers, and writing the word *cents* at the right. Thus, $625 = 62500 cents.

1. At $.07 a pound, how many pounds of sugar can be bought for $35?

$35 = 3500 cents
$.07 = 7 cents

OPERATION

7 cents)3500 cents
500

If 1 pound costs 7 cents, for 3500 cents there can be bought as many pounds as 7 cents are contained times in 3500 cents, or 500. That is, 500 pounds of sugar can be bought.

95. Dollars and cents may be changed to cents by omitting the symbol "$" and the decimal point (.). Thus, $437.26 = 43726 cents.

1. At \$.09 a quart, how many quarts of beans can be bought for \$120.78?

\$120.78 = 12078 cents

\$.09 = 9 cents

OPERATION

$$9 \text{ cents})\underline{12078 \text{ cents}}$$
$$1342$$

If 1 quart costs 9 cents, for 12078 cents there can be bought as many quarts as 9 cents are contained times in 12078 cents, or 1342. That is, 1342 quarts of beans can be bought.

PROBLEMS

96. 1. There are 2 pints in 1 quart. How many quarts are there in 3768 pints?

Since there are 2 pints in one quart, in 3768 pints there are as many quarts as 2 pints are contained times in 3768 pints, or 1884. Therefore in 3768 pints there are 1884 quarts.

OPERATION

$$2 \text{ pints})\underline{3768} \text{ pints}$$
$$1884$$

2. When flour is selling at \$7 a barrel, how many barrels can be bought for \$3255?

3. A clerk receives \$18 a week. If his expenses for board, washing, etc., are \$10 a week, how many weeks will it take him to save \$6448?

4. There are 5280 feet in 1 mile. How many revolutions will a carriage wheel 12 feet in circumference make in going 13 miles?

5. There are 3 feet in 1 yard, and 5280 feet in 1 mile. How many yards are there in 5 miles?

6. In 1 square yard there are 9 square feet. How much will it cost, at 17 cents a square yard, to plaster a room containing 1665 square feet?

7. How many five-cent pieces are equal to 8965 cents?

8. How many quarter-dollars are equal to 1780 five-cent pieces?

9. How many dollars are equal to 3564 quarter-dollars?

PROBLEMS

97. 1. There are 864 square inches in 6 square feet. How many square inches are there in 1 square foot?

Since there are 864 square inches in 6 square feet, 1 square foot contains $\frac{1}{6}$ of 864 square inches, or 144 square inches.

(To find $\frac{1}{6}$ of any number, practically means to divide it by 6.)

OPERATION

$6\overline{)864}$ square inches
144 square inches

2. An express train ran 522 miles in 9 hours. At the same rate, how far would it run in 27 hours?

3. If a family uses 2 pounds of coffee in 14 days, how many pounds would last the same family 42 days?

4. How many times will a wheel 6 feet in circumference go around in passing over 2 miles, or 10,560 feet?

5. If it costs $36,620 to build 5 miles of railroad, how much will it cost to build 17 miles, at the same price per mile?

6. How many quart bottles will be required to hold a barrel of cider, or $31\frac{1}{2}$ gallons?

98. 1. Divide 4628 by 10.

By canceling the right-hand figure in both dividend and divisor, we reduce the dividend from 4628 units to 462 units; and the divisor from 10 units to 1 unit. 1 unit is contained in 462 units 462 times. The canceled figure 8, being an undivided part of the dividend, is the remainder, and is usually expressed in the form of a fraction. Hence, the quotient of 4628 divided by 10 is $462\frac{8}{10}$.

OPERATION

$1\cancel{0}\overline{)462\cancel{8}}$
$462\frac{8}{10}$

2. Divide 4538 by 20.

By canceling the right-hand figure in both dividend and divisor, the dividend is reduced in value from 4538 units to 453 units; and the divisor from 20 units to 2 units. Dividing 453 units by 2 units we get 226, and a remainder of 1, which, prefixed to 8, makes 18, the entire remainder. Hence the quotient of 4538 divided by 20 is $226\frac{18}{20}$.

OPERATION

$2\cancel{0}\overline{)453\cancel{8}}$
$226\frac{18}{20}$

3. $43,869 \div 100 = 1\cancel{0}\cancel{0})438\cancel{6}\cancel{9}$
$438\frac{69}{100}$

4. $43,869 \div 1000 = 1\cancel{0}\cancel{0}\cancel{0})43\cancel{8}\cancel{6}\cancel{9}$
$43\frac{869}{1000}$

5. $43,869 \div 200 = 2\cancel{0}\cancel{0})438\cancel{6}\cancel{9}$
$219\frac{69}{200}$

6. $43,869 \div 2000 = 2\cancel{0}\cancel{0}\cancel{0})43\cancel{8}\cancel{6}\cancel{9}$
$21\frac{1869}{2000}$

7. Divide 3869 by 10; by 20; by 30; by 40; by 80.

8. Divide 79,638 by 100; by 600; by 700; by 1000; by 7000.

9. How much will 1 barrel of flour cost if 200 barrels cost $1150?

10. If 27,000 pounds of beef were divided equally among 9000 soldiers, how many pounds did each receive?

LONG DIVISION

99. When the divisor exceeds 12, the method called *Long Division* is employed. It is so called because the entire process is expressed.

1. Divide 5645 by 27.

OPERATION

Divisor Dividend Quotient

$$
\begin{array}{r}
27)5645(209\frac{2}{27} \\
\underline{54\phantom{45(209\frac{2}{27}}} \\
245\phantom{(209\frac{2}{27}} \\
\underline{243\phantom{(209\frac{2}{27}}} \\
2 \text{ Rem.}\phantom{9\frac{2}{27}}
\end{array}
$$

We arrange the dividend, divisor, and quotient as in the operation in the margin.

The divisor 27 is not contained any thousand times in 5 thousands; hence the quotient will contain no thousands; 5 thousands equal 50 hundreds, which, added to 6 hundreds, make 56 hundreds. 27 is contained in 56 hundreds 2 hundred times. Writing the 2 hundreds as the first figure of the quotient, and taking the product of the first quotient figure and the divisor, we have 54 hundreds, which, subtracted from 56 hundreds leave a remainder of 2 hundreds. The remainder 2 hundreds equal 20 tens, which, united with the 4 tens, make 24 tens. 27 is not contained in 24 tens, hence a cipher is written in

the quotient, and the 24 tens reduced to units and united with the 5 units, thus making 245 units. 27 is contained in 245 units 9 units times. Writing the 9 units in the quotient, and taking the product of the third quotient figure 9, and the divisor 27, we have 243 units, which, subtracted from 245 units, leave a remainder of 2 units. Therefore, the quotient of 5645 divided by 27 is 209, with a remainder of 2, or $209\frac{2}{27}$.

Divide:

2. 2358 by 23
3. 59,684 by 86
4. 168,349 by 127
5. 876,210 by 872
6. 105,056 yards by 74
7. 666,468 miles by 129
8. 31,317 bushels by 13 bushels
9. 145,890 feet by 90 feet

100. 1. Divide $ 7584 by $ .24.

OPERATION

$7584 = 758,400 cents ; $ .24 = 24 cents.
758,400 cents ÷ 24 cents = 31,600. *Ans.*

2. Divide $ 7050.85 by $ .83.

OPERATION

$7050.85 = 705,085 cents ; $ .83 = 83 cents.
705,085 cents ÷ 83 cents = 8495. *Ans.*

Divide:

3. $ 191,308 by $ .26
4. $ 227,772 by $ .27
5. $ 4543.38 by $ .86
6. $ 6021.75 by $ .93

7. There are 924,000 feet in 175 miles. Find how many feet there are in 1 mile.

8. If a nail factory turns out 87,624 kegs of nails in 313 days, how many kegs are turned out in 1 day ?

9. A cord of wood contains 128 cubic feet. How many cords are in a pile containing 41,472 cubic feet ?

10. A forest in the Adirondacks contains 2950 trees of which one in every 15 is a balsam. If the tourists cut down all the balsam trees, how many trees will be left ?

FACTORS AND MULTIPLES

PRIME AND COMPOSITE NUMBERS

101. Two times 6 make 12; therefore 2 and 6 are makers, or *Factors*, of 12. As every product is divisible by its factors, 12 is divisible by 2 and 6. Three times 4 make 12; therefore 3 and 4 are also makers, or factors, of 12, and 12 is divisible by 3 and 4.

102. A number which can be formed by multiplying together two or more integral numbers each greater than 1, or which can be divided without a remainder by one or more integral numbers besides itself and 1, is called a *Composite Number*. 12 is a composite number because it can be produced by multiplying together 2 and 6, or 3 and 4, and is divisible by all of its factors.

103. A few of the composite numbers in order are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, and 25.

104. A number which cannot be formed by multiplying together two or more integral numbers each greater than 1, or which cannot be exactly divided by any integral number except itself and 1, is called a *Prime Number*.

105. A few of the prime numbers in order are 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, and 41.

106. An *Even Number* is one which is exactly divisible by 2. Thus, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, etc., are

even numbers. All *even* numbers except "2," are composite numbers.

107. An *Odd Number* is one which cannot be exactly divided by 2. Thus, 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, etc., are *odd numbers.*

FACTORING

108. *Factoring* is the process of finding the factors, or makers, of a composite number.

109. The *Factors* of a composite number are the numbers which will produce it if multiplied together. Thus, 3 and 4, and 2 and 6, are factors of 12, because 3 times 4 equal 12, and 2 times 6 equal 12.

110. The *Prime Factors* of a composite number are the *prime numbers* which will produce it if multiplied together. The prime factors of 12 are 2, 2, and 3, because they are the prime numbers which will produce 12 if multiplied together.

111. Numbers are said to be prime to each other when they contain no factor which is common to all of them. The numbers 3, 4, 5, 7, 11, and 13 are prime to each other, because no greater number than 1 will exactly divide all of them.

112. When a composite number is formed by multiplying together two equal factors, one of these equal factors is called the *second* or *square root* of the number. Thus, 3 is the square root of 9, because it is one of the two equal factors of 9.

When a composite number is formed by multiplying together three equal factors, one of these factors is called the *third* or *cube root* of the number. In a number formed by multiplying together four equal factors, one of the factors is called the *fourth root* of the number. Thus, 4 is the third root of 64, because it is one of the three equal factors of 64; that is, $4 \times 4 \times 4 = 64$. Likewise it may be shown that 2 is the fourth root of 16.

113. A number is divisible:

1. By 2, when its right-hand term is zero, or an even digit.
2. By 3, when the sum of its digits is divisible by 3.
3. By 4, when the two right-hand terms are zeros, or when the number expressed by the last two figures is divisible by 4.
4. By 5, when its right-hand term is 0 or 5.
5. By 6, when it is an even number and is divisible by 3.
6. By 8, when its last three terms are zeros, or when the number expressed by the last three figures is divisible by 8.
7. By 9, when the sum of its digits is divisible by 9.
8. By 10, when its last digit is 0.

114. Every composite number equals the product of its prime factors.

115. Every composite number is divisible by its prime factors.

116. 1. Find the prime factors of 210.

OPERATION I	OPERATION II	OPERATION III
$\begin{array}{r\|r} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$	$\begin{array}{r\|r} 7 & 210 \\ \hline 5 & 30 \\ \hline 3 & 6 \\ \hline & 2 \end{array}$	$\begin{array}{r\|r} 5 & 210 \\ \hline 3 & 42 \\ \hline 2 & 14 \\ \hline & 7 \end{array}$

The prime factors of any composite number may be found by dividing the given number by any prime number that will exactly divide it, and the quotient by any prime number that will exactly divide it, and thus continuing to divide the resulting quotient until it becomes a prime number. The several divisors and the last quotient will be the required prime factors. The prime factors of the preceding number, 210, arranged according to their value, are 2, 3, 5, 7.

Find the prime factors of the following:

2. 40, 54, 72, 84, 92, 95, 110, 114, 132.
3. 160, 176, 210, 240, 360, 410, 476, 485.
4. 510, 576, 696, 876, 945, 1140, 1280, 8190.

117. When two or more composite numbers contain the same prime factor, that factor is said to be common to those numbers.

118. 1. Find the prime factors common to 60, 90, 210.

OPERATION

$60 = 2 \times 2 \times 3 \times 5$

$90 = 2 \times 3 \times 3 \times 5$

$210 = 2 \times 3 \times 5 \times 7$

By examining the prime factors of the numbers at the left, it will be seen that each of the numbers contains the prime factors 2, 3, and 5. Therefore 2, 3, and 5 are common to the numbers 60, 90, and 210.

Find the prime factors common to:

2. 40 and 54	6. 160 and 432
3. 72 and 84	7. 315 and 525
4. 110 and 112	8. 840 and 420
5. 140 and 176	9. 500 and 600

GREATEST COMMON DIVISOR

119. When a number will divide another without a remainder, it is called an *Exact Divisor* of that number. Thus, 9 is an exact divisor of 27.

120. When a number will divide two or more numbers without a remainder, it is called a *Common Divisor* of those numbers. Thus, 3 is a common divisor of 6, 9, 12, and 15, because it will exactly divide each of them.

121. The greatest number that will divide two or more numbers without a remainder, is called the *Greatest Common Divisor* of those numbers. Since 8 is the greatest number that will exactly divide 24, 32, 48, and 56, it is their *greatest common divisor.*

NOTE.—The initial letters G.C.D. are generally used to denote Greatest Common Divisor.

122. 1. Find the G. C. D. of 42, 126, and 168.

OPERATION

$$\begin{array}{r|rrr} 2 & 42 & 126 & 168 \\ \hline 3 & 21 & 63 & 84 \\ \hline 7 & 7 & 21 & 28 \\ \hline & 1 & 3 & 4 \end{array}$$

$2 \times 3 \times 7 = 42$, G. C. D.

We arrange the numbers as in the operation at the right. Since every composite number is divisible by its factors, and every possible product of them, any number that will exactly divide 42, 126, and 168, is a common factor of them, and, therefore, a factor of their G. C. D. Dividing by 2, we find it is a factor of each number, because it exactly divides each of them. Dividing the quotients by 3, we find it is a factor of each of them, because it is an exact divisor of each of them. In the same manner, we find 7 to be a factor of the G. C. D. As the remaining quotients 1, 3, and 4 contain no common factor, the divisors 2, 3, and 7 are the common factors of 42, 126, and 168, and, therefore, the factors of their G. C. D. Hence the G. C. D. required is the product of 2, 3, and 7, or 42.

Find the G. C. D. of the following:

2. 5, 10, 15	7. 120, 125, 255	12. 132, 330, 462
3. 25, 30, 40	8. 78, 104, 182	13. 565, 585, 910
4. 36, 48, 60	9. 84, 210, 360	14. 540, 270, 324
5. 72, 90, 126	10. 120, 360, 480	15. 216, 540, 864
6. 42, 84, 126	11. 315, 420, 630	16. 576, 1296, 1440

123. When the numbers are large, another method is usually employed. The principles involved in this method are:

1. *If a number divides two other numbers, it divides their sum and also their difference.*

For example, 7 divides 21 and 35. It also divides 56, the sum of 21 and 35. It also divides 14, the difference of 21 and 35.

2. *If a number divides another number, it will divide any number of times that number.*

For example, 7 divides 14; 7 also divides twice 14, or 28; it also divides 3 times 14, or 42, and so on.

3. *Any number which divides both dividend and divisor divides the remainder also.*

For example, 4 divides equally both 16 and 44. If we divide 44 by 16, we have a remainder of 12, which is divisible by 4. Since 4 divides 16 and 12, if we divide 16 by 12, we have the remainder 4, which itself is divisible by 4.

1. Find the G. C. D. of 546 and 609.

OPERATION

```
546)609(1
    546
    ---
     63)546(8
        504
        ---
         42)63(1
            42
            --
            21)42(2
               42
               --
                0
```

We first divide 609 by 546 and obtain the remainder 63. We then divide 546 by 63, and obtain the remainder 42. Again we divide 63 by 42, and we have the remainder 21. Lastly, we divide 42 by 21, and find that there is no remainder.

According to Prin. 1, whatever number divides 609 and 546 must also divide their difference 63, and therefore is a common divisor of 609, 546, and 63. Whatever number divides 546 and 63 must also divide their remainder 42, (Prin. 3), and hence is a common divisor of 609, 546, 63, and 42. Further, whatever number divides 63 and 42 must also divide their difference 21, (Prin. 1), and must therefore be a common divisor of 609, 546, 63, 42, and 21. But the G. C. D. of 42 and 21 is 21. Therefore the G. C. D. of 546 and 609 is 21.

If the G. C. D. of more than two numbers is required, first find the G. C. D. of any two of them, then find the G. C. D. of the number thus found, and a third number; then the G. C. D. of the last result and a fourth number, and so on.

Find the G. C. D. of the following:

2. 768 and 960
3. 576 and 720
4. 448 and 560
5. 1470 and 2415
6. 1332 and 2368
7. 540, 270, and 324
8. 784, 896, and 1008
9. 288, 1296, and 2232
10. 1020, 1887, and 2142
11. 1386, 1782, and 2178

PROBLEMS

124. 1. A coal operator has 1764 tons of No. 3 coal, and 2832 tons of No. 4, which he desires to ship to market in the largest cars of uniform size, without mixing the two kinds of coal. What is the capacity of each car?

2. A, B, and C have respectively $1127, $1281, and $1442 which they intend to invest in sheep. In order that each may get an exact number, what must be paid per head for the sheep? How many can each purchase?

3. A farmer owns a field 3598 feet long and 3374 feet wide, which he wishes to inclose with a fence 6 boards high. He wants to use the longest boards possible, and have an exact number of panels without cutting any of the boards. Find the length of the panels, the number of panels, and the number of boards required.

NOTE. — As *greatest common divisor* is used chiefly in reducing fractions to their lowest terms, only a few of the so-called "practical problems" are given.

LEAST COMMON MULTIPLE

125. When a number can be divided by another without a remainder, it is called a *Multiple* of that number. Thus, 9 is a multiple of 3.

126. When a number is divisible by two or more numbers without a remainder, it is said to be a *Common Multiple* of those numbers. Thus, 24 is a common multiple of 2, 4, 6, and 8, because 24 can be divided by each of them without a remainder.

127. The *smallest* number that can be divided by two or more numbers without a remainder, is called the *Least Common Multiple* of those numbers. Thus, 21 is the least common

multiple of 3 and 7, because 21 is the least number that can be divided by each of them without a remainder.

Note.—The initial letters L. C. M. are generally used to denote Least Common Multiple.

128. The *Least Common Multiple* of two or more numbers must contain no factor which is not found in any of those numbers.

For example, the least common multiple of 6, 9, and 24, must not contain 5 or 7, because neither 5 nor 7 is found in those numbers. Therefore, the least common multiple of two or more numbers must be composed only of the prime factors of each of the numbers. Thus, the least common multiple of 6, 9, and 24, must contain 2 and 3 in order to be a multiple of 6. It must contain two 3's in order to be a multiple of 9. It must contain three 2's and one 3 in order to be a multiple of 24. That is, the least common multiple of 6, 9, and 24 must be composed of the prime factors 2, 2, 2, 3, and 3.

The product of 2, 2, 2, 3, and 3, or $2 \times 2 \times 2 \times 3 \times 3$, which equals 72, is the least common multiple of 6, 9, and 24.

129. The least common multiple of numbers prime to each other is simply their product. Thus, the L. C. M. of 3, 5, and $7 = 3 \times 5 \times 7$, or 105.

First Method

130. This method is usually employed when the numbers are small and can be readily factored by inspection.

1. Find the L. C. M. of 16, 24, and 30.

OPERATION

$$\left.\begin{array}{l} 16 = 2 \times 2 \times 2 \times 2 \\ 24 = 2 \times 2 \times 2 \times 3 \\ 30 = 2 \times 3 \times 5 \end{array}\right\} 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 240, \text{ L. C. M.}$$

$$240 = \overbrace{2 \times \underbrace{2 \times 2 \times 2}_{24}}^{16} \overbrace{\times 3 \times 5}^{30}$$

By examining the prime factors of 240, the L. C. M. of 16, 24, and 30, we find that the prime factors of 16 are 2, 2, 2, 2; the prime factors of 24 are 2, 2, 2, 3; and the prime factors of 30 are 2, 3, 5.

As the L. C. M. of two or more numbers must be composed only of the prime factors of each of the numbers, the L. C. M. of 16, 24, and 30 equals $2 \times 2 \times 2 \times 2 \times 3 \times 5$, or 240.

Find the L. C. M. of the following:

2. 10, 20, and 30	7. 22, 44, and 88
3. 24, 36, and 48	8. 48, 72, and 120
4. 28, 56, and 70	9. 36, 42, and 54
5. 32, 48, and 72	10. 54, 72, and 90
6. 40, 50, and 75	11. 66, 165, and 154

Second Method

131. By this method we take the prime factors of the L. C. M. out of the numbers.

1. Find the L. C. M. of 28, 48, 56.

OPERATION

$$\begin{array}{r|rrr} 2 & 28 & 48 & 56 \\ \hline 2 & 14 & 24 & 28 \\ \hline 2 & 7 & 12 & 14 \\ \hline 7 & 7 & 6 & 7 \\ \hline & 1 & 6 & 1 \end{array}$$

$2 \times 2 \times 2 \times 7 \times 6 = 336$, L. C. M.

We write the numbers as in the operation at the right and then divide by any prime number that is a factor of two or more of them. We first divide by 2, which is contained in each of the numbers an exact number of times. Hence 2 must be a factor of the required L.C.M. By dividing the quotients by 2, we find it to be a factor of each. Hence 2 must be used twice as a factor of the required L. C. M. By dividing the second quotients by 2, we find it to be a factor of 12 and 14. Hence 2 must be used 3 times as a factor of the required L. C. M. Writing the quotients and the undivided number below the horizontal line, we next divide by 7, which we find to be a factor of 7. Hence 7 must be a factor of the required L. C. M. As the quotients and the undivided number 6 are prime to each other, the factors of the required L. C. M. are 2, 2, 2, 7, and 6. Hence the required L. C. M. equals $2 \times 2 \times 2 \times 7 \times 6$, or 336.

2. Find the L. C. M. of 14, 18, 22, and 24.

OPERATION

$$\begin{array}{r|rrrr} 2 & 14 & 18 & 22 & 24 \\ \hline 3 & 7 & 9 & 11 & 12 \\ \hline & 7 & 3 & 11 & 4 \end{array}$$

$2 \times 3 \times 7 \times 3 \times 11 \times 4 = 5544$, L. C. M. required.

In the same manner find the L. C. M. of:

3. 72, 96, and 144
4. 84, 126, and 168
5. 96, 144, and 192
6. 192, 288, and 360
7. 320, 160, and 280
8. 16, 24, 40, and 60
9. 20, 90, 150, and 180
10. 60, 130, 156, and 330
11. 48, 180, 420, and 630
12. 8, 240, 400, and 660

CANCELLATION

132. *Cancellation* is an operation to shorten work in division by striking out common factors from both dividend and divisor.

133. Striking out a factor from any number divides the number by that factor. Thus, to strike out, or to cancel, 7 from 21, is to divide 21 by 7.

134. Canceling equal factors from both dividend and divisor simply divides both dividend and divisor by those factors, which according to the principles of division does not alter the value of the quotient.

135. 1. Divide 210 by 30.

OPERATION BY CANCELLATION

$$210 \div 30 = \frac{210}{30} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{5}} \times 7}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}}} = \frac{7}{1} = 7. \textit{ Ans.}$$

It will be seen that we cancel the common factors 2, 3, and 5 from both dividend and divisor. Then taking the product of the remaining factors, we have

$$\frac{1 \times 1 \times 1 \times 7}{1 \times 1 \times 1} = \frac{7}{1}, \text{ or } 7 \quad \textit{Ans.}$$

2. Divide 336 by 84.

OPERATION BY CANCELLATION

$$336 \div 84 = \frac{336}{84} = \frac{\overset{2}{\cancel{6}} \times \overset{1}{\cancel{7}} \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{4}}} = \frac{4}{1} = 4 \quad \textit{Ans.}$$

3. Divide $8 \times 27 \times 49 \times 54$ by $4 \times 81 \times 7 \times 27$.

OPERATION BY CANCELLATION

$$\frac{\overset{2}{\cancel{8}} \times \overset{1}{\cancel{27}} \times \overset{7}{\cancel{49}} \times \overset{2}{\cancel{54}}}{\underset{1}{\cancel{4}} \times \underset{3}{\cancel{81}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{27}}} = \frac{28}{3} = 9\tfrac{1}{3} \quad \textit{Ans.}$$

4. Divide $24 \times 28 \times 32$ by $12 \times 14 \times 16$.
5. Divide $48 \times 18 \times 20$ by $12 \times 8 \times 10$.
6. Divide $40 \times 64 \times 70$ by $8 \times 10 \times 32$.
7. Divide $93 \times 27 \times 24$ by $6 \times 8 \times 9$.
8. Divide $99 \times 66 \times 21 \times 6$ by $33 \times 22 \times 7$.
9. Divide $36 \times 72 \times 96 \times 144$ by $24 \times 36 \times 48 \times 72$.
10. Divide $48 \times 27 \times 13 \times 9$ by $18 \times 16 \times 3 \times 26$.
11. Divide $125 \times 96 \times 57 \times 15$ by $16 \times 19 \times 25$.
12. Divide $342 \times 15 \times 56 \times 12$ by $18 \times 30 \times 14$.

13. $\dfrac{18 \times 36 \times 48}{24 \times 9 \times 12} = ?$

14. $\dfrac{30 \times 72 \times 81}{27 \times 36 \times 90} = ?$

15. $\dfrac{24 \times 18 \times 50}{10 \times 24 \times 6} = ?$

16. $\dfrac{90 \times 48 \times 45}{16 \times 45 \times 15} = ?$

17. $\dfrac{96 \times 98 \times 54}{9 \times 32 \times 63} = ?$

18. $\dfrac{78 \times 54 \times 45}{13 \times 81 \times 10} = ?$

PROBLEMS

136. 1. How many bushels of apples worth 90 cents a bushel will pay for 15 bushels of potatoes worth 60 cents a bushel?

If 1 bushel of potatoes is worth 60 cents, 15 bushels are worth 15 times 60 cents (15×60); and there will be required as many bushels of apples as 90 cents are contained times in 15×60 cents, which, according to the operation, we find to be 10.

OPERATION

$$\frac{\cancel{15} \times \overset{10}{\cancel{60}}}{\underset{\cancel{6}}{\cancel{90}}} = 10 \quad \textit{Ans.}$$

2. If 28 tons of coal cost \$84, how much will 35 tons cost?

If 28 tons cost \$84, *one* ton will cost $\frac{1}{28}$ of \$84, or \$$\frac{84}{28}$, and 35 tons will cost 35 times \$$\frac{84}{28}$, or $\frac{35 \times \$\overset{3}{\cancel{84}}}{\underset{1}{\cancel{28}}}$, or \$105.

OPERATION

$$\frac{35 \times \$\overset{3}{\cancel{84}}}{\cancel{28}} = \$105 \quad \textit{Ans.}$$

3. At \$3 a day, how many weeks must a stone mason work to pay for 21 tons of coal, at \$4 a ton?

4. How much must be paid for 42 gallons of molasses when \$18 are paid for 36 gallons?

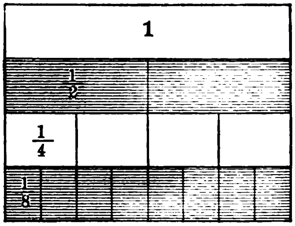
5. A farmer sold a barrel of cider, containing 31 gallons, for 14 cents a gallon. How many pounds of sugar, at 7 cents a pound, can he buy for what he receives for the cider?

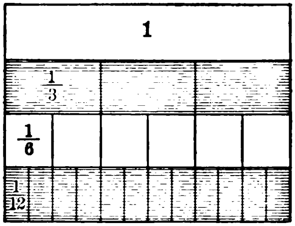
6. A huckster took to market 25 tubs of butter, each weighing 56 pounds, which he sold for 30 cents a pound. How many barrels of molasses, each containing 35 gallons, at 20 cents a gallon, was the butter worth?

7. How many dozen eggs, at 15 cents a dozen, are worth as much as 45 pounds of coffee, at 25 cents a pound?

8. A passenger on a train found that he passed 48 telegraph poles in 6 minutes. How many miles per hour was the train running, the poles being 110 yards apart?

COMMON FRACTIONS





137. If a unit is divided into *two equal* parts, *one* of these parts is called *one half,* written $\frac{1}{2}$.

138. If a unit is divided into *three equal* parts, *one* of these parts is called *one third,* written $\frac{1}{3}$. Two of these parts are called *two thirds,* written $\frac{2}{3}$.

139. One of the *four equal* parts of a unit is called *one fourth,* written $\frac{1}{4}$. *Two* of these parts are called *two fourths,* written $\frac{2}{4}$, and *three* of these parts are called *three fourths,* written $\frac{3}{4}$.

140. These equal parts of a unit are called *Fractions.*

141. Common fractions are expressed by two terms, called respectively *Numerator* and *Denominator.* The *Denominator* shows into how many equal parts the unit is divided, and is written below a short horizontal line. The *Numerator* shows the number of these equal parts taken, and is written above the line. In the fraction $\frac{6}{7}$, 7 shows that the unit

is divided into *seven* equal parts, and is called the denominator of the fraction. The 6 shows the number of these equal parts that are taken, and is called the numerator of the fraction.

142. Following are a few fractions and their names:

$\frac{1}{2}$, one half	$\frac{9}{13}$, nine thirteenths
$\frac{1}{3}$, one third	$\frac{11}{19}$, eleven nineteenths
$\frac{1}{4}$, one fourth	$\frac{14}{25}$, fourteen twenty-fifths
$\frac{1}{5}$, one fifth	$\frac{21}{46}$, twenty-one forty-sixths
$\frac{1}{6}$, one sixth	$\frac{39}{80}$, thirty-nine eightieths
$\frac{1}{7}$, one seventh	$\frac{47}{89}$, forty-seven eighty-ninths
$\frac{2}{3}$, two thirds	$\frac{59}{95}$, fifty-nine ninety-fifths
$\frac{4}{5}$, four fifths	$\frac{75}{110}$, seventy-five one-hundred-tenths
$\frac{7}{9}$, seven ninths	$\frac{92}{93}$, ninety-two ninety-thirds

143. Read:

1.	$\frac{5}{9}$	$\frac{9}{14}$	$\frac{6}{15}$	$\frac{11}{17}$	$\frac{14}{26}$	$\frac{8}{82}$	$\frac{24}{37}$	$\frac{27}{391}$
2.	$\frac{39}{49}$	$\frac{41}{54}$	$\frac{59}{61}$	$\frac{63}{84}$	$\frac{72}{87}$	$\frac{29}{105}$	$\frac{38}{144}$	$\frac{84}{172}$
3.	$\frac{89}{264}$	$\frac{98}{568}$	$\frac{172}{878}$	$\frac{346}{785}$	$\frac{462}{891}$	$\frac{579}{925}$	$\frac{641}{947}$	$\frac{728}{967}$

Express in figures:

4. Three fifths; four sevenths; five sevenths.
5. Seven twelfths; twenty-six fortieths.
6. Seven fifty-fourths; nine seventy-thirds.
7. Twenty-three eightieths; sixty-two hundredths.
8. Sixty-nine three-hundred-seventeenths.

144. When the numerator is smaller than the denominator, the expression is called a *Proper Fraction*. Thus, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{13}{25}$ are proper fractions.

145. When the numerator equals or exceeds the denominator, the expression is called an *Improper Fraction.* Thus, $\frac{5}{5}$, $\frac{9}{8}$, $\frac{23}{21}$, and $\frac{47}{39}$ are examples of improper fractions.

146. Expressions which consist of an integer (whole number) and a fraction, as $6\frac{3}{5}$, $7\frac{2}{9}$, and $9\frac{3}{7}$ are called *Mixed Numbers.*

147. A *Complex Fraction* is one whose numerator, or denominator, or both, contain a fraction. Thus,

$$\frac{7}{\frac{2}{3}},\ \frac{4\frac{3}{9}}{7},\ \frac{7\frac{3}{8}}{25\frac{1}{7}},\ \text{and}\ \frac{\frac{2}{3}\times\frac{4}{9}}{7\times 3\frac{1}{8}}$$

are examples of complex fractions. They are read: 7 divided by $\frac{2}{3}$; $4\frac{3}{9}$ divided by 7; $7\frac{3}{8}$ divided by $25\frac{1}{7}$; etc.

148. A *Compound Fraction* is a fraction of a fraction; as $\frac{3}{4}$ of $\frac{8}{11}$; $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{10}{11}$.

REDUCTION OF FRACTIONS

149. *Reduction of Fractions* is an operation to change the form of fractions without changing their value.

150. 1. Change $\frac{236}{15}$ to a mixed number.

OPERATION

$$\begin{array}{r l} 15)236(15\frac{11}{15}. & \textit{Ans.} \\ \underline{15} & \\ 86 & \\ \underline{75} & \\ 11 & \end{array}$$

$\frac{15}{15} = 1$, hence there are as many 1's in $\frac{236}{15}$ as 15 is contained times in 236, which is $15\frac{11}{15}$.

Change the following improper fractions to whole or mixed numbers:

2. $\frac{84}{63}$	5. $\frac{114}{38}$	8. $\frac{268}{9}$ rods	11. $\frac{3847}{268}$ tons
3. $\frac{87}{72}$	6. $\$\frac{172}{84}$	9. $\frac{878}{176}$ feet	12. $\frac{5688}{2819}$ days
4. $\frac{105}{29}$	7. $\$\frac{264}{89}$	10. $\frac{891}{462}$ miles	13. $\frac{6894}{3472}$ hours

151. 1. Change $21\frac{4}{5}$ to fifths.

OPERATION

$$\begin{array}{r} 21\frac{4}{5} \\ 5 \\ \hline \frac{109}{5} \end{array} \quad \textit{Ans.}$$

There are $\frac{5}{5}$ in 1, and in 21 there are 21 times $\frac{5}{5}$, or $\frac{105}{5}$, which added to $\frac{4}{5}$ make $\frac{109}{5}$.

NOTE. — In practice we multiply the whole number by the denominator of the fraction, add to the product the numerator of the fraction, and write the denominator of the fraction under the sum.

To change an integer to a fractional form, write 1 under it for a denominator; thus, $6 = \frac{6}{1}$; $14 = \frac{14}{1}$.

Change to improper fractions:

2. $22\frac{3}{4}$
3. $27\frac{9}{11}$
4. $31\frac{8}{15}$
5. $39\frac{5}{18}$
6. $42\frac{7}{25}$
7. $51\frac{9}{32}$
8. $63\frac{12}{19}$
9. $74\frac{13}{29}$
10. $82\frac{21}{37}$
11. $87\frac{25}{39}$
12. $92\frac{35}{41}$
13. $206\frac{39}{87}$

152. 1. Change $\frac{3}{4}$ to twelfths.

SOLUTION. — In 1 there are $\frac{12}{12}$, and in $\frac{3}{4}$ there are $\frac{3}{4}$ of $\frac{12}{12}$, or $\frac{9}{12}$.

2. Change $\frac{2}{3}$ to fifteenths.
3. Change $\frac{3}{8}$ to sixteenths.
4. Change $\frac{5}{6}$ to twenty-fourths.
5. How many 40ths in $\frac{3}{8}$?
6. How many 56ths in $\frac{7}{8}$?
7. How many 120ths in $\frac{7}{12}$?

153. 1. Change $\frac{12}{3}$ and $\frac{3}{4}$ to 12ths.

Both terms of each fraction are multiplied by that number which will raise the denominators of the given fractions to the required denominator.

OPERATION

$$\frac{12}{3} = \frac{12 \times 4}{3 \times 4} = \frac{48}{12}$$

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

NOTE. — To find the number by which to multiply, we divide the required denominator by the denominators of the several fractions.

2. Change $\frac{5}{12}$ and $\frac{7}{9}$ to 36ths.
3. Change $\frac{9}{10}$ and $\frac{3}{20}$ to 60ths.
4. Change $\frac{8}{9}$ and $\frac{11}{24}$ to 72ds.
5. How many 39ths in $\frac{9}{13}$?
6. How many 84ths in $\frac{13}{21}$?
7. How many 205ths in $\frac{39}{41}$?

154. 1. Reduce $\frac{16}{20}$ to 5ths.

SOLUTION. $\frac{1}{5} = \frac{4}{20}$, hence $\frac{16}{20}$ = as many $\frac{1}{5}$'s as 4 is contained times in 16, which are 4. Therefore $\frac{16}{20} = \frac{4}{5}$.

2. Reduce $\frac{12}{15}$ to 5ths.
3. Reduce $\frac{18}{20}$ to 10ths.
4. Reduce $\frac{15}{21}$ to 7ths.
5. Reduce $\frac{36}{48}$ to 4ths.
6. Reduce $\frac{21}{49}$ to 7ths.
7. Reduce $\frac{49}{56}$ to 8ths.

155. The value of a fraction is not changed if both numerator and denominator are divided by the same number.

156. A fraction is in its lowest terms when the numerator and denominator have no common divisor.

157. 1. Reduce $\frac{120}{168}$ to 7ths.

As a fraction is in its lowest terms when the numerator and denominator contain no common factor, we reduce $\frac{120}{168}$ to its lowest terms, either by dividing both numerator and denominator successively by common factors, as in operations I and II, or by dividing both terms by their *greatest common divisor*, as in operation III.

OPERATION I

$$3|\underline{\tfrac{120}{168}} = \tfrac{40}{56};\quad 8|\underline{\tfrac{40}{56}} = \tfrac{5}{7}$$

OPERATION II

$$4|\underline{\tfrac{120}{168}} = \tfrac{30}{42};\quad 6|\underline{\tfrac{30}{42}} = \tfrac{5}{7}$$

OPERATION III

$$24|\underline{\tfrac{120}{168}} = \tfrac{5}{7}$$

Reduce to lowest terms:

2. $\frac{72}{216}$	5. $\frac{432}{504}$	8. $\frac{600}{900}$	11. $\frac{990}{1980}$
3. $\frac{120}{180}$	6. $\frac{384}{432}$	9. $\frac{108}{756}$	12. $\frac{896}{1792}$
4. $\frac{216}{288}$	7. $\frac{360}{504}$	10. $\frac{462}{660}$	13. $\frac{512}{2560}$

THE LEAST COMMON DENOMINATOR OF FRACTIONS

158. When the denominators of several fractions are similar, they are said to have a common denominator. Thus, $\frac{5}{9}$ and $\frac{7}{9}$ have a common denominator.

159. The smallest common denominator which several fractions may have is called their *Least Common Denominator.*

The least common denominator which $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ may have is 12. Thus, $\frac{2}{3} = \frac{8}{12}$; $\frac{3}{4} = \frac{9}{12}$; $\frac{5}{6} = \frac{10}{12}$.

160. The least common denominator of several fractions is the least common multiple of their denominators.

161. 1. Change $\frac{5}{6}$, $\frac{7}{12}$, $\frac{13}{18}$ to their least common denominator.

OPERATION

$$\begin{array}{r|rrr} 2 & 6 & 12 & 18 \\ \hline 3 & 3 & 6 & 9 \\ \hline & 1 & 2 & 3 \end{array}$$

$$2 \times 3 \times 2 \times 3 = 36 = \text{L. C. M. of denominators}$$

$$(36 \div 6) \times 5 = 30, \text{ new numerator for } \tfrac{5}{6}$$

$$(36 \div 12) \times 7 = 21, \text{ new numerator for } \tfrac{7}{12}$$

$$(36 \div 18) \times 13 = 26, \text{ new numerator for } \tfrac{13}{18}$$

$$\text{Hence, } \tfrac{5}{6} = \tfrac{30}{36}$$

$$\tfrac{7}{12} = \tfrac{21}{36}$$

$$\tfrac{13}{18} = \tfrac{26}{36}$$

The L. C. M. of the denominators is 36, which is the least common denominator to which the fractions may be reduced. Dividing 36 by the denominator of each fraction, and multiplying the quotient by the numerator, we get the new numerator of each fraction, under which we write the least common denominator.

Change to least common denominator:

2. $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{5}{9}$, and $\frac{7}{15}$
3. $\frac{7}{9}$, $\frac{3}{10}$, $\frac{5}{8}$, $\frac{4}{15}$, and $\frac{23}{20}$
4. $\frac{1}{3}$, $\frac{9}{14}$, $\frac{7}{12}$, $\frac{16}{21}$, and $\frac{19}{28}$
5. $\frac{2}{3}$, $\frac{1}{4}$, $\frac{7}{9}$, $\frac{5}{6}$, and $\frac{5}{12}$
6. $\frac{3}{5}$, $\frac{9}{10}$, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{19}{20}$
7. $\frac{1}{3}$, $\frac{9}{14}$, $\frac{7}{12}$, $\frac{16}{21}$, and $\frac{19}{28}$
8. $\frac{4}{5}$, $\frac{7}{30}$, $\frac{7}{15}$, and $\frac{17}{60}$
9. $\frac{21}{3}$, $\frac{47}{14}$, $\frac{5}{7}$, and $\frac{23}{21}$
10. $\frac{17}{20}$, $\frac{16}{21}$, $\frac{27}{28}$, and $\frac{7}{15}$
11. $\frac{9}{7}$, $\frac{7}{9}$, $5\frac{2}{3}$, and $6\frac{4}{5}$
12. $\frac{17}{24}$, $\frac{7}{12}$, $\frac{23}{28}$, and $\frac{19}{36}$
13. $\frac{11}{18}$, $3\frac{5}{9}$, $\frac{14}{15}$, and $\frac{17}{20}$

ADDITION OF FRACTIONS

162. 1. Find the sum of $\frac{4}{5}$, $\frac{7}{10}$, and $\frac{11}{12}$.

OPERATION

Least Common Denominator = 60

$$\frac{4}{5}=\frac{4\times 12}{5\times 12}=\frac{48}{60}$$

$$\frac{7}{10}=\frac{7\times 6}{10\times 6}=\frac{42}{60}$$

$$\frac{11}{12}=\frac{11\times 5}{12\times 5}=\frac{55}{60}$$

$$\frac{48}{60}+\frac{42}{60}+\frac{55}{60}=\frac{145}{60}=2\tfrac{5}{12}.\ \textit{Ans.}$$

Reducing the given fractions to their least common denominator, which is 60, adding their numerators, and writing the sum over the least common denominator, we find the sum of $\frac{4}{5}$, $\frac{7}{10}$, and $\frac{11}{12}$ to be $\frac{145}{60}$, or $2\frac{5}{12}$.

NOTE. — Always express the sum in lowest terms.

2. Add $\frac{3}{5}$, $\frac{7}{8}$, and $\frac{9}{10}$.
3. Add $\frac{5}{6}$, $\frac{7}{18}$, and $\frac{3}{8}$.
4. Add $\frac{5}{12}$, $\frac{4}{9}$, and $\frac{5}{18}$.
5. Add $\frac{4}{5}$, $\frac{13}{20}$, and $\frac{19}{48}$.
6. Add $\frac{5}{7}$, $\frac{15}{28}$, and $\frac{31}{42}$.
7. Add $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{25}{32}$.
8. Add $\frac{5}{18}$, $\frac{5}{6}$, and $\frac{17}{81}$.
9. Add $\frac{5}{9}$, $\frac{11}{15}$, and $\frac{29}{45}$.
10. Add $\frac{15}{16}$, $\frac{5}{7}$, and $\frac{27}{56}$.
11. Add $\frac{2}{11}$, $\frac{8}{33}$, and $\frac{39}{44}$.

163. When mixed numbers are to be added, find the sum of the integers and fractions separately, and add the sums.

164. 1. Find the sum of $7\frac{3}{4}$, $8\frac{3}{16}$, and $18\frac{7}{12}$.

OPERATION I

$$7+8+18=33 \quad \text{sum of integers}$$

$$\frac{3}{4}+\frac{3}{16}+\frac{7}{12}=\frac{73}{48}=\underline{1\tfrac{25}{48}} \quad \text{sum of fractions}$$

$$\text{Entire sum}=34\tfrac{25}{48}$$

OPERATION II

$$\begin{aligned} 7\tfrac{3}{4} &= 7\tfrac{36}{48} \\ 8\tfrac{3}{16} &= 8\tfrac{9}{48} \\ 18\tfrac{7}{12} &= \underline{18\tfrac{28}{48}} \\ & \quad 33\tfrac{73}{48} = 34\tfrac{25}{48} \ \textit{Ans.} \end{aligned}$$

Find the sum of:

2. $4\frac{5}{8}$, $9\frac{5}{12}$, and $18\frac{7}{24}$
3. $25\frac{8}{9}$, $3\frac{5}{18}$, and $22\frac{2}{3}$
4. $18\frac{3}{4}$, $9\frac{3}{8}$, and $21\frac{5}{48}$
5. $14\frac{14}{15}$, $17\frac{18}{35}$, and $42\frac{25}{28}$
6. $18\frac{17}{20}$, $19\frac{11}{24}$, and $20\frac{31}{36}$
7. $32\frac{5}{8}$, $63\frac{4}{7}$, $45\frac{3}{5}$, and $53\frac{2}{3}$
8. $23\frac{8}{9}$, $73\frac{2}{3}$, $82\frac{2}{3}$, and $71\frac{19}{45}$
9. $81\frac{3}{5}$, $95\frac{3}{4}$, $45\frac{2}{7}$, and $61\frac{4}{5}$
10. $74\frac{7}{35}$, $61\frac{5}{7}$, $72\frac{4}{5}$, and $82\frac{3}{10}$
11. $84\frac{7}{12}$, $32\frac{5}{9}$, $63\frac{3}{7}$, and $89\frac{2}{3}$

PROBLEMS

165. 1. A grocer sold $\frac{5}{8}$ of a barrel of sugar to one man, and $\frac{2}{5}$ of it to another customer. What part of the barrel did he sell to both?

2. A man bought 3 loads of coal. The first load weighed $1\frac{1}{3}$ tons, the second $1\frac{1}{5}$ tons, and the third $1\frac{1}{4}$ tons. How many tons did he buy?

3. I bought an overcoat for \$ $35\frac{4}{5}$, trousers for \$ $9\frac{3}{8}$, shoes for \$ 8, and a hat for \$ $4\frac{3}{4}$. How much did I pay for all?

4. A man gave a farm to each of his three sons. The oldest received $124\frac{3}{10}$ acres, the second $128\frac{5}{8}$ acres, and the third $119\frac{11}{60}$ acres. How many acres did the three farms together contain?

5. A grocer bought $349\frac{5}{16}$ pounds of coffee in January, $336\frac{7}{8}$ pounds in March, and $438\frac{3}{4}$ pounds in April. How many pounds did he buy in the three months?

6. William Wilson sold from his farm corn for \$ $132\frac{4}{5}$, wheat for \$ $246\frac{3}{8}$, and barley for \$ $92\frac{13}{16}$. How much did he receive for all?

7. A merchant bought three pieces of silk. The first contained $36\frac{5}{8}$ yards, the second $29\frac{3}{4}$ yards, and the third $38\frac{5}{8}$ yards. How many yards were in the three pieces?

8. A certain town, A, is $13\frac{1}{2}$ miles west of B; C is $25\frac{5}{8}$ miles east of B, and D is $31\frac{1}{4}$ miles east of C. Draw a diagram and compute the distance between A and D.

SUBTRACTION OF FRACTIONS

166. 1. From $\frac{8}{9}$ take $\frac{7}{10}$.

OPERATION

Least Common Denominator $= 90$

$$\frac{8}{9} = \frac{8 \times 10}{9 \times 10} = \frac{80}{90}$$

$$\frac{7}{10} = \frac{7 \times 9}{10 \times 9} = \frac{63}{90}$$

$$\frac{80}{90} - \frac{63}{90} = \frac{17}{90} \textit{ Ans.}$$

Reducing the given fractions to their least common denominator, 90, we find $\frac{8}{9}$ to equal $\frac{80}{90}$, and $\frac{7}{10}$ to equal $\frac{63}{90}$. Hence their difference equals $\frac{80}{90} - \frac{63}{90}$, or $\frac{17}{90}$.

2. $\frac{7}{12} - \frac{2}{9} = ?$
3. $\frac{7}{9} - \frac{3}{7} = ?$
4. $\frac{17}{21} - \frac{5}{7} = ?$
5. $\frac{10}{11} - \frac{7}{8} = ?$
6. $\frac{11}{12} - \frac{2}{3} = ?$
7. $\frac{17}{18} - \frac{5}{6} = ?$
8. $\frac{18}{25} - \frac{7}{10} = ?$
9. $\frac{28}{35} - \frac{3}{10} = ?$
10. $\frac{16}{31} - \frac{1}{2} = ?$
11. $\frac{29}{37} - \frac{1}{3} = ?$
12. $\frac{54}{59} - \frac{1}{6} = ?$
13. $\frac{69}{80} - \frac{3}{7} = ?$
14. $\frac{85}{96} - \frac{5}{24} = ?$
15. $\frac{93}{108} - \frac{13}{36} = ?$
16. $\frac{125}{165} - \frac{29}{55} = ?$

167. 1. From 12 take $\frac{5}{9}$.

OPERATION

$$12 = 11\frac{9}{9};\ 11\frac{9}{9} - \frac{5}{9} = 11\frac{4}{9} \textit{ Ans.}$$

2. $13 - \frac{5}{6} = ?$
3. $18 - \frac{7}{12} = ?$
4. $22 - \frac{8}{13} = ?$
5. $28 - \frac{12}{13} = ?$
6. $32 - \frac{14}{15} = ?$
7. $36 - \frac{18}{19} = ?$
8. $39 - \frac{19}{21} = ?$
9. $46 - \frac{21}{23} = ?$
10. $54 - \frac{39}{41} = ?$
11. $62 - \frac{41}{54} = ?$
12. $71 - \frac{57}{68} = ?$
13. $82 - \frac{61}{75} = ?$

168. 1. From 29 take $16\frac{7}{8}$.

OPERATION

$$29 = 28\frac{8}{8}$$

$$28\frac{8}{8} - 16\frac{7}{8} = 12\frac{1}{8}\ \textit{Ans.}$$

2. From 49 take $18\frac{9}{11}$.
3. From 51 take $49\frac{7}{18}$.
4. From 56 take $54\frac{8}{15}$.

169. 1. From $17\frac{3}{5}$ take $\frac{7}{9}$.

OPERATION

$$17\frac{3}{5} = 16\frac{72}{45};\ \frac{7}{9} = \frac{35}{45}$$

$$16\frac{72}{45} - \frac{35}{45} = 16\frac{37}{45}\ \textit{Ans.}$$

2. From $29\frac{5}{12}$ take $\frac{7}{8}$.
3. From $46\frac{3}{13}$ take $\frac{8}{26}$.
4. From $59\frac{7}{18}$ take $\frac{8}{9}$.

170. 1. From $26\frac{7}{36}$ take $18\frac{5}{9}$.

OPERATION

$$26\frac{7}{36} = 25\frac{43}{36};\quad 18\frac{5}{9} = 18\frac{20}{36};\quad 25\frac{43}{36} - 18\frac{20}{36} = 7\frac{23}{36}\ \textit{Ans.}$$

2. From $39\frac{8}{11}$ take $18\frac{19}{22}$.
3. From $43\frac{9}{11}$ take $15\frac{4}{7}$.
4. From $27\frac{8}{9}$ take $12\frac{5}{6}$.
5. From $91\frac{2}{21}$ take $87\frac{6}{7}$.
6. From $79\frac{1}{32}$ take $23\frac{13}{16}$.
7. From $77\frac{11}{14}$ take $43\frac{15}{98}$.
8. From $69\frac{3}{7}$ take $32\frac{13}{18}$.
9. From $96\frac{1}{18}$ take $79\frac{19}{24}$.
10. From $95\frac{5}{8}$ take $45\frac{2}{7}$.
11. From $99\frac{11}{99}$ take $98\frac{8}{11}$.

171. Perform the operations indicated (see § 63):

1. $(\frac{7}{8} + \frac{5}{9} + \frac{7}{12}) - (\frac{2}{3} + \frac{4}{5}) = ?$
2. $(8 + \frac{2}{3}) - 7 + (6\frac{3}{4} - 3\frac{5}{8}) = ?$
3. $(9\frac{5}{6} + 4\frac{2}{3}) - (7\frac{8}{11} - 6\frac{1}{2}) + \frac{4}{7} = ?$
4. $54 - (19\frac{5}{7} - 4\frac{1}{2}) + 16\frac{1}{3} = ?$
5. $98\frac{7}{20} + \overline{19\frac{1}{2} - 9\frac{7}{10}} - 4\frac{1}{3} = ?$
6. $(8 - 4\frac{5}{7}) + (21\frac{5}{14} - 7\frac{1}{2}) = ?$
7. $59\frac{2}{3} + (84 - 42\frac{7}{10}) - 16\frac{8}{15} = ?$
8. $(172 - 85\frac{2}{9}) + (88\frac{11}{27} - \frac{5}{9}) = ?$

PROBLEMS

172. 1. From a bin containing $231\frac{3}{4}$ bushels of wheat $125\frac{5}{6}$ bushels were sold. How many bushels remained in the bin?

2. From a piece of cambric containing $65\frac{3}{8}$ yards there were sold at one time $16\frac{3}{4}$ yards, and at another time $17\frac{5}{9}$ yards. How many yards remained?

3. A farmer raised $1246\frac{3}{4}$ bushels of potatoes. He sold all but $25\frac{5}{6}$ bushels. How many bushels did he sell?

4. The Lehigh and Wilkesbarre Coal Co. bought $1272\frac{9}{10}$ acres of coal land, at four different times. If the first purchase contained $463\frac{5}{6}$ acres, the second $289\frac{1}{4}$ acres, and the third $389\frac{31}{80}$ acres, how many acres were contained in the fourth purchase?

5. If I buy a house and lot for \$4675, and pay \$$847\frac{3}{5}$ for improvements and \$$12\frac{3}{4}$ for taxes, how much must I ask for it in order to gain what I paid out for repairs and taxes?

6. Five bees in succession visited a clover blossom. The first carried away $\frac{5}{80}$ of the pollen, the second $\frac{6}{32}$, the third $\frac{2}{10}$, and the fourth $\frac{3}{8}$. How much remained for the fifth?

7. There were delivered at a school building three loads of coal, weighing upon the scales $38\frac{18}{25}$ hundredweight, $39\frac{81}{100}$ hundredweight, and $37\frac{47}{50}$ hundredweight, respectively. How much coal was delivered if the weight of the wagon was $14\frac{73}{100}$ hundredweight?

8. From a barrel of cider containing $31\frac{1}{2}$ gallons there were drawn, at three different times, $5\frac{3}{4}$ gallons, $6\frac{5}{16}$ gallons, and $12\frac{3}{8}$ gallons. If the barrel was then filled with water, how much cider and how much water did the mixture contain?

MULTIPLICATION OF FRACTIONS

173. 1. How much will 4 yards of cashmere cost at \$$\frac{7}{8}$ a yard?

ANALYSIS. — If 1 yard is worth \$$\frac{7}{8}$, 4 yards are worth 4 times as much, which is \$$\frac{28}{8}$, or \$$3\frac{4}{8}$, or \$$3\frac{1}{2}$.

If we had divided the *denominator* by 4 instead of multiplying the *numerator*, we should have obtained \$$\frac{7}{2}$, or \$$3\frac{1}{2}$, the same as before.

From these illustrations we deduce the following:

PRINCIPLE. — *A fraction may be multiplied either by multiplying its numerator, or by dividing its denominator.*

2. Multiply $\frac{13}{14}$ by 7.

OPERATION

$$\frac{13 \times 7}{14} = \frac{91}{14} = 6\tfrac{1}{2} \quad \textit{Ans.}$$

Multiplying the numerator of the fraction by 7, we get $\frac{91}{14}$, or $6\frac{1}{2}$. Or, dividing the denominator by 7, we get $\frac{13}{2}$, or $6\frac{1}{2}$.

OPERATION II

$$\frac{13}{14 \div 7} = \frac{13}{2} = 6\tfrac{1}{2} \quad \textit{Ans.}$$

NOTE. — A factor common to the whole number and the denominator of the fraction should be canceled before the multiplication. For, it is evident that canceling a factor common to both before the multiplication, is equivalent to dividing both terms of the resulting fraction by the same factor after the multiplication. (Arts. 133, 134.)

ILLUSTRATION. — $\dfrac{13 \times \cancel{7}}{\underset{2}{\cancel{14}}} = \dfrac{13}{2}$; $\dfrac{13 \times 7}{14} = \dfrac{91}{14}$; $7 \,\Big|\, \dfrac{91}{14} = \dfrac{13}{2}$.

Multiply:

3. $\frac{14}{15}$ by 9	8. $\frac{12}{45}$ by 20	13. $\frac{38}{45}$ by 60
4. $\frac{13}{18}$ by 27	9. $\frac{19}{27}$ by 33	14. $\frac{59}{75}$ by 85
5. $\frac{27}{35}$ by 40	10. $\frac{13}{49}$ by 14	15. $\frac{125}{148}$ by 74
6. $\frac{5}{44}$ by 22	11. $\frac{21}{32}$ by 40	16. $\frac{226}{345}$ by 115
7. $\frac{7}{15}$ by 35	12. $\frac{29}{64}$ by 24	17. $\frac{475}{1062}$ by 59

174. 1. Multiply $36\frac{7}{9}$ by 6.

OPERATION I

$$\begin{array}{rl} 36\tfrac{7}{9} & \\ 6 & \\ \hline 4\tfrac{2}{3} & = 6 \text{ times } \tfrac{7}{9} \\ 216 & = 6 \text{ times } 36 \\ \hline 220\tfrac{2}{3} & = 6 \text{ times } 36\tfrac{7}{9} \end{array}$$

OPERATION II

$$36\tfrac{7}{9} = \tfrac{331}{9}$$

$$\frac{331 \times \overset{2}{\cancel{6}}}{\underset{3}{\cancel{9}}} = \frac{662}{3} = 220\tfrac{2}{3}$$

It will be seen, in operation I, that we first multiply $\frac{7}{9}$ by 6, which gives us $4\frac{2}{3}$. We next multiply 36 by 6, which gives us 216. Then by adding the partial products $4\frac{2}{3}$ and 216, we find the entire product to be $220\frac{2}{3}$.

In operation II, we change $36\frac{7}{9}$ to an improper fraction and multiply as in Art. 173.

Multiply:

2. $28\frac{7}{9}$ by 12
3. $36\frac{6}{7}$ by 14
4. $59\frac{9}{10}$ by 45
5. $64\frac{5}{8}$ by 36
6. $69\frac{4}{11}$ by 55
7. $78\frac{19}{20}$ by 35
8. $87\frac{65}{72}$ by 36
9. $91\frac{68}{81}$ by 54
10. $159\frac{31}{64}$ by 72
11. $539\frac{55}{182}$ by 66

PROBLEMS

175. 1. If a family uses $5\frac{5}{6}$ barrels of flour in a year, how many barrels will 8 families use in the same time?

2. There are $24\frac{3}{4}$ cubic feet in a perch of stone. How many cubic feet are there in 84 perches?

3. If a man can build $8\frac{3}{4}$ perches of stone in one day, how many perches can he build in 1 week?

4. At $\$5\frac{7}{10}$ a box, how much will 65 boxes of raisins cost?

5. At the rate of $29\frac{11}{12}$ bushels per acre, how many bushels of wheat will a field of 9 acres yield?

6. If a load of hay weighs $\frac{5}{8}$ of a ton, find the weight of 18 loads of the same weight.

7. How many yards are there in 24 rolls of carpet each containing $16\frac{2}{3}$ yards?

8. Find the cost of 138 bushels of potatoes at $\$.46\frac{2}{3}$ per bushel.

9. A coal train moves at the rate of $21\frac{3}{8}$ miles an hour. How far will it go in 24 hours?

10. When bread is 5 cents a loaf, butter 30 cents a pound, and ham 35 cents a pound, how much will 12 sandwiches cost me, if I use for each $\frac{1}{15}$ of a loaf of bread, $\frac{1}{25}$ of a pound of ham, and $\frac{1}{40}$ of a pound of butter?

176. 1. Multiply 36 by $\frac{8}{9}$.

36 multiplied by $\frac{8}{9}$ means the same as $\frac{8}{9}$ of 36. Taking $\frac{8}{9}$ of 36, we have 32.

Or, by canceling the common factor, 9, from both dividend and divisor, and multiplying together the remaining factors we get 32.

OPERATION

$\frac{1}{9}$ of $36 = 4$,

$\frac{8}{9}$ of $36 = 8 \times 4$, or 32

Or,

$$\overset{4}{\cancel{36}} \times \frac{8}{\underset{1}{\cancel{9}}} = \frac{32}{1}, \text{ or } 32$$

Multiply:

2. 36 by $\frac{3}{4}$	5. 74 by $\frac{11}{12}$	8. 320 by $\frac{35}{72}$
3. 45 by $\frac{8}{9}$	6. 81 by $\frac{17}{18}$	9. 816 by $\frac{47}{96}$
4. 54 by $\frac{5}{12}$	7. 63 by $\frac{13}{14}$	10. 792 by $\frac{71}{144}$

PROBLEMS

177. 1. If an acre of land costs $ 160, how much will $\frac{4}{5}$ of an acre cost?

2. A boy rode 90 miles on his bicycle one day. At the same rate, how far would he ride in $\frac{8}{15}$ of a day?

3. A drover bought 1278 sheep, and sold $\frac{5}{9}$ of them. How many sheep did he sell?

4. A man picked 90 bushels of apples in 3 days. At the same rate, how many bushels would he pick in $\frac{13}{15}$ of a day?

5. Of a farm consisting of 140 acres, $\frac{3}{7}$ were used for wheat, $\frac{3}{4}$ of the remainder for corn, and the rest for grazing. How many acres were used for grazing?

178. 1. Multiply 81 by $7\frac{8}{9}$.

OPERATION I

$$\begin{array}{rl} 81 & \\ 7\frac{8}{9} & \\ \hline 72 & = \frac{8}{9} \text{ times } 81 \\ 567 & = 7 \text{ times } 81 \\ \hline 639 & = 7\frac{8}{9} \text{ times } 81. \end{array}$$

OPERATION II

$7\frac{8}{9} = \frac{71}{9}$

$$\overset{9}{\cancel{81}} \times \frac{71}{\underset{1}{\cancel{9}}} = \frac{639}{1} = 639 \textit{ Ans.}$$

In operation I we first multiply 81 by $\frac{8}{9}$, which gives us 72. We next multiply 81 by 7, which gives us 567. Then we add the partial products 72 and 567, and find the required product to be 639.

In operation II we change the multiplier $7\frac{8}{9}$ to an improper fraction, and multiply as in Art. 176.

Multiply:

2. 48 by $6\frac{3}{8}$	5. 594 by $36\frac{5}{6}$	8. 768 by $56\frac{5}{11}$
3. 108 by $8\frac{5}{9}$	6. 580 by $46\frac{9}{10}$	9. 900 by $75\frac{23}{24}$
4. 608 by $16\frac{5}{8}$	7. 360 by $45\frac{4}{5}$	10. 980 by $79\frac{9}{16}$

PROBLEMS

179. 1. If 2 men can plow 5 acres of land in a day, how many acres can they plow in $11\frac{3}{5}$ days?

2. If a bushel of wheat makes 48 lb. of flour, how many pounds of flour will $12\frac{5}{6}$ bushels make?

3. If a boy can run 5 miles an hour, how far can he run in $21\frac{7}{8}$ hours?

4. A man earns \$.45 an hour. At the same rate how much can he earn in $26\frac{3}{5}$ hours?

5. Find the cost of $89\frac{7}{8}$ yd. of cloth, at \$2.70 a yard?

6. A vessel sails 130 miles a day. How far, at the same rate, will she sail in $22\frac{3}{5}$ days?

7. Find the cost of $125\frac{5}{8}$ tons of guano, at \$40 a ton.

8. A farmer raised 36 bu. of corn on an acre of land. At the same rate, how many bushels would $36\frac{3}{4}$ acres yield?

9. How much will $234\frac{11}{60}$ bu. of wheat cost, at \$.85 a bushel?

10. John saves, after paying his board, \$.88 a day. How much will he save in $310\frac{4}{5}$ days?

180. 1. Multiply $\frac{5}{12}$ by $\frac{3}{4}$.

To multiply $\frac{5}{12}$ by $\frac{3}{4}$ is practically the same as to find $\frac{3}{4}$ of $\frac{5}{12}$.

The word "of" placed between fractions signifies the same as the sign of multiplication. Such expressions as $\frac{2}{3}$ of $\frac{4}{5}$ and $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ are generally called compound fractions.

OPERATIONS

$\frac{1}{4}$ of $\frac{5}{12} = \frac{1}{4} \times \frac{5}{12} = \frac{5}{48}$.

$\frac{3}{4}$ of $\frac{5}{12} = 3 \times \frac{5}{48} = \frac{15}{48} = \frac{5}{16}$ *Ans.*

By Cancellation,

$$\frac{5}{\underset{4}{\cancel{12}}} \times \frac{\cancel{3}}{4} = \frac{5}{16} \quad \textit{Ans.}$$

Multiply:

2. $\frac{10}{11}$ by $\frac{3}{5}$
3. $\frac{11}{12}$ by $\frac{6}{7}$
4. $\frac{38}{49}$ by $\frac{7}{19}$
5. $\frac{27}{54}$ by $\frac{8}{9}$
6. $\frac{46}{63}$ by $\frac{7}{23}$
7. $\frac{51}{72}$ by $\frac{12}{17}$
8. $\frac{64}{125}$ by $\frac{15}{84}$
9. $\frac{126}{325}$ by $\frac{120}{267}$
10. $\frac{345}{572}$ by $\frac{286}{1035}$
11. $\frac{427}{634}$ by $\frac{217}{1281}$
12. What is $\frac{7}{8}$ of $\frac{24}{25}$?
13. What is $\frac{9}{11}$ of $\frac{33}{54}$?
14. What is $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$?
15. What is $\frac{4}{5}$ of $\frac{10}{11}$ of $\frac{12}{16}$ of $\frac{25}{36}$?
16. What is $\frac{9}{25}$ of $\frac{45}{36}$ of $\frac{81}{90}$ of $\frac{125}{180}$?

181. 1. Multiply $\frac{10}{21}$ by $3\frac{3}{5}$.

We first multiply $\frac{10}{21}$ by $\frac{3}{5}$, which gives us $\frac{2}{7}$. We next multiply $\frac{10}{21}$ by 3, which gives us $\frac{10}{7}$. Then by adding the partial products $\frac{2}{7}$ and $\frac{10}{7}$, we have $\frac{12}{7}$, or $1\frac{5}{7}$.

Or, we reduce the multiplier $3\frac{3}{5}$ to an improper fraction, and multiply as in Art. 180.

OPERATION I

$$\frac{\overset{2}{\cancel{10}}}{\underset{7}{\cancel{21}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} = \frac{2}{7}$$

$$\frac{10}{\underset{7}{\cancel{21}}} \times \overset{1}{\cancel{3}} = \frac{10}{7}$$

$$\frac{2}{7} + \frac{10}{7} = \frac{12}{7} = 1\frac{5}{7} \quad \textit{Ans.}$$

OPERATION II

$$3\frac{3}{5} = \frac{18}{5}$$

$$\frac{\overset{2}{\cancel{10}}}{\underset{7}{\cancel{21}}} \times \frac{\overset{6}{\cancel{18}}}{\underset{1}{\cancel{5}}} = \frac{12}{7} = 1\frac{5}{7} \quad \textit{Ans.}$$

Multiply:

2. $\frac{8}{15}$ by $12\frac{4}{5}$	5. $\frac{13}{30}$ by $25\frac{8}{13}$	8. $\frac{3}{4}$ of $\frac{8}{27}$ by $13\frac{3}{7}$
3. $\frac{13}{21}$ by $28\frac{5}{7}$	6. $\frac{28}{35}$ by $45\frac{9}{14}$	9. $\frac{4}{5}$ of $\frac{35}{36}$ by $46\frac{5}{7}$
4. $\frac{11}{20}$ by $24\frac{7}{8}$	7. $\frac{49}{51}$ by $53\frac{5}{7}$	10. $\frac{7}{9}$ of $\frac{63}{64}$ by $82\frac{3}{4}$

182. 1. Multiply $6\frac{3}{8}$ by $5\frac{1}{4}$.

We reduce the mixed numbers to improper fractions, and multiply as shown in the operation.

OPERATION

$$6\frac{3}{8} = \frac{51}{8}. \qquad 5\frac{1}{4} = \frac{21}{4}$$

$$\frac{51}{8} \times \frac{21}{4} = \frac{1071}{32} = 33\frac{15}{32} \quad \textit{Ans.}$$

Multiply:

2. $9\frac{3}{7}$ by $8\frac{4}{5}$	5. $19\frac{5}{8}$ by $16\frac{3}{5}$	8. $\frac{2}{3}$ of $18\frac{3}{5}$ by $8\frac{4}{9}$
3. $12\frac{5}{9}$ by $9\frac{3}{7}$	6. $26\frac{7}{9}$ by $18\frac{3}{4}$	9. $\frac{3}{4}$ of $12\frac{4}{5}$ by $9\frac{3}{8}$
4. $18\frac{3}{11}$ by $12\frac{8}{9}$	7. $29\frac{3}{5}$ by $21\frac{2}{3}$	10. $\frac{5}{6}$ of $21\frac{3}{8}$ by $13\frac{3}{7}$

PROBLEMS

183. Find the cost of:

1. 25 cords of wood at $\$7\frac{7}{8}$ a cord.
2. $36\frac{3}{8}$ yards of oil-cloth at $33\frac{1}{3}$ cents a yard.
3. $14\frac{4}{5}$ tons of hay at \$18.50 a ton.
4. 18 barrels of cider at $\$7\frac{3}{5}$ a barrel.
5. $25\frac{4}{5}$ pounds of tea at \$.73 a pound.
6. $30\frac{13}{30}$ bushels of rye at \$.60 a bushel.
7. 50 pairs of shoes at $\$6.37\frac{1}{2}$ a pair.
8. $25\frac{2}{3}$ days' labor at \$.84 a day.
9. $40\frac{3}{4}$ dozen eggs at $\$.22\frac{1}{2}$ a dozen.
10. $56\frac{3}{8}$ pounds of beef at $\$.12\frac{4}{5}$ a pound.
11. $30\frac{1}{2}$ yards of ribbon at $\$.12\frac{1}{2}$ a yard.
12. $42\frac{1}{2}$ pounds of coffee at $\$.27\frac{1}{2}$ a pound.
13. 6 dozen magazines at $\$.33\frac{1}{3}$ each.
14. $51\frac{1}{2}$ yards of braid at $22\frac{1}{2}$ cents a yard.

DIVISION OF FRACTIONS

184. 1. Divide $\frac{12}{13}$ by 6.

Since dividing a quantity by 6 is the same as finding $\frac{1}{6}$ of that quantity, we proceed as in multiplication. By canceling the common factor 6, and multiplying, we get $\frac{2}{13}$, the required quotient.

OPERATION

$$\frac{12}{13} \div 6 = \frac{1}{\cancel{6}} \times \frac{\overset{2}{\cancel{12}}}{13} = \frac{2}{13} \quad \textit{Ans.}$$

Divide:

2. $\frac{15}{16}$ by 5	6. $\frac{21}{25}$ by 7	10. $\frac{81}{129}$ by 108
3. $\frac{16}{17}$ by 8	7. $\frac{15}{16}$ by 6	11. $\frac{8}{9}$ of $\frac{189}{864}$ by 117
4. $\frac{18}{19}$ by 9	8. $\frac{16}{17}$ by 12	12. $\frac{25}{87}$ by 35
5. $\frac{20}{28}$ by 5	9. $\frac{49}{56}$ by 63	13. $\frac{7}{8}$ of $\frac{408}{558}$ by 153

185. 1. Divide $23\frac{4}{5}$ by 14.

We first change $23\frac{4}{5}$ to an improper fraction. Then, as dividing by 14 is practically the same as finding $\frac{1}{14}$ of the quantity, we proceed as in Art. 184.

OPERATION I

$$23\tfrac{4}{5} = \tfrac{119}{5}$$

$$\frac{119}{5} \div 14 = \frac{1}{\underset{2}{\cancel{14}}} \times \frac{\overset{17}{\cancel{119}}}{5} = \frac{17}{10} = 1\tfrac{7}{10} \quad \textit{Ans.}$$

Or, we first divide $23\frac{4}{5}$ by 14, and find the quotient 1 and a remainder of $9\frac{4}{5}$, which, divided by 14 =

$$9\tfrac{4}{5} \div 14 = \frac{49}{5} \div 14 = \frac{1}{\underset{2}{\cancel{14}}} \times \frac{\overset{7}{\cancel{49}}}{5} = \frac{7}{10}.$$

OPERATION II

$$14)23\tfrac{4}{5}(1\tfrac{7}{10} \quad \textit{Ans.}$$
$$\underline{14}$$
$$\frac{9\frac{4}{5}}{14} = \frac{7}{10}$$

Uniting the two partial quotients 1 and $\frac{7}{10}$, we have $1\frac{7}{10}$, the required quotient.

Divide:

2. $4\frac{3}{8}$ by 20	6. $129\frac{3}{7}$ by 43
3. $14\frac{3}{7}$ by 14	7. $246\frac{5}{9}$ by 51
4. $21\frac{6}{7}$ by 17	8. $364\frac{8}{11}$ by 64
5. $42\frac{5}{9}$ by 21	9. $429\frac{7}{9}$ by 72

PROBLEMS

186. 1. If a vessel sails $420\frac{3}{4}$ miles in 24 hours, how far will she sail in 5 hours?

2. A pole 28 feet high casts a shadow $64\frac{3}{10}$ feet long. How long a shadow will a pole 9 feet high cast at the same time of day?

3. A pipe discharges $842\frac{3}{16}$ gallons of water in 10 hours. How much does it discharge in 7 hours?

4. A farmer raised $464\frac{3}{4}$ bushels of wheat on 13 acres. How much is that per acre?

5. If it takes $43\frac{1}{3}$ yards of cloth to make 5 suits of clothes, how many yards will be needed to make 12 suits?

6. A boy picked $15\frac{3}{4}$ quarts of berries in 2 hours. At the same rate, how many quarts would he pick in 5 hours?

7. From a barrel containing $36\frac{1}{2}$ gallons of oil $8\frac{7}{8}$ gallons leaked out. How many gallons remained?

8. A man paid $\$\frac{11}{12}$ for 3 bushels of oats. How much was that a bushel?

187. 1. Divide 56 by $\frac{7}{8}$.

56 divided by 1 equals 56, hence 56 divided by $\frac{1}{8}$ equals 8 times 56, and 56 divided by $\frac{7}{8}$ equals $\frac{1}{7}$ of 8 times 56, or $\frac{8}{7}$ of 56, or $56 \times \frac{8}{7}$, which by cancellation equals

$$\overset{8}{\cancel{56}} \times \frac{8}{\cancel{7}} = 64 \textit{ Ans.}$$

OPERATION

$$56 \div 1 = 56$$
$$56 \div \tfrac{1}{8} = 8 \times 56, \text{ or } 448$$
$$56 \div \tfrac{7}{8} = \tfrac{1}{7} \times 448, \text{ or } 64 \textit{ Ans.}$$

Therefore, to divide by a fraction, we invert the divisor and proceed as in multiplication of fractions.

Divide:

2. 18 by $\frac{3}{4}$	6. 28 by $\frac{21}{25}$	10. 72 by $\frac{22}{61}$
3. 21 by $\frac{7}{8}$	7. 48 by $\frac{20}{29}$	11. 86 by $\frac{41}{56}$
4. 24 by $\frac{6}{7}$	8. 54 by $\frac{26}{31}$	12. 125 by $\frac{25}{73}$
5. 27 by $\frac{12}{13}$	9. 63 by $\frac{27}{47}$	13. 246 by $\frac{82}{95}$

188. 1. Divide 48 by $6\frac{2}{3}$.

We change the divisor, $6\frac{2}{3}$, to an improper fraction, and proceed as in Art. 187.

OPERATION

$$6\tfrac{2}{3} = \tfrac{20}{3}$$

$$48 \div \frac{20}{3} = \overset{12}{\cancel{48}} \times \frac{3}{\underset{5}{\cancel{20}}} = \frac{36}{5} = 7\tfrac{1}{5} \ \textit{Ans.}$$

Divide:

2. 25 by $8\frac{3}{4}$	6. 42 by $13\frac{5}{7}$	10. 75 by $28\frac{3}{8}$
3. 28 by $9\frac{3}{7}$	7. 48 by $15\frac{3}{5}$	11. 88 by $32\frac{4}{5}$
4. 32 by $10\frac{5}{8}$	8. 54 by $18\frac{3}{8}$	12. 125 by $14\frac{3}{7}$
5. 36 by $12\frac{4}{9}$	9. 63 by $21\frac{7}{8}$	13. 428 by $12\frac{4}{9}$

189. 1. Divide $\frac{12}{13}$ by $\frac{6}{7}$.

$\frac{12}{13}$ divided by 1 equals $\frac{12}{13}$. $\frac{12}{13}$ divided by $\frac{1}{7}$ equals 7 times $\frac{12}{13}$, and $\frac{12}{13}$ divided by $\frac{6}{7}$ equals $\frac{1}{6}$ of 7 times $\frac{12}{13}$, or $\frac{7}{6}$ of $\frac{12}{13}$, or $\frac{12}{13} \times \frac{7}{6}$, which by cancellation equals

$$\frac{\overset{2}{\cancel{12}}}{13} \times \frac{7}{\cancel{6}} = \frac{14}{13} = 1\tfrac{1}{13} \ \textit{Ans.}$$

OPERATION

$$\tfrac{12}{13} \div 1 = \tfrac{12}{13}$$

$$\tfrac{12}{13} \div \tfrac{1}{7} = 7 \times \tfrac{12}{13} = \tfrac{84}{13}$$

$$\tfrac{12}{13} \div \tfrac{6}{7} = \tfrac{1}{6} \times \tfrac{84}{13} = \tfrac{14}{13} = 1\tfrac{1}{13} \ \textit{Ans.}$$

Therefore to divide by a fraction we invert the divisor, and proceed as in multiplication of fractions.

Divide:

2. $\frac{14}{15}$ by $\frac{7}{11}$	5. $\frac{36}{49}$ by $\frac{9}{56}$	8. $\frac{65}{84}$ by $\frac{26}{63}$	11. $\frac{96}{125}$ by $\frac{48}{95}$
3. $\frac{18}{20}$ by $\frac{6}{13}$	6. $\frac{32}{45}$ by $\frac{24}{25}$	9. $\frac{72}{88}$ by $\frac{45}{84}$	12. $\frac{92}{184}$ by $\frac{23}{268}$
4. $\frac{25}{36}$ by $\frac{15}{12}$	7. $\frac{54}{63}$ by $\frac{27}{49}$	10. $\frac{84}{95}$ by $\frac{36}{85}$	13. $\frac{110}{268}$ by $\frac{56}{184}$

190. 1. Divide $8\frac{3}{4}$ by $\frac{7}{11}$.

We change the mixed number $8\frac{3}{4}$ to an improper fraction, and proceed as in Art. 189.

OPERATION

$$8\tfrac{3}{4} = \tfrac{35}{4}$$

$$\frac{35}{4} \div \frac{7}{11} = \frac{\overset{5}{\cancel{35}}}{4} \times \frac{11}{\cancel{7}} = \frac{55}{4} = 13\tfrac{3}{4} \ \textit{Ans.}$$

Divide:

2. $6\frac{3}{5}$ by $\frac{11}{12}$	5. $9\frac{7}{9}$ by $\frac{22}{27}$	8. $42\frac{5}{6}$ by $\frac{7}{8}$
3. $7\frac{4}{5}$ by $\frac{13}{14}$	6. $33\frac{1}{3}$ by $\frac{20}{33}$	9. $48\frac{3}{4}$ by $\frac{4}{9}$
4. $8\frac{3}{4}$ by $\frac{15}{16}$	7. $40\frac{2}{3}$ by $\frac{61}{93}$	10. $54\frac{3}{5}$ by $\frac{2}{3}$ of $\frac{3}{4}$

191. 1. Divide $9\frac{3}{5}$ by $4\frac{2}{3}$.

OPERATION

$$9\tfrac{3}{5} = \tfrac{48}{5}$$

$$4\tfrac{2}{3} = \tfrac{14}{3}$$

$$\frac{48}{5} \div \frac{14}{3} = \frac{\overset{24}{\cancel{48}}}{5} \times \frac{3}{\underset{7}{\cancel{14}}} = \frac{72}{35} = 2\tfrac{2}{35} \quad \textit{Ans.}$$

We change the mixed numbers $9\frac{3}{5}$ and $4\frac{2}{3}$ to improper fractions, invert the divisor, and proceed as in multiplication.

Divide:

2. $9\frac{3}{4}$ by $2\frac{1}{8}$	7. $8\frac{19}{24}$ by $14\frac{2}{3}$	12. $30\frac{1}{2}$ by $14\frac{2}{3}$
3. $16\frac{5}{8}$ by $8\frac{1}{3}$	8. $10\frac{9}{10}$ by $6\frac{5}{8}$	13. $24\frac{9}{10}$ by $8\frac{3}{5}$
4. $12\frac{2}{3}$ by $8\frac{3}{5}$	9. $22\frac{1}{4}$ by $3\frac{1}{5}$	14. $21\frac{4}{7}$ by $\frac{4}{7}$ of 8
5. $14\frac{1}{2}$ by $15\frac{2}{3}$	10. $18\frac{1}{2}$ by $7\frac{1}{8}$	15. $82\frac{3}{5}$ by $\frac{3}{5}$ of 61
6. $18\frac{3}{16}$ by $2\frac{3}{4}$	11. $14\frac{1}{9}$ by $9\frac{2}{7}$	16. $95\frac{1}{2}$ by $\frac{3}{4}$ of 75

192. 1. Reduce $\dfrac{\frac{2}{3} \text{ of } 6}{2 \times \frac{3}{5}}$ to its simplest form.

The above fractional expression, generally called a *Complex Fraction*, is read $\frac{2}{3}$ of 6, divided by 2 multiplied by $\frac{3}{5}$.

OPERATION

$$\frac{\frac{2}{3} \text{ of } 6}{2 \times \frac{3}{5}} = \frac{4}{\frac{6}{5}} = 4 \div \frac{6}{5} = \overset{2}{\cancel{4}} \times \frac{5}{\underset{3}{\cancel{6}}} = \frac{10}{3} = 3\tfrac{1}{3} \quad \textit{Ans.}$$

2. Reduce $\dfrac{\frac{3}{4} + 5\frac{1}{2}}{8 - 2\frac{1}{8}}$ to its simplest form.

OPERATION

$$\frac{\frac{3}{4} + 5\frac{1}{2}}{8 - 2\frac{1}{8}} = \frac{6\frac{1}{4}}{5\frac{7}{8}} = \frac{\frac{25}{4}}{\frac{47}{8}} = \frac{25}{4} \div \frac{47}{8} = \frac{25}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{47} = \frac{50}{47} = 1\tfrac{3}{47} \quad \textit{Ans.}$$

Reduce to their simplest form:

3. $\dfrac{4\frac{1}{2} \times \frac{3}{4}}{\frac{7}{8} \text{ of } \frac{9}{10}}$ **4.** $\dfrac{12 - 3\frac{2}{3}}{8 + \frac{1}{3}}$ **5.** $\dfrac{1 \div \frac{1}{4}}{1 \times \frac{1}{4}}$ **6.** $\dfrac{3\frac{1}{2} + 4\frac{1}{6}}{\frac{2}{3} \times \frac{3}{4} \times \frac{5}{6}}$ **7.** $\dfrac{2\frac{2}{3} \times \frac{3}{4} \text{ of } \frac{5}{6}}{7\frac{1}{2} - \frac{5}{6} \text{ of } 8}$

REVIEW PROBLEMS

193. 1. If 21 acres of land yield $735\frac{3}{5}$ bushels of corn, how many bushels will 40 acres yield at the same rate?

2. A boat sails $184\frac{3}{4}$ miles in 6 hours. How far does it go in $\frac{2}{3}$ of an hour?

3. A owned $\frac{2}{3}$ of 160 acres of land and sold $\frac{3}{4}$ of what he owned. How many acres had he left?

4. I bought a tract of land consisting of $212\frac{1}{2}$ acres, at \$$75\frac{3}{4}$ an acre. If I sell $\frac{2}{3}$ of it at \$$70\frac{1}{4}$ an acre, and the remainder at cost, what will be my loss?

5. A man bought 90 head of cattle at \$$45\frac{3}{5}$ each, and 126 head at \$$44\frac{1}{7}$ each. What must be the average selling price per head to yield a gain of \$1242?

6. William has \$$612\frac{3}{4}$, Henry has \$$125\frac{2}{3}$ more than William, and John has \$$26\frac{3}{5}$ less than both. How much money have they together?

7. If 5 is added to both terms of the fraction $\frac{30}{35}$, will the value of the fraction be increased or diminished? How much?

8. If 5 is subtracted from both terms of the fraction $\frac{30}{35}$, will the value of the fraction be increased or diminished? How much?

9. Will the value of $\frac{30}{35}$ be increased or diminished if both terms are divided by 5?

10. What effect will multiplying both terms of the fraction $\frac{30}{35}$ by 5, have on its value?

11. The product of 3 numbers is 14,848. Two of the numbers are $21\frac{3}{4}$ and $25\frac{3}{5}$. What is the third number?

12. The dividend is $26\frac{5}{9}$, the quotient $48\frac{17}{18}$. What is the divisor?

13. The distance from Wilkesbarre to New York is 162 miles. If a passenger train running at the rate of $32\frac{2}{5}$ miles an hour leaves Wilkesbarre at 9 A.M., and a freight train running at the rate of $21\frac{3}{5}$ miles an hour leaves New York at the same time, when the passenger train arrives at New York, how far will the freight train be from Wilkesbarre?

14. Find the amount of the following bill:

CHICAGO, ILL., Jan. 29, 1898.

C. B. WHITNEY,

Bought of WILLIAM STODDART AND CO.

126 lb. Black Tea,	@	$\$.82\frac{1}{2}$		
$128\frac{5}{16}$ lb. Sugar,	"	$.05\frac{3}{4}$		
$72\frac{5}{8}$ lb. Coffee,	"	$.32\frac{1}{4}$		
$142\frac{3}{8}$ lb. Rope,	"	$.15\frac{3}{8}$		
$132\frac{15}{16}$ lb. Ham,	"	$.11\frac{3}{4}$		
$49\frac{1}{2}$ lb. Raisins,	"	$.16\frac{1}{2}$		
50 Seamless Bags,	"	$.31\frac{5}{8}$		
$29\frac{3}{8}$ yd. Calico,	"	$.09\frac{1}{4}$		
Amt. of Bill,				

RELATION OF NUMBERS

194. The relation of one number to another is ascertained by division. When we wish to find *what part* a smaller number is of a larger number, we divide the smaller number by the larger. What part of 12 is 4? $4 \div 12 = \frac{4}{12}$, or $\frac{1}{3}$. Therefore 4 is $\frac{1}{3}$ of 12.

But when we wish to find how *many times* a smaller number equals a larger, we divide the larger by the smaller. 12 is how many times 4? $12 \div 4 = 3$. Therefore 12 is 3 times 4.

195. 1. What part of 16 is 4?

$4 \div 16 = \frac{4}{16} = \frac{1}{4}$. Therefore 4 is $\frac{1}{4}$ of 16.

What part of:

2. 48 is 18?	5. 158 is 52?	8. 240 is 84?
3. 108 is 42?	6. 162 is 54?	9. 273 is 104?
4. 123 is 36?	7. 216 is 63?	10. 294 is 112?

11. 40 is how many times 8? How many times 10?

12. 56 is how many times 7? How many times 28?

13. 84 is how many times 8? How many times 24?

14. 126 is how many times 84? How many times 105?

15. 217 is how many times 124? How many times 155?

16. How much will 120 steers cost, if 15 cost $525?

17. If 39 pocket knives cost $33.15, how much will 26 cost?

18. From a barrel of sirup containing 45 gallons, there were sold 20 gallons. What part of the whole barrel remained?

19. If 56 buggies cost $4200, find the cost of 84.

196. 1. What part of 70 is $\frac{7}{16}$?

$$\frac{7}{16} \div 70 = \frac{\cancel{7}}{16} \times \frac{1}{\underset{10}{\cancel{70}}} = \frac{1}{160}.$$ Therefore $\frac{7}{16}$ is $\frac{1}{160}$ of 70.

What part of:

2. 84 is $\frac{56}{57}$?	4. 125 is $\frac{25}{26}$?	6. 256 is $6\frac{2}{7}$?
3. 100 is $\frac{5}{9}$?	5. 185 is $\frac{50}{51}$?	7. 288 is $38\frac{2}{5}$?

8. 144 is how many times $\frac{16}{25}$? How many times $\frac{32}{35}$?

9. 160 is how many times $26\frac{2}{3}$? How many times $33\frac{1}{3}$?

10. If 8 gallons of oil cost 80 cents, how much will $\frac{4}{5}$ of a gallon cost?

SUGGESTION. $\frac{4}{5}$ is what part of 8?

11. If $5\frac{1}{2}$ yards of muslin cost $$1\frac{1}{4}$, how much will 15 yards cost?

197. 1. What part of $\frac{9}{10}$ is $\frac{3}{5}$?

$$\frac{3}{5} \div \frac{9}{10} = \frac{\cancel{3}}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}. \quad \text{Therefore } \tfrac{3}{5} \text{ is } \tfrac{2}{3} \text{ of } \tfrac{9}{10}.$$

What part of:

2. $\frac{9}{10}$ is $\frac{3}{10}$?
3. $\frac{35}{54}$ is $\frac{7}{8}$?
4. $\frac{34}{63}$ is $\frac{17}{21}$?
5. $3\frac{1}{2}$ is $\frac{5}{6}$?
6. $18\frac{3}{4}$ is $\frac{25}{81}$?
7. $22\frac{2}{5}$ is $\frac{56}{79}$?
8. $21\frac{1}{4}$ is $5\frac{5}{6}$?
9. $44\frac{2}{3}$ is $\frac{3}{4}$ of $16\frac{4}{5}$?
10. $86\frac{3}{7}$ is $\frac{2}{3}$ of $27\frac{9}{11}$?

11. $\frac{28}{35}$ is how many times $\frac{2}{7}$? How many times $\frac{14}{45}$?

12. A bought $\frac{5}{6}$ of a cargo of bananas, and sold $\frac{3}{4}$ of them. What part of the cargo did he sell?

REVIEW PROBLEMS

198. 1. Sarah bought 10 yards of silk, and used $6\frac{2}{3}$ yards. What part of the silk then remained?

2. John owns 20 sheep, and William owns $\frac{3}{5}$ as many plus 3 sheep. What part of John's number of sheep equals William's number?

3. A laborer earns \$ $12\frac{1}{2}$ in 10 days. How much can he earn in 150 days at the same rate?

4. If 12 men can do a piece of work in $13\frac{1}{3}$ days, how long will it take 20 men to do it?

5. A boy shared 9 apples with his companions, giving to each $\frac{3}{4}$ of an apple. Find the number of companions.

6. If 7 men can mow a field of grass in $4\frac{2}{7}$ days, how long will it take 5 men to mow the field?

7. If \$ 96 will buy 120 bushels of wheat, how many bushels will \$ 84 buy?

8. There are 128 cubic feet in a cord of wood. What part of $2\frac{2}{3}$ cords is $12\frac{3}{4}$ cubic feet?

9. If $\frac{8}{13}$ of 2 times the value of a pile of lumber is \$2368, what is the value of the pile?

10. $1609\frac{2}{3}$ feet equals $\frac{11}{12}$ of the distance around a square lot. What is the distance around the lot in rods? ($16\frac{1}{2}$ feet = 1 rod.)

11. If 9 barrels of apples cost \$21.60, how much will 23 barrels cost?

12. How much hay will $25\frac{1}{2}$ acres yield, if 16 acres yield $53\frac{1}{3}$ tons?

13. How many bushels of wheat will make 4650 pounds of flour, if 60 pounds of wheat will make $48\frac{7}{16}$ pounds of flour?

14. If 28 bales of hay last 18 horses 12 days, how many bales will be required for 35 horses for the same time?

15. If a pole 45 feet high casts a shadow $73\frac{3}{4}$ feet long, how long a shadow will a pole 36 feet high cast?

16. What must be paid for 25 boxes of oranges when $\frac{2}{3}$ of a box costs $\$1\frac{5}{6}$?

17. \$28 is $\frac{5}{6}$ of what I paid for a sleigh, and 4 times its cost is what I paid for a horse. How much did the horse cost me?

18. If from $\frac{3}{32}$ of an acre of tobacco I realize \$80, how much should I realize from 4 acres?

19. Mr. Wilson mowed 12 acres of grass in $3\frac{1}{2}$ days. How many acres can he mow in 12 days?

20. A passenger train ran $83\frac{1}{3}$ miles in $3\frac{1}{3}$ hours. At the same rate, how far would it go in 8 hours?

21. If $10\frac{4}{5}$ miles equal $\frac{2}{3}$ of the distance from Kingston to Shickshinny, and 4 times the distance to Shickshinny equals the distance from Kingston to Northumberland, what is the distance to Northumberland?

22. If \$ $25\frac{1}{20}$ equals $\frac{7}{8}$ of my gain on 10 pigs, what is the average gain on each pig?

23. Find the cost of 15 yards of cloth if $5\frac{5}{8}$ yards cost \$ $20\frac{5}{8}$.

24. $\frac{3}{4}$ of 22 yards of silk is $\frac{11}{12}$ of the number of yards required for a dress. Find the number of yards of silk in the dress.

25. If \$ $46\frac{7}{8}$ is $\frac{5}{16}$ of the price I pay for a buggy, what would be the cost of 8 buggies at the same rate?

26. How far will a man walk in $4\frac{1}{8}$ hours, at the rate of 14 miles in $3\frac{1}{2}$ hours?

27. A man exchanged $20\frac{1}{2}$ pounds of butter, at 30 cents a pound, for muslin worth $6\frac{1}{4}$ cents a yard. How many yards did he receive?

28. A farmer exchanged $10\frac{3}{4}$ tons of hay for $57\frac{1}{8}$ tons of coal. At the same rate how many tons of coal would he get in exchange for $14\frac{1}{2}$ tons of hay?

29. How many bushels of wheat will \$ $512\frac{3}{4}$ buy when \$ $74\frac{1}{4}$ will buy 77 bushels?

30. A pole $30\frac{3}{4}$ feet high casts a shadow $46\frac{1}{8}$ feet long. Find the length of a pole that will cast a shadow $23\frac{1}{16}$ feet long at the same time of day.

MISCELLANEOUS PROBLEMS

199. 1. A boy spent $\frac{1}{2}$ of his money for a top, and $\frac{1}{3}$ of the remainder for a knife. What part of his money had he left?

2. If I pay $33\frac{1}{3}$ cents for a knife, what fraction of a dollar do I pay?

3. Mary sold 3 eggs more than $\frac{2}{3}$ of a dozen, and had $\frac{1}{3}$ of a dozen left. How many eggs had she before she sold any?

4. If 12 men can do a piece of work in 5 days, what part of the work can they do in 1 day? What part in 2 days? In 3 days? In 4 days?

5. How long will it take 1 man to do what 12 men will do in 5 days?

6. If John can make a door in 5 hours, what part of the work can he do in 1 hour? In 3 hours?

7. If John can make $\frac{3}{5}$ of a door in 3 hours, how long will it take him to make the whole door?

8. Walter Johnson can make a gate in $\frac{3}{5}$ of a day. How many gates can he make in 6 days?

9. If $\frac{9}{10}$ of a ton of coal is worth \$3, how much are 3 tons worth?

10. A capitalist invested $\frac{1}{3}$ of his money in real estate; $\frac{1}{3}$ of the remainder in railroad stock; and $\frac{1}{2}$ of what still remained in city bonds. How much less than $\frac{1}{2}$ of his money was not invested?

11. If $\frac{2}{3}$ of a yard of lace costs 10 cents, what fraction of a dollar will 1 yard cost?

12. If $\frac{2}{3}$ of a barrel of flour costs $\frac{1}{2}$ of \$7, how much is that a barrel?

13. If \$90 is added to $\frac{3}{4}$ of \$120 the sum will equal $\frac{1}{2}$ of what a gentleman paid for board for 1 year, or 12 months. How much did he pay a month for his board?

14. If I can do a piece of work in 20 days, how much of the work can I do in $3\frac{1}{2}$ days?

15. In a school, $\frac{1}{6}$ of the pupils read in the first reader, $\frac{1}{8}$ in the second reader, $\frac{1}{4}$ in the third reader, $\frac{1}{8}$ in the fourth reader, and the remainder, which is 80, read in the fifth reader. How many pupils are there in the school?

16. A man divided a certain sum of money among his three sons. To the eldest he gave $\frac{2}{5}$ of the sum; to the

second, $\frac{3}{5}$ of the remainder; and to the third, the rest, which was \$60. How much money was divided, and what was each son's share?

17. A can dig an acre of potatoes in 4 days, and D in 5 days. What part of an acre can each dig in one day? How long will it take both working together to dig 1 acre? 2 acres?

18. If it takes 4 men $6\frac{2}{3}$ days to build a fence, how long will it take 12 men to build $\frac{1}{2}$ of the fence?

19. If $\frac{3}{4}$ of my weight is increased by $12\frac{1}{2}$ pounds it will equal 125 pounds. What is my weight?

20. Five times a certain fraction, minus $2\frac{1}{3}$, equals $\frac{2}{3}$. What is the fraction?

21. William's overcoat cost \$50, and $\frac{2}{3}$ of its cost is \$5 more than $3\frac{1}{3}$ times the cost of his shoes. How much did his shoes cost?

22. If 8 men build a shed in $3\frac{3}{4}$ days, how long will it require 4 men to build 4 similar sheds?

23. A man invested $\frac{1}{4}$ of his money in a house and lot, $\frac{1}{4}$ of the remainder in the lumber business, and $\frac{1}{4}$ of what still remained in coal lands, and still had left \$2700. How much money had he at first?

24. Sallie has 15 peaches more than Hattie, and $\frac{2}{3}$ of Sallie's number equals Hattie's number. How many peaches has each?

25. If George can dig $\frac{3}{5}$ of a ditch in a day, how long will it take him to do $\frac{1}{5}$ of the work? $\frac{4}{5}$ of the work? $\frac{5}{6}$ of the work?

26. I spent $\frac{1}{4}$ of my money on Monday, $\frac{1}{3}$ of the remainder on Tuesday, and $\frac{1}{2}$ of what then remained on Wednesday. On Thursday I earned $\frac{1}{3}$ as much as I spent on Tuesday and Wednesday together, and then had 15 cents. How much money had I at first?

27. $\frac{1}{3}$ of the difference between two numbers is 12. If $\frac{3}{4}$ of their difference is added to $\frac{2}{5}$ of the larger number, the sum will equal the larger number. Find the two numbers.

28. $\frac{2}{3}$ of the difference between two numbers is 8, and $\frac{1}{2}$ of the larger number equals $\frac{2}{3}$ of the smaller. What are the numbers?

29. I sold my sleigh to Mr. Randall for $1\frac{1}{4}$ times what it cost me. Mr. Randall sold it to Mr. Detrick for $ 32, which was $\frac{1}{5}$ less than he paid for it. How much did the sleigh cost me?

30. The sum of $\frac{1}{2}$ and $\frac{1}{3}$ of a number is 5 more than $\frac{3}{4}$ of the number. What is the number?

31. A owns $32\frac{2}{3}$ acres of land, B $45\frac{1}{2}$ acres, and C owns $\frac{1}{2}$ as many acres as A and B together. How many acres does C own, and how many acres have they all?

32. In the preceding example, if A should sell $\frac{1}{3}$ of his land at $ $33\frac{1}{3}$ an acre, and the remainder at $ 35 an acre, how much would he receive for it?

33. There are $16\frac{1}{2}$ feet in 1 rod. How many rods are there in $255\frac{3}{4}$ feet?

34. There are $5\frac{1}{2}$ yards in 1 rod. How many rods are there in $173\frac{1}{4}$ yards?

35. Wells & Co. bought a bill of goods amounting to $ $93\frac{3}{25}$, and another amounting to $ $122\frac{11}{20}$. They gave in payment 4 fifty-dollar bills and a twenty-dollar bill. How much change should they receive?

36. A and B start from the same place and travel in the same direction. A travels at the rate of $165\frac{3}{5}$ mi. in $4\frac{3}{5}$ da., and B $225\frac{3}{4}$ mi. in $5\frac{3}{8}$ da. How far apart will they be at the end of 18 da.?

37. A cuts $3\frac{3}{4}$ acres of grass while B cuts $3\frac{3}{10}$ acres. How many acres will B have finished when A has cut $13\frac{3}{4}$ acres?

38. Hartman and Fissel built a barn. Hartman did $\frac{14}{25}$ of the work, and received \$ $310\frac{1}{2}$ more than Fissel. How much did the baru cost, and what was the share of each?

39. Three men, A, B, and C, built a mill. A furnished $\frac{3}{7}$ of the money, B $\frac{3}{8}$, and C the remainder. C's share of the gain the first year was \$ $247\frac{1}{2}$. Find the gain of A and B respectively.

40. How many times will a carriage wheel $9\frac{5}{12}$ feet in circumference turn in going $3\frac{1}{2}$ miles, there being 5280 feet in a mile?

41. If a man walks $51\frac{3}{4}$ miles in $13\frac{1}{2}$ hours, how many hours will he require to walk $973\frac{7}{8}$ miles?

42. A ship sails $18\frac{3}{5}$ miles in a day, or 24 hours. At the same rate how far will it sail in $10\frac{1}{2}$ hours.

43. Out of a chest of tea containing 72 pounds, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ were sold. What part of the whole chest remained?

44. What is the cost of $13\frac{1}{5}$ pounds of tea at the rate of $19\frac{1}{3}$ pounds for \$ $17.59\frac{1}{3}$?

45. A man sold $\frac{3}{7}$ of his sheep, and $\frac{4}{5}$ of the remainder were killed by dogs. If he then had 24 sheep left, how many had he at first?

46. A farmer sold 72 sheep from one field, which was $\frac{3}{4}$ of the number in the field at first, and $\frac{6}{25}$ of the number in another field. How many sheep must be taken from the second field, and put into the first, so that the number in each field may be equal?

47. Jacob Read sold $\frac{3}{5}$ of his potatoes, and then bought $\frac{2}{3}$ as many as he had remaining, and then had $113\frac{1}{3}$ bushels less than he had at first. How many bushels had he at first?

48. If $12\frac{1}{2}$ tons of iron cost \$ 190, how much will $\frac{1}{5}$ of $172\frac{1}{2}$ tons cost?

49. What number diminished by $\frac{7}{15}$ of itself equals $321\frac{7}{9}$?

50. Twice a certain number, plus $\frac{1}{4}$ of $1\frac{3}{5}$ times the same number, equals $1553\frac{1}{7}$. What is the number?

51. William Toomb gathered 1813 bushels of apples from two orchards. $\frac{2}{9}$ of what he gathered from the larger orchard equaled $\frac{3}{5}$ of what he gathered from the smaller. How many bushels were gathered from each?

52. A man commenced business with a capital of $ 74,800. At the end of two years he found he had gained $\frac{2}{5}$ of his capital. Find each year's gain if his gain the second year was $1\frac{1}{2}$ times what it was the first year.

53. A man bought $25\frac{1}{4}$ yards of cloth at $ $2\frac{2}{5}$ a yard. How many bushels of rye at $ $\frac{4}{5}$ a bushel will be required to pay for it?

54. Two men travel from the same point in opposite directions. One travels at the rate of $2\frac{3}{4}$ miles an hour, and the other at the rate of $3\frac{2}{3}$ miles. How far will they be apart at the end of 3 days of $9\frac{3}{5}$ hours each?

55. What number, increased by its $\frac{2}{3}$, equals 60 more than 840?

56. A carpenter built a shed in $17\frac{1}{2}$ days of $8\frac{2}{5}$ hours each. Had he worked $9\frac{1}{2}$ hours a day, how many days would he have required to do the work?

57. By selling apples at $ 2.80 a barrel, I gain $\frac{3}{11}$ of their cost. At what price per barrel must I sell them to gain $\frac{2}{5}$ of their cost?

58. A man can plant a field of corn in 14 hours, and his son can do the same work in 25 hours. In what time can they do the work together?

59. If a boy can do $\frac{36}{225}$ of a piece of work in a day, how long will it take him to do $\frac{1}{2}$ of it?

60. A teacher spent $\frac{1}{3}$ of his salary for board and clothes, $\frac{1}{6}$ of it for books, $\frac{1}{22}$ of it for pleasure and incidentals, and the remainder he deposited in bank. What part of his money did he deposit in bank?

61. John can shoe 4 horses in $\frac{3}{4}$ of a day, and Henry can shoe them in $\frac{4}{5}$ of a day. In what time can they shoe them working together?

62. A grain dealer received \$49 for 42 bushels of corn and 35 bushels of rye. What was the selling price per bushel, if he received 16 cents a bushel more for the rye than for the corn?

63. A can build $\frac{1}{4}$ of a certain wall in 8 days. B can build $\frac{1}{3}$ of the same wall in 10 days. How long will it take them to build the wall working together?

64. If I buy potatoes at the rate of 8 bushels for \$$3\frac{3}{5}$, and sell them at the rate of 9 bushels for \$4.50, how many bushels must I sell to gain \$10.50?

65. A man agreed to do a job of work in 15 days. What part of it ought he to do in $8\frac{1}{4}$ days?

66. John can do a piece of work in 10 days, Charlie can do it in 12 days, and William in 8 days. In what time can they do it working together?

67. If a man travels 8 miles in $\frac{3}{5}$ of an hour, how far could he travel in $1\frac{1}{2}$ hours?

68. John lost $\frac{1}{2}$ of his kite string, and then added 40 feet. It was then $\frac{3}{5}$ of its original length. Find its length at first.

69. James and John are $32\frac{3}{8}$ miles apart, and travel toward each other. When they meet John has traveled $2\frac{3}{4}$ miles more than James. How far has each traveled?

70. A dealer bought a lot of tinware for $\frac{19}{16}$ of its value, and sold it for $\frac{11}{12}$ of its value. If his loss was \$$32\frac{1}{2}$, how much did he pay for it?

USE OF SIGNS

200. The *Parenthesis* ( ), the *Brackets* [ ], the *Braces* { }, and the *Vinculum* $\overline{}$ are called *Signs of Aggregation,* and are used to show that all numbers inclosed are to be considered together as one number. Thus, $24 \div (9-3)$, $24 \div [9-3]$, $24 \div \{9-3\}$, and $24 \div \overline{9-3}$ all mean that 24 is to be divided by $9-3$, or 6.

The signs + and − always indicate points of separation, and the parts between these signs are *terms.* The expression $25 - 3 \times 5 + 14 \div 7$ consists of three terms, 25, 3×5, and $14 \div 7$. Each term should be simplified before the operations of addition and subtraction are performed. Hence the value of the above expression is $25 - 15 + 2 = 12$.

Operations of multiplication and division are to be performed in the order of their occurrence, from left to right.

Thus, $40 \div 8 \times 4 = 20$
$40 \times 8 \div 4 = 80$
$40 \div 4 \div 2 = 5$

In writing a series of arithmetical operations it is best to employ signs of aggregation to avoid uncertainty as to the order of the operations.

For example, $64 \div 4 \times 2 + 36 \div 3 - 2 \times 4$ is better written, $64 \div (4 \times 2) + (36 \div 3) - (2 \times 4)$.

In order to simplify when there is more than one sign of aggregation employed, it will be found most convenient to remove the innermost one first.

Thus $\overline{(16 \times 4) \div 8} \div 2 = \overline{64 \div 8} \div 2 = 8 \div 2 = 4,$

and $28 \div [7 - (3 + 2)] = 28 \div [7 - 5] = 28 \div 2 = 14.$

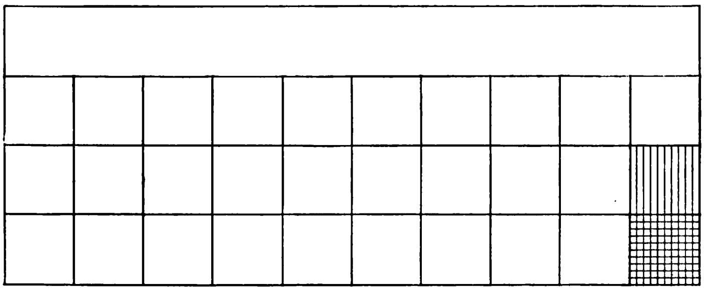
EXAMPLES

201. Find the value of:

1. $(14 \times 2) + (16 \div 4) - 8$
2. $8 \times 6 - (9 + 13)$
3. $26 \times (9 \div 3) - 21$
4. $8 \times (9 + 6) - (6 \times 8)$
5. $(4 + 5) \times 20 - \overline{9 \times 6}$
6. $64 - [(6 \times 4) - 10]$
7. $(8 + 9) \times 6 - \overline{8 - 5}$
8. $34 + \{9 \times \overline{6 - 3}\}$
9. $46 - \overline{18 - (4 \times 3)}$
10. $56 \div [7 - (3 + 2)]$
11. $(7 \times \overline{14 - 4}) - \overline{4 \times (16 - 12)}$
12. $(\frac{9}{10} + \frac{7}{10}) \times (\frac{1}{2} + 1\frac{1}{10})$
13. $\overline{14 \times (6 + 18)} - \overline{5 \times (24 - 16\frac{1}{2})}$
14. $36 + \{34 - 4 - (30 - 8)\}$
15. $\overline{16 \times (3\frac{2}{5} - 2\frac{1}{5})} \times (2\frac{3}{5} - \frac{3}{25})$
16. $(56 \div 7) - \overline{3 + 2}$
17. $56 \div (7 - 3) + 2$
18. $56 \div [7 - (3 + 2)]$
19. $10 \times [26 + 4 \times (3 + 4) - 10]$
20. $\overline{(6\frac{1}{4} + 5\frac{1}{2}) \div (6\frac{1}{4} - 5\frac{1}{2})} + \overline{(6\frac{1}{4} \times 5\frac{1}{2}) \div (6\frac{1}{4} + 5\frac{1}{2})}$
21. $\left(\dfrac{\frac{1}{3} + \frac{8}{10}}{6\frac{1}{3}} + \dfrac{5\frac{5}{7}}{4\frac{4}{9}}\right) \div \left(11 \times \dfrac{15\frac{2}{5}}{8\frac{1}{12}}\right)$
22. $\overline{(1 - \frac{1}{5}) \div 2} \times \overline{(2 - \frac{1}{5}) \div 3} \times \overline{(3 - \frac{1}{5}) \div 4}$
23. $\overline{(\frac{4}{7} \text{ of } \frac{3}{8}) \div (5\frac{1}{14} - 4\frac{51}{56})} - \overline{(\frac{5}{11} \text{ of } \frac{13}{15}) \div (7\frac{15}{22} - 6\frac{1}{2})}$

DECIMAL FRACTIONS

NOTATION AND NUMERATION



202. If a unit is divided into 10 equal parts, *one* of these parts is called $\frac{1}{10}$ of a unit.

If $\frac{1}{10}$ of a unit is divided into 10 equal parts, *one* part is $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$ of a unit.

If $\frac{1}{100}$ of a unit is divided into 10 equal parts, *one* part is $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$ of a unit, and so on.

From these continuous tenth or decimal divisions of the unit, there arises a class of fractions called *Decimal Fractions.*

203. A *Decimal Fraction,* therefore, may be defined as a number of the decimal divisions of a unit; as $\frac{1}{10}$, $\frac{4}{100}$, $\frac{5}{1000}$, $\frac{728}{100000}$, $\frac{45}{1000000}$, $\frac{1248}{10000000}$.

204. The denominator of a decimal fraction is always 10, or the product arising from using 10 two or more times as a factor. That is, it is always 10, 100, 1000, 10,000, 100,000, 1,000,000, etc.

205. By the use of the decimal point (.) decimal fractions may be expressed without writing the denominator. They are then usually called *Decimals.* Thus, $\frac{5}{10}$ written as a decimal becomes .5; $\frac{9}{10}$ becomes .9; $\frac{6}{100}$ becomes .06; $\frac{253}{1000}$ becomes .253.

206. As the decimal system of fractions is but an extension of the system for integers continued below the unit, the uniform law of increase and decrease in a tenfold ratio is continued through the decimal divisions of the unit also. That is, *one* of any order equals $\frac{1}{10}$ of the value of *one* of the next order to the left, and 10 times the value of *one* of the next order to the right. Thus, $\frac{1}{10}$ equals *one tenth* of 1, and *ten* times $\frac{1}{100}$; $\frac{1}{100}$ equals *one tenth* of $\frac{1}{10}$, and *ten* times $\frac{1}{1000}$; $\frac{1}{1000}$ equals *one tenth* of $\frac{1}{100}$, and *ten* times $\frac{1}{10000}$, etc.

207. The following shows the extension of the system for integers continued below the unit, by the uniform scale of 10, to millionths:

Decimal Fractions,—

$$1,\ \frac{1}{10},\ \frac{1}{100},\ \frac{1}{1000},\ \frac{1}{10000},\ \frac{1}{100000},\ \frac{1}{1000000}.$$

Decimals, —

1, .1, .01, .001, .0001, .00001, .000001.

208. By examining the following table the student will perceive that every decimal is composed of as many *decimal places* as there are ciphers in the denominator of the corresponding decimal fraction; that the first decimal place to the right of the decimal point is tenths; the second place, hundredths; the third place, thousandths; the fourth, ten-thousandths, etc.

DECIMAL FRACTIONS	DECIMALS	HOW READ
$\frac{5}{10}$	.5	5 tenths
$\frac{9}{10}$	.9	9 tenths
$\frac{8}{100}$	.08	8 hundredths
$\frac{95}{100}$	.95	95 hundredths
$\frac{7}{1000}$	.007	7 thousandths
$\frac{84}{1000}$	.084	84 thousandths
$\frac{327}{1000}$	.327	327 thousandths
$\frac{6}{10000}$	.0006	6 ten-thousandths
$\frac{128}{10000}$	.0128	128 ten-thousandths
$\frac{1536}{10000}$	.1536	1536 ten-thousandths
$\frac{306}{100000}$	.00306	306 hundred-thousandths
$\frac{8006}{1000000}$	.008006	8006 millionths

209. Express the following decimal fractions in decimal form:

1.	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$
2.	$\frac{2}{100}$	$\frac{8}{100}$	$\frac{25}{100}$	$\frac{32}{100}$	$\frac{89}{100}$
3.	$\frac{8}{1000}$	$\frac{19}{1000}$	$\frac{86}{1000}$	$\frac{126}{1000}$	$\frac{496}{1000}$
4.	$\frac{6}{10000}$	$\frac{98}{10000}$	$\frac{267}{10000}$	$\frac{938}{10000}$	$\frac{1689}{10000}$
5.	$\frac{5}{100000}$	$\frac{72}{100000}$	$\frac{896}{100000}$	$\frac{5689}{100000}$	$\frac{72896}{100000}$
6.	$\frac{7}{1000000}$	$\frac{65}{1000000}$	$\frac{389}{1000000}$	$\frac{5968}{1000000}$	$\frac{98645}{1000000}$

210. The denominator of every decimal fraction is 1, followed by as many ciphers as there are decimal places in the decimal.

211. Write the following decimals in fractional form:

1.	.3	.4	.07	.22	.98
2.	.005	.063	.198	.105	.896
3.	.0007	.0067	.0159	.1061	.9006
4.	.00005	.00056	.00906	.01601	.96019
5.	.000009	.000036	.000106	.001709	.015657

212. In expressing decimals orally, a short pause should be made immediately after reading the numerator when there might be a doubt as to the meaning. Thus, .500 should be read five hundred —— thousandths, in order to distinguish it from .00005, which should be read five —— hundred-thousandths.

213. Read the following decimals:

1.	2.	3.
.5	.8006	.003896
.05	.00007	.004007
.16	.00039	.709038
.007	.00604	.040709
.049	.09301	.601601
.389	.19601	.700386
.0006	.000009	.600101
.0049	.000068	.100016
.0467	.000704	.393906

214. When a mixed number is expressed by the method of decimal notation, the decimal point separates the whole number from the decimal, and indicates the value of the decimal. Thus, $8\frac{9}{10}$ expressed decimally is 8.9; $17\frac{9}{100}$ is 17.09.

215. Express in decimal form:

1. $5\frac{8}{10}$	6. $329\frac{17}{1000}$
2. $8\frac{7}{10}$	7. $416\frac{128}{100000}$
3. $12\frac{8}{100}$	8. $169\frac{829}{100000}$
4. $27\frac{47}{100}$	9. $874\frac{5384}{1000000}$
5. $146\frac{7}{1000}$	10. $729\frac{72084}{10000000}$

216. Express in fractional form:

1. 8.4	6. 92.031
2. 9.8	7. 124.107
3. 7.07	8. 167.0101
4. 29.36	9. 246.1809
5. 84.002	10. 396.06041

217. The word *and* should be used where the decimal point is, in reading expressions consisting of integers and decimals.

Thus, 4.06 is read, 4 *and* 6 hundredths, with a short pause after 4. 17.043 is read 17 *and* 43 thousandths, with a short pause after 17.

218. Write the following in figures:

Twenty-one, and twenty-one hundredths. One hundred, and twenty-four thousandths. One hundred twenty-four thousandths. Three hundred eighty-four ten-thousandths. Three hundred, and eighty-four ten-thousandths. One thousand, and eight hundred-thousandths. One thousand eight hundred-thousandths. Ninety-two millionths. Nine thousand nine millionths. Nine thousand, and nine millionths. Twenty-nine, and three thousand three hundred forty-six hundred-thousandths. Eight, and seventy thousand fifty-four millionths. Seventeen, and ninety thousand nine ten-millionths. Eighty-three, and one hundred fifty-six ten-thousandths.

219. The value of a decimal is determined by the position of the decimal point. Hence, changing the position of the decimal point changes the value of the decimal.

Moving the decimal point one place to the right multiplies the value of the decimal by 10; moving it two places to the right multiplies by 100; moving it three places multiplies by 1000, etc. If, in the decimal .008, the point is moved *one* place to the right, we have 0.08, which is 10 times .008. If the decimal point is moved *two* places to the right, we have 00.8, which is 100 times .008. Conversely, moving the point one place to the left divides the value of the fraction by 10; moving it two places divides by 100; moving it three places divides by 1000, etc. If the point in the decimal .8 is moved *one* space to the left, and its place filled with a cipher, we have .08, which is $\frac{1}{10}$ the value of .8. If the point is moved *two* spaces to the left, and the vacant places filled with ciphers, we have .008, which is $\frac{1}{100}$ of the value of .8. It will be seen from the preceding that placing a cipher between

the decimal point and the decimal divides the value of the decimal by 10.

REDUCTION OF DECIMALS

220. Annexing ciphers to the right of a decimal does not change the value of the decimal. Thus,

$$.5 = \tfrac{5}{10};\ .50 = \tfrac{50}{100} = \tfrac{5}{10};\ .500 = \tfrac{500}{1000} = \tfrac{5}{10}.$$

221. 1. Change .35 to a common fraction in its lowest terms.

OPERATION

$$.35 = \tfrac{35}{100} = \tfrac{7}{20}\ \textit{Ans.}$$

Change to common fractions in their lowest terms:

2. .45	4. .55	6. .125	8. .008	10. .0065
3. .48	5. .75	7. .045	9. .525	11. .00845

222. 1. Change 8.05 to a mixed number in its simplest form.

OPERATION

$$8.05 = 8\tfrac{5}{100} = 8\tfrac{1}{20}\ \textit{Ans.}$$

SOLUTION. — 8.05 expressed in the form of a mixed number is $8\frac{5}{100}$. Reducing $\frac{5}{100}$ to its lowest terms, which is $\frac{1}{20}$, and annexing it to 8, we have $8\frac{1}{20}$.

Change to mixed numbers in their simplest form:

2. 7.25	4. 9.85	6. 18.028	8. 36.325	10. 84.0006
3. 8.35	5. 16.125	7. 25.008	9. 41.0045	11. 93.1015

223. 1. Change $.4\frac{4}{5}$ to a common fraction in its lowest terms.

OPERATION

$$.4\tfrac{4}{5} = \frac{4\frac{4}{5}}{10} = \frac{\frac{24}{5}}{10} = \frac{24}{5} \times \frac{1}{10} = \frac{24}{50} = \frac{12}{25}\ \textit{Ans.}$$

Change to common fractions in their lowest terms:

2. $.6\frac{1}{4}$	5. $.12\frac{1}{2}$	8. $.62\frac{1}{2}$	11. $.212\frac{1}{2}$
3. $.06\frac{1}{4}$	6. $.16\frac{2}{3}$	9. $.66\frac{2}{3}$	12. $.433\frac{1}{3}$
4. $.08\frac{1}{3}$	7. $.33\frac{1}{3}$	10. $.87\frac{1}{2}$	13. $.087\frac{1}{2}$

224. 1. Change $9.8\frac{3}{4}$ to a mixed number in its lowest terms.

OPERATION

$$.8\tfrac{3}{4}=\frac{\frac{35}{4}}{10}=\frac{\overset{7}{\cancel{35}}}{4}\times\frac{1}{\underset{2}{\cancel{10}}}=\frac{7}{8}.\quad 9+\tfrac{7}{8}=9\tfrac{7}{8}\ \textit{Ans.}$$

Change to mixed numbers in their lowest terms:

2. $9.36\frac{2}{3}$	4. $8.87\frac{1}{2}$	6. $27.08\frac{1}{4}$
3. $7.41\frac{2}{3}$	5. $19.28\frac{4}{7}$	7. $13.0\frac{5}{9}$

225. 1. Change $\frac{3}{8}$ to a decimal.

OPERATION

$$\tfrac{3}{8}=3\div 8=8\overline{)3.000}\quad .375\ \textit{Ans.}$$

$\frac{3}{8}$ is the same as $\frac{1}{8}$ of 3. 3 units = 30 tenths. $\frac{1}{8}$ of 30 tenths = 3 tenths and 6 tenths remaining. 6 tenths = 60 hundredths. $\frac{1}{8}$ of 60 hundredths = 7 hundredths and 4 hundredths remaining. 4 hundredths = 40 thousandths. $\frac{1}{8}$ of 40 thousandths = 5 thousandths. Hence $\frac{3}{8}$, when reduced to the decimal form equals 3 tenths + 7 hundredths + 5 thousandths, or .375.

NOTE.—Frequently the division will not terminate, and the common fraction cannot be exactly expressed decimally. In such cases, the remainder is usually expressed by a common fraction. Thus,

$$\tfrac{2}{7}=2\div 7=.285\tfrac{5}{7};\quad \tfrac{8}{15}=8\div 15=.53\tfrac{1}{3}.$$

Change to decimals:

2. $\frac{1}{2}$	9. $\frac{7}{20}$	16. $\frac{15}{16}$
3. $\frac{1}{5}$	10. $\frac{3}{8}$	17. $\frac{101}{250}$
4. $\frac{1}{4}$	11. $\frac{1}{40}$	18. $\frac{141}{400}$
5. $\frac{3}{4}$	12. $\frac{21}{40}$	19. $\frac{171}{500}$
6. $\frac{1}{8}$	13. $\frac{9}{25}$	20. $\frac{216}{400}$
7. $\frac{5}{8}$	14. $\frac{81}{125}$	21. $\frac{175}{360}$
8. $\frac{7}{8}$	15. $\frac{11}{400}$	22. $\frac{125}{650}$

226. Memorize the decimal equivalents of the following common fractions:

1. $\frac{1}{2} = .5$	7. $\frac{2}{5} = .4$	13. $\frac{3}{8} = .37\frac{1}{2}$
2. $\frac{1}{3} = .33\frac{1}{3}$	8. $\frac{3}{5} = .6$	14. $\frac{5}{8} = .62\frac{1}{2}$
3. $\frac{2}{3} = .66\frac{2}{3}$	9. $\frac{4}{5} = .8$	15. $\frac{7}{8} = .87\frac{1}{2}$
4. $\frac{1}{4} = .25$	10. $\frac{1}{6} = .16\frac{2}{3}$	16. $\frac{1}{12} = .08\frac{1}{3}$
5. $\frac{3}{4} = .75$	11. $\frac{5}{6} = .83\frac{1}{3}$	17. $\frac{5}{12} = .41\frac{2}{3}$
6. $\frac{1}{5} = .2$	12. $\frac{1}{8} = .12\frac{1}{2}$	18. $\frac{1}{16} = .06\frac{1}{4}$

ADDITION OF DECIMALS

227. Addition of decimals does not differ from addition of simple whole numbers. Since only similar units can be added, we write tenths in *one* column, hundredths in another, and thousandths in another, etc., and add and carry as in whole numbers, and for the same reasons.

228. 1. Find the sum of 18.3, .04, 2.106, and .0051.

OPERATION

$$\begin{array}{r} 18.3 \\ .04 \\ 2.106 \\ .0051 \\ \hline 20.4511 \end{array} \textit{ Ans.}$$

We write tenths under tenths, hundredths under hundredths, thousandths under thousandths, etc., and add as in simple whole numbers.

Find the sum of:

2. 3.42, .006, .41, 46.81, and 281.004.

3. .08, 1.641, 1.71, 72.301, 463.1, and .0004.

4. 1.665, .3842, 98.005, .5101, 84.3, and 91.91.

5. .006, 4.011, 3.8406, .004, 9.2, and .31365.

6. \$$.66\frac{2}{3}$, \$$.37\frac{1}{2}$, \$$.62\frac{1}{2}$, \$$.83\frac{1}{3}$, \$$24.33\frac{1}{3}$, and \$$94.87\frac{1}{2}$.

7. Find the sum of 8 tenths; twenty-one hundredths; 16 thousandths; 31 ten-thousandths; 62 millionths.

8. Find the sum of 9, and 3 tenths; 42, and 16 hundredths; 33 thousandths; 18, and 5 thousandths; 21, and 21 ten-thousandths; 160, and 316 ten-thousandths.

9. Find the sum of $16\frac{2}{3}$ thousandths; $66\frac{2}{3}$ thousandths; 29, and $87\frac{1}{2}$ hundredths; 321, and $85\frac{5}{9}$ thousandths; 725, and 325 ten-thousandths.

SUBTRACTION OF DECIMALS

229. 1. From 92.61 take 84.006.

OPERATION

$$\begin{array}{r} 92.610 \\ 84.006 \\ \hline 8.604 \end{array}$$ *Ans.*

The minuend and subtrahend are written so that units of the same order stand in the same column. When the number of decimal places in the subtrahend exceeds the number in the minuend, ciphers are generally annexed to the minuend to make them equal.

2. From 32.06 take 18.03.
3. From 29.1 take .0101.
4. From 381.064 take 260.009.
5. From 182.606 take 38.406.
6. From 3.009 take 1.666.
7. From 9.4 take .8460.
8. From 16.001 take 3.8749.
9. From 5.0014 take 4.9999.
10. From 1.1 take .99999.
11. From 6.09 take 5.00001.
12. From $.7\frac{3}{4}$ take $23\frac{3}{5}$ ten-millionths.
13. From $9\frac{2}{5}$ take $9\frac{1}{2}$ ten-thousandths.
14. From $.08\frac{3}{5}$ take $15\frac{1}{2}$ ten-thousandths.

MULTIPLICATION OF DECIMALS

230. 1. $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = .01$; or, $.1 \times .1 = .01$.

2. $\frac{7}{10} \times \frac{3}{10} = \frac{21}{100} = .21$; or, $.7 \times .3 = .21$.

3. $\frac{3}{10} \times \frac{9}{100} = \frac{27}{1000} = .027$; or, $.3 \times .09 = .027$.

4. $\frac{8}{100} \times \frac{7}{100} = \frac{56}{10000} = .0056$; or, $.08 \times .07 = .0056$.

5. $\frac{5}{10} \times 7 = \frac{35}{10} = 3.5$; or, $.5 \times 7 = 3.5$.

231. 1. Multiply 3.2 by 2.4.

OPERATION

$$\begin{array}{r} 3.2 \\ \underline{2.4} \\ 128 \\ \underline{64} \\ 7.68 \end{array}$$

EXPLANATION

$3.2 = 3\frac{2}{10} = \frac{32}{10}$

$2.4 = 2\frac{4}{10} = \frac{24}{10}$

$\frac{32}{10} \times \frac{24}{10} = \frac{768}{100} = 7\frac{68}{100} = 7.68 = 7.68$ *Ans.*

From the preceding it will be seen that we multiply decimals like whole numbers, and point off from the right of the product as many decimal places as there are in the multiplicand and multiplier together, prefixing ciphers when the number of figures in the product does not equal the number of decimal places in both factors.

Multiply:

2. 8.06 by .04	7. 161.3 by .401	12. $4\frac{1}{2}$ by .001
3. 24.38 by 1.65	8. 32.001 by .3016	13. $3.2\frac{1}{4}$ by $.01\frac{1}{2}$
4. .004 by 3.02	9. 0.604 by .31	14. $8.01\frac{3}{8}$ by $.00\frac{1}{8}$
5. 1.01 by .002	10. .009 by .3031	15. $21\frac{1}{2}$ by $.21\frac{1}{2}$
6. 30.03 by 28.3	11. .001 by 1.1	16. $.000\frac{1}{8}$ by $.62\frac{1}{2}$

DIVISION OF DECIMALS

232. 1. Divide 1282.56 by 76.8.

We divide as in whole numbers. As the dividend is the product of the divisor and quotient, the number of decimal places in the dividend should equal the number in the divisor and quotient taken together. Since there are *two* decimal places in the dividend and *one* in the divisor, there must be the difference between *two* and *one*, or *one* decimal place in the quotient.

OPERATION

$$\begin{array}{l} 76.8)1282.56(16.7\ \textit{Ans.} \\ \underline{768} \\ 5145 \\ \underline{4608} \\ 5376 \\ \underline{5376} \end{array}$$

233. 1. Divide 136.318 by 3.185.

OPERATION

```
3.185)136.3180(42.8 Ans.
      12740
      -----
        8918
        6370
       -----
       25480
       25480
       -----
```

When there is a remainder, ciphers should be annexed to the dividend and the division continued, as shown in the operation.

234. When the division will not terminate, and an exact quotient cannot be expressed decimally, the remainder may be expressed by a common fraction (Note, Art. 225); or, when a number of decimal places sufficient for practical purposes has been obtained, the sign "+" may be annexed to indicate that the division is not complete.

(See operations following.)

1. Divide 29.62 by 24

OPERATION

```
24)29.620(1.234⅙
   24
   --
    56
    48
    --
     82
     72
     --
     100
      96
      --
       4/24 = 1/6
```

2. Divide 56.285 by 6.3

OPERATION

```
6.3)56.28500(8.9341 +
    504
    ---
     588
     567
     ---
      215
      189
      ---
       260
       252
       ---
        80
        63
        --
        17
```

235. 1. Divide 16.384 by .00256.

OPERATION

```
.00256)16.38400(6400
        1536
        ----
         1024
         1024
         ----
           00
```

When the number of decimal places in the divisor exceeds the number in the dividend, we make them equal by annexing ciphers to the dividend.

236. 1. Divide .054 by 28.8.

OPERATION

```
28.8).0540000(.001875
       288
       ----
       2520
       2304
       ----
        2160
        2016
        ----
         1440
         1440
         ----
```

Sometimes ciphers must be prefixed to the quotient figures in order to make the number of decimal places equal the excess of the number in the dividend over the number in the divisor.

237. Divide:

1. 21.872 by .32
2. 1.367 by .02
3. 2579.85 by 409.05
4. .21512 by 70.35
5. 21.3615 by 3.15
6. 2.13615 by 31.5
7. 1.38768 by 2.12
8. 138.768 by .212
9. 819.873 by 18.45
10. .819873 by 184.5
11. 24 by .032
12. 9.6 by .00032
13. 4687.5 by .00075
14. 11.52 by .0096
15. .225112 by 70.35
16. 1.1848896 by 17.28
17. .25 by 3.1416
18. 6.72 by .175
19. .035 by .00105
20. $.8\frac{1}{3}$ by $6\frac{1}{4}$
21. .231 by .0654
22. .0356 by .754
23. 1.2345 by .018
24. 62.125 by .0007

238. Find the value of:

1. $2.325 \times .0125 \times .08$
2. $3.8 \times .0225 \times 6.5$
3. $3.225 \times 50 \times .3 \times \frac{1}{4}$
4. $10.5 \times 333\frac{1}{3} \times .003\frac{1}{3}$
5. $(131.45 \div .28) \times .0042$
6. $(68.52\frac{1}{2} \div 50) \times .0\frac{3}{8}$
7. $(212\frac{1}{2} \times .003) \div 500$
8. $(3.84375\frac{1}{2} \div 93.75) \times 2\frac{9}{16}$
9. $(30.688 \div 1.37) \times 5$
10. $(5.8\frac{1}{2} \div 650) \times .004\frac{1}{2}$

239. As U.S. money is written in the decimal scale, its denominations increase and decrease in a ten-fold ratio. Hence all operations in addition, subtraction, multiplication, and division of U.S. money are performed in the same way as corresponding operations in decimals.

240. *Decimals applied in business transactions.*

1. At \$.37½ per bushel, how many bushels of oats can be bought for \$13.12½?

2. If 10⅖ tons of coal cost \$50, how much will 2.6 tons cost?

3. How many suits of clothes can be made from 390.6 yd. of cloth, if 9.3 yd. are required to make one suit?

4. Find the cost of 390.6 yd. of cloth, if 9.3 yd. cost \$20.925.

5. Find the cost of 101¾ pounds of tea at the rate of 46.5 pounds for \$17.43¾.

241. 1. How much will 424 watermelons cost at \$28 a hundred?

To change a number to hundreds we divide it by 100, which is the same as pointing off two places at the right. Hence 424 equals 4.24 hundreds. If *one* hundred melons cost \$28, 4.24 hundreds will cost 4.24 times \$28, or \$118.72.

STATEMENT

$(424 \div 100) \times \$28 = Ans.$

OPERATION

$424 \div 100 = 4.24$ hundreds

$4.24 \times \$28 = \$118.72$ *Ans.*

NOTE. — In business transactions C is frequently used to denote *one hundred*; and M, *one thousand*.

2. How much will 3846 pounds of freight cost at \$.875 per C?

3. How much are 328 fence posts worth at \$22.33$\frac{1}{3}$ per C?

4. Find the value of 4628 cabbages at \$4.62$\frac{1}{2}$ per C.

5. Find the value of 1462 pounds of buckwheat flour at \$2.35 a hundred.

6. If I pay \$27.46$\frac{1}{4}$ for 845 pickets for my front fence, how much is that per C?

242. 1. Find the cost of 5385 bricks at \$7.35 per M.

To change a number to thousands we divide by 1000, which is the same as pointing off three places at the right. Hence 5385 equals 5.385 thousands. If *one* thousand bricks cost \$7.35, the cost of 5.385 thousands will be 5.385 times \$7.35, or \$39.58.

STATEMENT

$$(5385 \div 1000) \times \$7.35 = \textit{Ans.}$$

OPERATION

$$5385 \div 1000 = 5.385 \text{ thousands}$$

$$5.385 \times \$7.35 = \$39.58 \quad \textit{Ans.}$$

2. Find the cost of 8384 feet of boards at \$13.35 per M.

3. Find the cost of 36,845 shingles at \$6.75 per M.

4. Find the cost of 7540 envelopes at \$2.33$\frac{1}{3}$ per M.

5. If I buy boards at \$11.87$\frac{1}{2}$ per M, and sell at \$13.625 per M, how many must I buy and sell to gain \$12.95?

243. 1. Find the cost of 9846 pounds of hay at \$11.25 a ton.

2000 pounds make 1 ton. Hence to change pounds to tons we divide the number representing the quantity by 2000, which is the same as pointing off three places at the right for decimals, and dividing by 2. 9846 pounds equal 4.923 tons. Since the cost of 1 ton is \$11.25, the cost of 4.923 tons is 4.923 times \$11.25, or \$55.38$\frac{3}{8}$.

STATEMENT

$$(9846 \div 2000) \times \$11.25 = \textit{Ans.}$$

OPERATION

$$9846 \div 2000 = 4.923 \text{ tons}$$

$$4.923 \times \$11.25 = \$55.38\tfrac{3}{8} \quad \textit{Ans.}$$

2. What is the value of 7384 pounds of timothy hay at $ 18.75 a ton?

3. How much will 8760 pounds of phosphate cost at $ 33⅛ a ton?

4. If .75 ton of clover hay is worth $ 11.62½, find the value of 8963 pounds.

PROBLEMS

244. 1. A merchant bought goods for $ 1280, and sold them for .15 more than he paid for them. How much did he receive for them?

2. At $ 2.25 per ton, how many pounds of coal can be bought for $ 11.25?

3. Find the value of 240 sacks of Peruvian guano, at $ 63.75 a ton, if each sack contains 150 pounds.

4. If I buy a horse for $ 175, and sell it at a gain of $ 25, the gain is what part of the cost? Express the answer in a decimal.

5. How many bushels will fill a bin whose capacity is 22579.41 cubic inches, there being 2150.42 cubic inches in a bushel?

6. A miller bought 435.75 bushels of wheat at $ .88 a bushel. He sold .2 of it to one man at $ .93 a bushel, and .25 of the remainder to another, at $ .95 a bushel. At what price per bushel must he sell the remainder in order that his entire gain may be $ 20.916?

7. I bought 90.44 yards of cloth for $ 194.446. I found .25 of it damaged so that I was obliged to sell it at a loss of $ .25 a yard. For how much per yard must I sell the remainder that I may neither gain nor lose by the transaction?

8. A farmer sold .12½ of his crop of hay in January, .37½ in February, and the remainder, 24,684 pounds, he kept for his own use. Of how many tons did his crop consist?

9. A merchant paid $ $116\frac{2}{3}$ for a piece of carpet containing $87\frac{1}{4}$ yd. He sold .4 of it at a gain of $ $.16\frac{2}{3}$ a yard, and the remainder at a gain of $ $.12\frac{1}{2}$ a yard. Find the average selling price per yard.

10. If I gained 34 cents by selling $21\frac{1}{4}$ pounds of rice at the rate of 16 pounds for $ 2, how much did the rice cost me per pound?

11. If 38.4 bu. of wheat are worth 86.4 bu. of corn, how many bushels of wheat are worth 199.8 bu. of corn?

12. If I sell a certain number of bushels of wheat at $ .85 a bushel, I will gain $ 3, but if I sell it at $ .78 a bushel, I will lose $ 1.20. Find the number of bushels.

13. I sold B $\frac{13}{40}$ of my apples, and C $\frac{2}{9}$ of the remainder. If C received $262\frac{1}{2}$ bushels less than B, how many bushels had I left, and how much did I receive for what I sold if the selling price was 70¢ per bushel?

14. A miller invested $ $77\frac{5}{8}$ in an equal number of bushels of oats, wheat, and corn, paying $ $\frac{2}{5}$ per bushel for the oats, $ $\frac{7}{8}$ for the wheat, and $ $\frac{9}{20}$ for the corn. How many bushels of each did he get?

15. A, B, and C purchased a tract of wood land for $ 9314.25; $\frac{1}{2}$ of what A paid equaled what B paid, and $\frac{2}{3}$ of what B paid equaled what C paid. Find how much each paid respectively.

16. Johnson and White can dig a well in 24 days. Johnson can dig $\frac{3}{5}$ as much as White. How long will it take each to do the work?

BILLS

245. A *Bill* is a statement in detail of goods sold or delivered, or of services rendered. It shows the place, time, names of the parties concerned, the quantity, price, etc.

246. When a person buys anything for which he does not pay at the time, we say he goes in debt for it, and he is therefore called a *Debtor*.

247. When a person sells anything for which he does not receive pay at the time, he is said to give credit for it, and he is therefore called a *Creditor*.

248. A bill is receipted when the creditor, or some one authorized to act for him, writes "Received Payment," or "Paid," at the bottom of the bill, and signs his name.

249. Following are a few of the abbreviations and symbols commonly used in bills and accounts.

@	at.	doz.	dozen.
Acct., %c	account.	lb.	pound.
Amt.	amount.	mdse.	merchandise.
bal.	balance.	No., #	number.
bbl.	barrel.	payt.	payment.
¢	cents.	pd.	paid.
Co.	company.	per.	by.
Cr.	creditor.	pc.	pieces.
Dr.	debtor.	recd.	received.
do., "	the same.	yd.	yard.

Note. — It is customary for the creditor to render an itemized bill, and if it is not paid, a second form of bill called a *Statement*, is made out. The statement contains only the words, "To Bill Rendered," or "To Merchandise," together with the amount.

(Form 1)

250. New York, Jan. 27, 1898.

Mr. JOHN H. PAYNE,

Bought of W. M. Miller & Co.

3 bbl. Salt,	@	$1.30	$3	90
60 lb. Crushed Sugar,	"	.11	6	60
14 Hams (170 lb.),	"	.14	23	80
25 bu. of Potatoes,	"	.90	22	50
9 bbl. Apples,	"	1.40	12	60
20 lb. of Honey,	"	.12	2	40
			$71	80

Recd. Payment,
W. M. Miller & Co.
Per Beers.

(Form 2)

251. Philadelphia, Jan. 30, 1898.

Mr. JACOB READ,

To Johnson & Smith, *Dr.*

1897.						
Nov.	9	To 30 lb. Dried Apples,	@	$.11	$3	30
"	"	" 100 lb. Codfish,	"	.06	6	00
Dec.	12	" 8 boxes Starch,	"	.20	1	60
"	"	" 3 gal. Sirup,	"	1.10	3	30
"	"	" ½ bu. Cracked Corn,	"	1.50		75
1898.						
Jan.	17	" 3 doz. Eggs,	"	.26		78
"	"	" 1 gal. Kerosene Oil,	"			35
"	"	" 9½ bu. Meal,	"	.80	7	60
					$23	68

Received Payment,
Johnson & Smith.
Per Johnson.

PROBLEMS

(Form 1)

252. 1. Miss E. L. Verlenden bought of Isaac Long, Richmond, Va., Jan. 31, 1898: 14 yd. calico, at $9\frac{1}{2}$¢; $3\frac{1}{4}$ yd. velvet, at $\$2\frac{3}{4}$; $\frac{1}{2}$ doz. linen handkerchiefs, at $37\frac{1}{2}$¢ apiece; 3 pr. kid gloves, at $1.50; $\frac{1}{2}$ doz. buttons, at 20¢; $9\frac{1}{2}$ yd. blk. silk, at $2.45; $22\frac{1}{2}$ yd. bleached muslin, at 11¢. Make out the bill in proper form and receipt it.

(Form 1)

2. Harrison & Goodall, of Chicago, sold Charles Sauermilch, Feb. 2, 1898, 1 piece black mohair serge, $65\frac{1}{4}$ yd., at $\$.37\frac{1}{2}$; 1 piece sateen sleeve lining, $27\frac{3}{4}$ yd. at 25¢; 1 piece black Italian cloth, $31\frac{1}{2}$ yd., at $\$1.12\frac{1}{2}$; 1 piece twilled sleeve lining, $56\frac{1}{2}$ yd., at 15¢; 2 pieces black silesia, $107\frac{1}{4}$ yd. at 15¢; 2 pieces canvas, 93 yd., at 19¢; 1 piece red padding, 60 yd., at 65¢. Make out the bill.

(Form 2)

3. William Butler bought of the American Book Co., New York, Sept. 15, 1898,

325 Robinson's New Practical Arithmetic, at 65¢;
400 Barnes' Fifth Reader, at 90¢;
500 Barnes' Fourth Reader, at 70¢;
600 Barnes' Third Reader, at 50¢;
600 Barnes' Second Reader, at 35¢;
600 Barnes' First Reader, at 20¢;
100 Jepson's Music Reader, No. 3, at 50¢;
200 Jepson's Music Reader, No. 2, at 35¢;
500 Jepson's Music Reader, No. 1, at 30¢.

Make out a receipted bill.

(Form 2)

4. Wilson J. Smith bought of Bright & Harris, of Pittsburg, Feb. 6, 1898, 3856 ft. white pine boards, at $48.75 per M; 8365 shingles, No. 1, at $8.75 per M. March 23, 1898, 8462 ft. hemlock boards, at $12 per M; 24,673 ft. hemlock plank, at $11.25 per M; 7364 ft. white pine siding, at $38.75 per M. May 9, 1898, 374 chestnut fence posts, at $18¾ per C; 9375 ft. yellow pine flooring, at $22 per M; 7386 fence pickets, at 87½ ¢ per C.

Make out a receipted bill.

ACCOUNTS

253. An *Account Current* is a detailed record of unsettled business transactions, embracing both debits and credits. It shows the place and date of each transaction, the names of the parties concerned, the items bought and sold, or services rendered, together with quantity, price, etc.

(Form 1)

254. 1. Buffalo, N.Y., July 1, 1898.

HENRY BADDERS,

In Account with Stevens & Co.

1898		*Dr.*				
Jan.	5	To 3 tubs Butter (168 lb.), @ $.31	52	08		
"	14	" 14 bu. Meal, " .87½	12	25		
Mar.	22	" 9 bbl. Potatoes, " 1.75	15	75		
June	17	" 3 sacks Salt, " 1.90	5	70	85	78
		Cr.				
Jan.	11	By 12 tons Coal, @ $2.25	27	00		
May	21	" 1 day's Work, with team	2	00		
June	25	" 2 loads Kindling Wood, @ $1.30	2	60	31	60
		Bal. due Stevens & Co.			54	18

2. Arrange, according to the preceding form, the following business transactions of E. C. Williams, a farmer, in account with Myron Jones, who keeps a general store in Wilsontown, Ohio:

Jan. 1, 1898. — Jones sold Williams 15½ lb. granulated sugar, at 9¢; 7 lb. coffee, at 45¢; 3 lb. tea, at 85¢.

Feb. 5. — Williams bought of Jones 3 gal. kerosene oil, at 20¢; 5 lb. cheese, at 15¢; 1 pr. boots, at $3.25.

March 21. — Williams delivered Jones 10 bu. potatoes, at 45¢; 8 doz. eggs, at 14¢; 25 lb. dried apples, at 10¢; and bought of him, 14 yd. calico, at 8¢; 1 box yeast powder, at 20¢.

(Form 2)

255. 1. St. Louis, Mo., June 9, 1898.

JAMES A. GROVE,

In Acct. with Francis Ward. *Dr.* *Cr.*

1898							
Jan.	30	To 6 gal. Oil,	@ $1.05	$6	30		
Feb.	7	" 10 lb. Soda,	" .09		90		
"	17	By 2 cd. Hickory Wood,	" 4.50			$9	00
Mar.	9	To 18 bu. Wheat,	" 1.00	18	00		
April	13	By cash on %,				3	00
"	"	" bal. due,				13	20
				$25	20	$25	20
		Received Payment,					
		June 12, 1898. Francis Ward.					

Arrange the following transactions according to the preceding form:

2. W. A. Powers, a contractor and builder, in acct. with Joseph Wilson, a lumber dealer of Scranton, Pa.

Jan. 9, 1898. — Wilson sold Powers 3865 ft. hemlock scantling, at $13 per M; 3846 ft. white pine boards, at $45 per M.

Jan. 26, 1898. — Powers repaired lumber shed for Wilson, for which Wilson credited Powers on account \$18.50, as per contract.

Feb. 9, 1898. — Powers delivered Wilson a walnut office table, for which Wilson agreed to pay \$14.

March 30, 1898. — Powers bought of Wilson 3864 shingles, No. 1, at \$8.75 per M; 8672 ft. siding, at \$32.50 per M; 3972 ft. N. C. pine, at \$25 per M.

April 1, 1898. — Powers paid Wilson \$175 on acct.

PROBLEMS

256. 1. J. J. Sweet, a miller, in account with R. S. Robbins, a merchant, of Detroit, Mich.

Jan. 1, 1898. — Sweet bought of Robbins 8¾ yd. muslin, at 9¢; 13 yd. alpaca, at 75¢; 1 pr. men's boots, at \$3.60.

Jan. 12, 1898. — Robbins sold Sweet 45 lb. sugar, at 9¢; 6 linen handkerchiefs, at 40¢.

Jan. 14, 1898. — Sweet delivered Robbins 350 lb. buckwheat flour, at \$2.75 per C; 785 lb. corn meal, at \$1.90 per C; 20 sacks wheat flour, 25 lb. each, at \$3.12 per C.

Jan. 21, 1898. — Sweet bought of Robbins 65 bu. wheat, at 87½¢; 26 bu. rye, at 65¢; 60 bu. oats, at 35¢.

Jan. 30, 1898. — Sweet paid Robbins \$25 on account. Arrange the preceding account according to "Form 1." Show balance due Robbins Jan. 30, 1898.

2. Thomas Winder, a carpenter, in account with William Keeler, a bookseller, of Cincinnati, Ohio.

March 1, 1898. — Winder bought of Keeler, 1 Arithmetic, at 60¢; 1 Dictionary, at \$10; 1 Geography, at \$1.20; 1 Lessons in English, at 60¢.

March 7, 1898. — Winder put up shelves in Keeler's store, for which Keeler credited him on account \$5.

April 16, 1898. — Winder paid Keeler \$2.25 on account.

May 1, 1898. — Winder paid cash to bal. account. Make out the preceding account according to "Form 2," and receipt it.

Order for Goods

257. $\$10\frac{00}{100}$. Wilkesbarre, Pa., Feb. 7, 1898.

To William Stoddart & Co.

Please deliver to James Hanna, or order, goods from your store to the amount of Ten Dollars, and charge the same to my account.

William Eaton.

Order for Money

258. $\$15\frac{00}{100}$. Baltimore, Md., June 11, 1898.

To James Morrison.

Please pay Uriah James, or order, Fifteen Dollars in cash, and charge the same to my account.

Henry Wood.

Due Bill Payable in Cash

259. $\$21\frac{17}{100}$. Richmond, Va., March 29, 1898.

For value received, due Henry Weaver, or order, Twenty-one and $\frac{17}{100}$ Dollars.

Myron B. Slocum.

Due Bill Payable in Goods

260. $\$25\frac{86}{100}$. Elmira, N.Y., July 11, 1898.

For value received, due John Anderson, or order, Twenty-five and $\frac{86}{100}$ Dollars, to be paid in goods from my store.

James Fulton's Sons.

A Receipt in Full of All Demands

261. $\$26\frac{81}{100}$. Kingston, Pa., Feb. 19, 1898.

Received of William Edwards Twenty-six and $\frac{81}{100}$ Dollars, in full of all demands.

Henry Tyrrell.

PERCENTAGE

262. Instead of saying $\frac{2}{100}$, $\frac{5}{100}$, $\frac{9}{100}$, etc., we may use the term *per cent* for the denominator, ***hundredths;*** thus, 2 per cent, 5 per cent, 9 per cent, etc.

263. The sign % is frequently used for the term ***per cent;*** thus 2%, 5%, 9%, etc. Therefore 1% means $\frac{1}{100}$; 2%, $\frac{2}{100}$; 3%, $\frac{3}{100}$; 4%, $\frac{4}{100}$, etc.

Example.—If a man had 400 sheep and sold 9% of them, he sold $\frac{9}{100}$ of 400 sheep. $\frac{9}{100}$ of 400 sheep = 36 sheep. Therefore he sold 36 sheep.

264. This process of computing by hundredths is called *Percentage,* from *per cent,* which means by the hundred.

265. The *per cent* may be expressed in the form of a common fraction, or of a decimal. Thus, $5\% = \frac{5}{100} = .05$. In oral work it is most convenient to express the per cent in fractional form, and in written work in decimal form.

266. 1. Express 6% in decimal form.

Model. 6% = .06.

In like manner express:

2. 4%	5. 25%	8. 47%	11. 72%
3. 8%	6. 36%	9. 54%	12. 83%
4. 11%	7. 43%	10. 60%	13. 98%

267. 1. What % is .75?

MODEL. $.75 = 75\%$.

Likewise express:

2. .08	4. .14	6. .64	8. .85
3. .09	5. .21	7. .76	9. .92

268. 1. Express $5\frac{1}{2}\%$ in decimal form.

MODEL. $5\frac{1}{2}\% = .05\frac{1}{2} = .055$.

Likewise express:

2. $6\frac{1}{4}\%$	4. $12\frac{1}{2}\%$	6. $14\frac{3}{4}\%$	8. $46\frac{3}{8}\%$
3. $8\frac{1}{3}\%$	5. $16\frac{2}{3}\%$	7. $22\frac{1}{8}\%$	9. $54\frac{4}{5}\%$

269. 1. What % is .245?

MODEL. $.245 = .24\frac{1}{2} = 24\frac{1}{2}\%$.

What % is:

2. .375?	4. .0675?	6. $.08\frac{2}{3}$?
3. .084?	5. $.163\frac{1}{3}$?	7. $.126\frac{3}{7}$?

270. 1. Express $\frac{3}{5}\%$ decimally.

MODEL. $\frac{3}{5}\% = .00\frac{3}{5} = .006$.

Express decimally:

2. $\frac{3}{4}\%$	4. $\frac{4}{25}\%$	6. $\frac{7}{9}\%$	8. $\frac{5}{8}\%$
3. $\frac{3}{8}\%$	5. $\frac{5}{6}\%$	7. $\frac{3}{16}\%$	9. $\frac{7}{16}\%$

271. 1. Express 125% decimally.

MODEL. $125\% = \frac{125}{100} = 125 \div 100 = 1.25$.

Express decimally:

2. 132%	4. 185%	6. $137\frac{1}{2}\%$	8. $262\frac{3}{7}\%$
3. 175%	5. 168%	7. $118\frac{3}{5}\%$	9. $108\frac{1}{3}\%$

272. 1. What is 8% of 385?

OPERATION

$$385 \times .08 = 30.8 \quad Ans.$$

What is:

2. 8% of 750?	6. $12\frac{1}{2}$% of \$1400?
3. 10% of 326?	7. $66\frac{2}{3}$% of \$845.60?
4. 12% of 429?	8. $112\frac{1}{2}$% of \$380.50?
5. $8\frac{1}{4}$% of 487 miles?	9. 5.5% of 4386 feet?

273. The principal quantities considered in percentage are the Base, the Rate, the Percentage, the Amount, and the Difference.

1. From 740 bushels of wheat a miller sold 5% and converted the rest into flour. Find the number of bushels sold, and the number made into flour. In this example 740 bu. is the *base;* 5% the *rate;* the result obtained by multiplying 740 bu. by .05 (740 bu. $\times$ .05 = 37 bu.), or 37 bu., is the *percentage.*

274. The *Base* is the number on which the percentage is computed.

275. The *Rate* is the number which denotes how many hundredths of the base are to be taken.

276. The *Percentage* is the result obtained by multiplying the base by the rate.

277. The *Amount* is the sum of the base and the percentage.

278. The *Difference* equals the base minus the percentage.

PROBLEMS

279. 1. A drover purchased 630 head of cattle, and sold 40% of them. How many remained?

STATEMENT I	STATEMENT II
$630 - (630 \times .40) =$ answer	$630 \times (1.00 - .40) =$ answer

EXPLANATION OF STATEMENT I

$$630 \times .40 = 252, \text{ number sold}$$
$$630 - 252 = 378, \text{ number remaining}$$

It will be seen that the number sold is represented by what is included within the parenthesis, according to statement I. By subtracting the number sold from the number purchased, we get the number that remained.

EXPLANATION OF STATEMENT II

The whole of anything equals 100%, or 1.

What the drover sold equals 40%, or .40 of the whole number. What remained equals 100% – 40%, or 1.00 – .40, or .60 of the whole number. Hence $630 \times (1.00 - .40) = 630 \times .60 = 378$, the number that remained.

NOTE.—Pupils should be encouraged to make as many different statements of the same problem as possible. The following problems more fully illustrate the plan. Pupils should be required to explain their statements in a manner somewhat similar to the two preceding explanations.

2. If a man bought a farm for $8500, and sold it at a gain of 8%, how much did he receive for it?

STATEMENT I

$\$8500 + (\$8500 \times .08) = \text{answer}$

STATEMENT II

$\$8500 \times 1.08 = \text{answer}$

In statement I, the gain is expressed by what is included within the parenthesis.

Since the selling price always equals the cost plus the gain, or the cost minus the loss, it is evident if we add the gain to the cost we shall obtain the amount received for the farm.

Since the gain was 8%, the farm was sold for 100% + 8%, or 108%, or 1.08 of the cost. Hence if we multiply the cost by 1.08, as indicated in statement II, we shall obtain the amount received for the farm.

3. Mr. Riley failed in business, and paid only 35% of his debts. How much did Mr. Koerner lose if Riley owed him $3750?

STATEMENT

$$\$3750 \times (1.00 - .35) = \text{answer}$$

4. A miller owning $\frac{3}{8}$ of a mill, valued at \$5000, sold his share for 20% more than its value. How much did he receive?

STATEMENT

$$(\$5000 \times \tfrac{3}{8}) \times 1.20 = \text{answer}$$

5. For how much must I rent my farm of 85 acres, worth \$75 an acre, to realize an annual income of 8% from it?

STATEMENT

$$(\$75 \times 85) \times .08 = \text{answer}$$

6. A bookkeeper's annual income is \$1600. If he pays 25% of it for rent, and $33\frac{1}{3}$% of it for other expenses, how much can he save in 5 yr.?

DEVELOPMENT OF THE STATEMENT

$(.25 + .33\frac{1}{3})$ = part of income paid for rent and other expenses.

$\overline{1.00 - (.25 + .33\frac{1}{3})}$ = part of income left after paying rent and other expenses.

$\$1600 \times \overline{1.00 - (.25 + .33\frac{1}{3})}$ = part of income saved in 1 yr.

$\$1600 \times \overline{1.00 - (.25 + .33\frac{1}{3})} \times 5$ = ans., amt. he saves in 5 yr.

Statements may frequently be simplified by using equivalent common fractions.

Thus, $\$1600 \times \overline{1.00 - (.25 + .33\frac{1}{3})} \times 5 = \$1600 \times \overline{1 - (\frac{1}{4} + \frac{1}{3})} \times 5.$

7. A laborer agreed to dig a mill race 360 ft. long, 8 ft. wide, and 4 ft. deep, at 30¢ a cubic yard. He lost $6\frac{1}{4}$% on the contract price. Find his loss.

DEVELOPMENT OF THE STATEMENT

$(360 \times 8 \times 4)$ = number cubic feet to be removed.

$\overline{(360 \times 8 \times 4) \div 27}$ = number cubic yards to be removed.

$\$.30 \times \overline{(360 \times 8 \times 4) \div 27}$ = contract price for doing the work.

$\$.30 \times \overline{(360 \times 8 \times 4) \div 27} \times .06\frac{1}{4}$ = answer, what he lost.

8. A coal dealer bought 80 tons of coal for \$300. After allowing 10% for waste, he retailed the remainder so as to realize 20% on his investment. Find the retail price per ton.

STATEMENT

$$(\$300 \times 1.20) \div (80 \times .90) = \text{answer}$$

9. Two brothers, William and John, each bought a farm. William's farm contained 240 acres, and John's contained $12\frac{1}{2}$% more acres than William's. William paid \$30 an acre, and John $16\frac{2}{3}$% less per acre than William. How much more did William pay for his farm than John?

STATEMENT

$$(\$30 \times 240) - \overline{(\$30 \times \tfrac{5}{6}) \times (240 \times \tfrac{9}{8})} = \text{answer}$$

10. A grain dealer bought 8640 bu. of wheat at \$1.10 per bu., and sold 25% of it at $\$1.12\frac{1}{2}$, and 40% at \$1.25, and the remainder, 35%, at \$.95. How much more did he receive for it than he paid?

STATEMENT

$$\$1.12\tfrac{1}{2} \times (8640 \times .25) = \text{sum received for 1st lot}$$
$$\$1.25 \times (8640 \times .40) = \text{sum received for 2d lot}$$
$$\$\ .95 \times (8640 \times .35) = \text{sum received for 3d lot}$$
$$(\text{1st} + \text{2d} + \text{3d}) - (\$1.10 \times 8640) = \text{answer}$$

11. Mr. Smith received \$8600 from his father's estate. He invested 35% of it in railroad stock, 40% in bonds, and the remainder in real estate. Find his annual income from each if he realizes $8\frac{3}{4}$% on the railroad stock, 5% on the bonds, and $5\frac{1}{2}$% on the real estate.

STATEMENT

$$(\$8600 \times .35) \times .08\tfrac{3}{4} = \text{income from railroad stock}$$
$$(\$8600 \times .40) \times .05 = \text{income from city bonds}$$
$$\$8600 \times \overline{1.00 - (.35 + .40)} \times .05\tfrac{1}{2} = \text{income from real estate}$$

NOTE.—After carefully studying these statements pupils will have no difficulty in using the plan in all applications of percentage.

280. 1. 75 is 25% of what number?

OPERATION

$75 \div .25 = 300$ *Ans.*

If 75 is 25% of a certain number, it is .25 of it. If a certain number multiplied by .25 gives a product of 75, it is evident the number must equal 75 divided by .25, or 300.

2. 116 is 8% of what number?
3. 119 is $33\frac{1}{3}$% of what number?
4. $93\frac{1}{4}$ is 32% of what number?
5. $129\frac{3}{7}$ is 56% of what number?
6. 46.5 is 45% of what number?
7. 353.3 is 49.6% of what number?
8. A grain speculator gained $5460 in 1899, which was 75% of what he gained in 1898. How much did he gain in 1898?

STATEMENT

$\$5460 \div .75 = \text{answer}$

9. A coal dealer sold 37,800 tons of coal in December, which was $87\frac{1}{2}$% of what he sold in January, and $62\frac{1}{2}$% of what he sold in February. How many tons did he sell in January and February, respectively? (Statement only required.)

10. If I own 45% of the stock of a mining company and sell $66\frac{2}{3}$% of my share at full value for $19,500, what is the value of the entire stock of the company?

STATEMENT

$\$19{,}500 \div (.45 \times .66\frac{2}{3}) = \text{answer}$

281. 1. What number increased by 40% of itself equals 847?

OPERATION

$847 \div 1.40 = 605$ *Ans.*

A number increased by 40% of itself equals 140% of the number, or 1.40 times the number. Hence the number equals $847 \div 1.40$, or 605.

2. 588 is 20% more than what number?

3. 1020 is $133\frac{1}{3}\%$ of what number?

4. What number increased by 3.5% of itself equals 414?

5. What fraction increased by 45% of itself equals $\frac{29}{30}$?

282. 1. What number diminished by 20% of itself equals 936?

A number diminished by 20% of itself equals 80%, or .80 of the number. Therefore, 936 must equal .80 of the number. The number equals 936 ÷ .80, or 1170.

OPERATION

$$936 \div .80 = 1170 \textit{ Ans.}$$

2. 3750 is $33\frac{1}{3}\%$ less than what number?

3. 4946.05 is $22\frac{1}{2}\%$ less than what number?

4. What number diminished by 5.3% of itself equals 7765.4?

5. 32 mi. 112 rd. 12 ft. equal $11\frac{1}{9}\%$ less than what distance?

6. 81 mi. 120 rd. equal $12\frac{1}{2}\%$ less than the distance from New York to Philadelphia. What is the entire distance?

7. $42\frac{1}{2}\%$ of \$8400 is 40% less than a man paid for 100 acres of farm land. How much per acre did he pay?

STATEMENT

$$(\overline{\$8400 \times .42\tfrac{1}{2}}) \div .60 \div 100 = \text{answer}$$

8. A speculator bought 380 tons of tan bark, and was forced to sell it for 8% less than he paid for it. Find the cost per ton if he received \$7691.20 for it. (Statement only required.)

283. 1. What per cent of 450 is 90?

We find by division that 90 is .20 of 450, and .20 of anything is 20% of it.

OPERATION

$$90 \div 450 = .20 = 20\%$$

What per cent of:

2. \$180 is \$60?

3. \$130 is \$19.50?

4. 4600 lb. is 575 lb.?

5. 365 tons is 146 tons?

6. 360 days is 300 days?

7. 184 qt. is $5\frac{3}{4}$ gal.?

8. 385 ft. is 161.7 ft.?

9. $530 is $257.05?

10. $\frac{25}{36}$ is $\frac{2}{9}$?

11. $\frac{85}{64}$ is $\frac{14}{45}$?

12. What per cent of 18 hr. 30 min. is 3 hr. 42 min.?

13. Goods to the value of $4086 were destroyed by water. The insurance was $3268.80. The insurance was what per cent of the value of the goods?

14. The population of a city 10 years ago was 25,380. Its population now is 30,456. Find the per cent of increase.

15. An importer failed in business. His property was valued at $15,354.90. What per cent can he pay if he owes $25,380?

16. There were mined from a coal mine 10,860 tons of coal in January, and 9774 tons in February. The amount mined in February is what per cent less than that mined in Jan.?

PROFIT AND LOSS

284. *Profit* and *Loss* are terms used to denote the gain or loss in business transactions.

NOTE 1. — Gains and losses are reckoned on the *cost* of goods. This point should be emphasized by the teacher, as pupils too often estimate the gain or loss on the selling price.

NOTE 2. — A statement, only, of the method of solving the following problems is required, as illustrated in Examples 1, 2, and 3. For further suggestions, see Statements, Art. 279.

PROBLEMS

285. 1. Find the gain on flour bought at $5.50 a barrel and sold at an advance of 20%.

STATEMENT

$$\$5.50 \times .20 = \text{answer}$$

Find the selling price.

STATEMENT

$$\$5.50 \times 1.20 = \text{answer}$$

2. Flour bought at \$5.50 a barrel was sold at \$6.60 a barrel. Find the rate of gain.

STATEMENT

$$(\$6.60 - \$5.50) \div \$5.50 = \text{answer}$$

3. By selling flour at an advance of 20%, \$1.10 is gained on each barrel. Find the cost per barrel.

STATEMENT

$$\$1.10 \div .20 = \text{answer}$$

Note. — In order to stimulate pupils to correct habits of thought, the problems under Profit and Loss are not arranged in cases, thus requiring a change in the process of reasoning in passing from one problem to another.

4. A building lot was sold for \$2765, which was $16\frac{2}{3}\%$ more than it cost. How much did it cost?

5. A merchant realizes 20% profit by selling tea at \$1.50 a pound. What per cent would he lose by selling it at $\$1.13\frac{1}{2}$ per pound?

STATEMENT

$$\$1.50 \div 1.20 = \text{cost}$$
$$(\text{Cost} - \$1.13\tfrac{1}{2}) \div \text{cost} = \text{answer}$$

6. A man bought 2 dozen turkeys and sold them at \$1.75 a pair, gaining 25% on the cost. Find the cost of the 2 dozen.

7. A dealer sold 20 head of cattle for \$1380, thereby gaining 15%. For how much per head should he have sold them to realize a gain of 20%?

8. By selling a quantity of goods for \$7700, a merchant lost 12% of the cost. Find the cost.

9. A merchant tailor made \$144 profit on 24 suits of clothes. If his profit was $33\frac{1}{3}\%$, what was the average selling price per suit?

10. A jeweler sold 8 gold watches for \$690, thereby gaining 15%. How much did the watches cost him apiece?

11. A miller bought 2400 bu. of grain, but owing to a decline in the price he was obliged to sell at a loss of 5%. If he received $1960.80 for it, how much did the grain cost him per bushel?

12. If 80% of a bill of goods is sold for what the whole bill costs, what is the gain per cent?

13. Hats that cost $36 a dozen, are sold at $2.25 apiece. Find the loss per cent.

14. A man lost 5% by selling 50 spring chickens for $28.50. How much apiece did the chickens cost?

15. I bought a carriage for $84, which was 40% less than its value, and sold it for 5% less than its value. What was my gain?

STATEMENT I

$$\$84 \div .60 = \text{value}$$
$$\text{Value} \times .95 = \text{selling price}$$
$$\text{Selling price} - \$84 = \text{gain}$$

STATEMENT II

$$\$84 \div .60 = \text{value}$$
$$(.40 - .05) \times \text{value} = \text{gain}$$

SIMPLE INTEREST

286. *Interest* is money paid for the use of money.

287. The *Principal* is the sum loaned.

288. The *Amount* is the sum of the principal and interest.

289. The *Rate* is the per cent of the principal paid for its use for 1 year.

290. The *Time* is the period for which interest is charged.

NOTE 1.—The rate of interest varies in different states. When no rate is given the legal rate is always understood.

NOTE 2.—By the *common method* of reckoning interest it is customary to regard 30 days as a month and 360 days as a year.

PROBLEMS

291. 1. What is the interest of \$ 800 for 3 yr. at 6%?

STATEMENT

$$\$800 \times .06 = \$48 = \text{int. for 1 yr.}$$
$$\$48 \times 3 = \$144 = \text{int. for 3 yr.}$$

2. What is the interest of \$ 860 for 5 yr. at 6% ?
3. What is the interest of \$ 890 for 4 yr. at $5\frac{1}{2}$% ?
4. What is the interest of \$ 725 for $3\frac{1}{2}$ yr. at 5% ?
5. What is the interest of \$ 850.25 for 2 yr. at 6% ?

292. 1. What is the interest of \$ 500 for 2 yr. 3 mo. at 6% ?

STATEMENT

$$\text{Time} = 2\tfrac{1}{4} \text{ yr.}$$
$$\$500 \times .06 = \$30 = \text{int. for 1 yr.}$$
$$\$30 \times 2\tfrac{1}{4} = \$67.50 = \text{int. for 2 yr. 3 mo.}$$

2. What is the interest of \$ 435 for 4 yr. 4 mo. at $4\frac{1}{2}$% ?
3. What is the interest of \$ 363 for 3 yr. 8 mo. at 6% ?
4. What is the interest of \$ 472.50 for 5 yr. 9 mo. at 8% ?
5. What is the interest of \$ 963.40 for 2 yr. 10 mo. at 6% ?
6. What is the interest of \$ 835.25 for 3 yr. 5 mo. at $5\frac{1}{2}$% ?

293. 1. What is the interest of \$ 700 for 4 yr. 3 mo. 18 da. at 6% ?

SUGGESTION. — 18 da. = $\frac{18}{30}$, or $\frac{3}{5}$ mo., which added to 3 mo. = $3\frac{3}{5}$ mo.; $3\frac{3}{5}$ mo. = $\frac{18}{5}$ mo.; 1 mo. = $\frac{1}{12}$ of a year, and $\frac{18}{5}$ mo. = $\frac{18}{5} \times \frac{1}{12} = \frac{18}{60}$, or $\frac{3}{10}$ yr., which added to 4 yr. = $4\frac{3}{10}$ yr.

2. What is the interest of \$ 900 for 4 yr. 2 mo. 20 da. at 5% ?
3. Find the interest of \$ 386 for 3 yr. 5 mo. 15 da. at 6%.
4. Find the amount of \$ 783 for 2 yr. 7 mo. 10 da. at $4\frac{1}{2}$%.
5. Find the amount of \$ 836 for 5 yr. 8 mo. 5 da. at $5\frac{1}{2}$%.

DENOMINATE NUMBERS

294. A *Denominate Number* is a concrete number whose unit is a fixed measure of quantity. Thus, 5 pounds is a denominate number, because the *pound* is a unit used to measure quantity of weight. Likewise, 4 feet is a denominate number, the *foot* being used to measure quantity of length.

295. Denominate numbers are regarded either as simple or compound.

296. Quantity expressed in a single unit is called a *Simple Quantity*, or a *Simple Denominate Number.* Thus, 5 pounds is a simple denominate number, also 14 gallons, 9 feet, etc.

297. Quantity expressed in several different units is called a *Compound Quantity*, or a *Compound Denominate Number.* Thus, 4 rd. 3 yd. 2 ft. 7 in. is an example of a compound quantity, or compound denominate number.

Long Measure

298. *Long Measure* is used in measuring lines or estimating distances.

299. Table of Long Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

1 mi. = 1760 yd. = 5280 ft. = 63,360 in.

OTHER MEASURES. — 12 lines = 1 in.; 3 barleycorns = 1 in.; 4 in. = 1 hand, used in measuring the height of horses. 3.3 ft. = 1 pace; 6 ft. = 1 fathom, used in measuring depths at sea; 120 fathoms = 1 cable length; 6086.7 ft. = 1 knot, a nautical or geographical mile; 60 geographical, or 69.16 common miles = 1 degree of latitude or longitude at the equator; 3 miles = 1 league; 40 rods = 1 furlong; 8 furlongs = 1 mi.

REDUCTION OF DENOMINATE NUMBERS

300. *Reduction* of Denominate Numbers consists in changing the unit of quantity from one denomination to another without changing the value. Thus, 1 rod = $16\frac{1}{2}$ ft.; 36 in. = 3 ft., are examples of reduction.

301. If the unit is changed to a lower denomination, as 3 yd. = 9 ft., the process is called Reduction Descending; but if the change is to a higher denomination, as 48 in. = 4 ft., the process is called Reduction Ascending.

302. 1. Change 12 rd. 4 yd. 2 ft. to feet.

In 1 rd. there are $5\frac{1}{2}$ yd., and in 12 rd. and 4 yd. there are 12 times $5\frac{1}{2}$ yd., plus 4 yd. $(12 \times 5\frac{1}{2}$ yd.$) + 4$ yd., or 70 yd. In 1 yd. there are 3 ft., and in 70 yd. and 2 ft. there are 70 times 3 ft., plus 2 ft., $(70 \times 3$ ft.$) + 2$ ft., or 212 ft.

OPERATION

rd.	yd.	ft.
12	4	2
$5\frac{1}{2}$		
70		
3		
212 *Ans.*		

303. 1. Change 20 mi. 46 rd. 14 ft. 7 in. to inches.

In 1 mi. there are 320 rd., and in 20 mi. and 46 rd. there are 20 times 320 rd., plus 46 rd. $(20 \times 320$ rd.$) + 46$ rd., or 6446 rd. In 1 rd. there are $16\frac{1}{2}$ ft., and in 6446 rd. and 14 ft. there are 6446 times $16\frac{1}{2}$ ft., plus 14 ft. $(6446 \times 16\frac{1}{2}$ ft.$) + 14$ ft., or 106,373 ft. In 1 ft. there are 12 in., and in 106,373 ft. and 7 in. there are 106,373 times 12 in., plus 7 in. $(106{,}373 \times 12$ in.$) + 7$ in., or 1,276,483 in.

OPERATION

mi.	rd.	ft.	in.
20	46	14	7
320			
6446			
$16\frac{1}{2}$			
106373			
12			
1276483 *Ans.*			

304. Change to feet:

1. 16 rd. 8 ft.
2. 25 rd. 3 yd. 5 ft.
3. 2 mi. 12 rd. 8 ft.
4. 7 mi. 46 rd. 3 yd.
5. 9 mi. 84 rd. $2\frac{1}{2}$ yd.

305. Change to inches:

1. 21 mi. 80 rd. 5 yd.
2. 125 rd. 2 yd. 1 ft.
3. 28 mi. 24 rd. 2 in.
4. 40 mi. 64 rd. 3 yd. 4 in.
5. 51 mi. 89 rd. $4\frac{1}{2}$ yd.

306. 1. Change 11,232 feet to higher denominations.

In 1 yd. there are 3 ft., and in 11,232 ft. there are as many yards as 3 is contained times in 11,232, or 3744 yd. In 1 rd. there are $5\frac{1}{2}$ yd., and in 3744 yd. there are as many rods as $5\frac{1}{2}$ is contained times in 3744, or 680 rd., and 4 yd. remaining. In 1 mi. there are 320 rd., and in 680 rd. there are as many miles as 320 is contained times in 680, or 2 mi., and 40 rd. remaining. Therefore, 11,232 ft. = 2 mi. 40 rd. 4 yd.

OPERATION

$$\begin{array}{r|l} 3 & 11232 \\ \hline 5\frac{1}{2} & 3744 \\ \hline 320 & 680 \;.\;.\;.\; 4\text{ yd.} \\ \hline & \quad\;\; 2 \;.\;.\;.\; 40\text{ rd.} \end{array}$$

2 mi. 40 rd. 4 yd. *Ans.*

307. 1. Change 131,588 inches to higher denominations.

In 1 ft. there are 12 in., and in 131,588 in. there are as many feet as 12 is contained times in 131,588, or 10,965 ft., and 8 in. remaining. In 1 rd. there are $16\frac{1}{2}$ ft., and in 10,965 ft. there are as many rods as $16\frac{1}{2}$ is contained times in 10,965, or 664 rd., and 9 ft. remaining. In 1 mile there are 320 rd., and in 664 rd. there are as many miles as 320 is contained times in 664, or 2 mi., and 24 rd. remaining. Therefore, 131,588 in. = 2 mi. 24 rd. 9 ft. 8 in.

OPERATION

$$\begin{array}{r|l} 12 & 131588 \\ \hline 16\frac{1}{2} & 10965 \;.\;.\;.\; 8\text{ in.} \\ \hline 320 & \quad 664 \;.\;.\;.\; 9\text{ ft.} \\ \hline & \quad\;\;\; 2 \;.\;.\;.\; 24\text{ rd.} \end{array}$$

2 mi. 24 rd. 9 ft. 8 in. *Ans.*

308. Change to higher denominations:

1. 6380 ft.	6. 38,469 in.	11. 384,690 in.
2. 14,685 yd.	7. 43,092 in.	12. 642,071 in.
3. 9684 rd.	8. 38,672 ft.	13. 764,328 ft.
4. 89,643 ft.	9. 683,721 yd.	14. 848,429 ft.
5. 72,865 in.	10. 393,641 in.	15. 972,864 ft.

ADDITION OF DENOMINATE NUMBERS

309. 1. Find the sum of 18 mi. 120 rd. 2 yd. 2 ft. 6 in.; 10 mi. 240 rd. 1 yd. 2 ft. 5 in.; 12 mi. 179 rd. 1 ft. 8 in.; 126 rd. 4 yd. 7 in.

OPERATION

mi.	rd.	yd.	ft.	in.	
18	120	2	2	6	
10	240	1	2	5	
12	179	0	1	8	
	126	4	0	7	
42	26	$3\frac{1}{2}$	1	2	
		$\frac{1}{2}$=	1	6	
42	26	3	2	8	*Ans.*

The numbers are written so that units of the same denomination stand in the same column. The sum of the inches = 26; 26 in. = 2 ft. 2 in. We write the 2 in. under the column of inches, and carry the 2 ft. to the column of feet. The sum of the feet = 7; 7 ft. = 2 yd. 1 ft. We write the 1 ft. under the column of feet, and carry the 2 yd. to the column of yards. The sum of the yards = 9; 9 yd. = 1 rd. $3\frac{1}{2}$ yd. We write the $3\frac{1}{2}$ yd. under the column of yards, and carry the 1 rd. to the column of rods. The sum of the rods = 666; 666 rd. = 2 mi. 26 rd. We write the 26 rd. under the column of rods, and carry the 2 mi. to the column of miles. The sum of the miles = 42, which we write under the column of miles. The $\frac{1}{2}$ yd. is reduced to 1 ft. 6 in. and added to 1 ft. 2 in.

2.

rd.	yd.	ft.	in.
12	4	2	8
16	2	1	10
19	5	0	11
9	3	2	9

3.

rd.	yd.	ft.	in.
14	2	0	5
8	4	1	11
29	5	2	9
46	3	1	8

4.

rd.	ft.	in.
31	12	9
64	7	6
132	3	9
120	15	8

5. Find the sum of 21 mi. 42 rd. 3 yd. 2 ft. 9 in.; 32 mi. 14 rd. 13 ft. 8 in.; 32 rd. 16 ft. 7 in.; 18 mi. 9 ft. 9 in.; 46 mi. 3 yd. 8 in.; 14 ft. 1 yd. 7 in.; 17 mi. 146 rd. 1 in.; 3 mi. 3 yd. 3 ft. 5 in.

6. To 15 mi. 6 yd. 1 ft. 5 in. add 65 mi. 12 in. and 36 rd. 2 ft.

SUBTRACTION OF DENOMINATE NUMBERS

310. 1. From 21 mi. 125 rd. 5 yd. 1 ft. 9 in. take 10 mi. 136 rd. 2 yd. 2 ft. 7 in.

OPERATION

mi.	rd.	yd.	ft.	in.	
21	125	5	1	9	
10	136	2	2	7	
10	309	2	2	2	*Ans.*

The subtrahend is written under the minuend so that units of the same denomination are in the same column. Beginning at inches to subtract, we proceed as follows: 7 in. from 9 in. leave 2 in., which we write under the column of inches. Since 2 ft. cannot be taken from 1 ft., we take 1 yd. (1 yd. = 3 ft.) and add it to the 1 ft., thus making 4 ft.; 2 ft. from 4 ft. leave 2 ft., which we write under the column of feet. 2 yd. from 4 yd. leave 2 yd., which we write under the column of yards. Since 136 rd. cannot be taken from 125 rd., we take 1 mi. (1 mi. = 320 rd.) and add it to the 125 rd., thus making 445 rd.; 136 rd. from 445 rd. leave 309 rd., which we write under the column of rods. 10 mi. from 20 mi. leave 10 mi.

2.

rd.	ft.	in.
28	9	6
12	14	4

3.

rd.	yd.	ft.	in.
84	2	1	6
21	4	2	8

4.

mi.	rd.	yd.	in.
49	38	4	9
18	147	5	11

5. Subtract 32 mi. 310 rd. 3 yd. 2 ft. 9 in. from 84 mi. 5 yd.

6. Subtract 39 mi. 12 ft. 6 in. from 126 mi. 1 rd. 1 yd. 1 ft. 1 in.

MULTIPLICATION OF DENOMINATE NUMBERS

311. 1. Multiply 3 mi. 59 rd. 3 yd. 5 ft. 8 in. by 7.

OPERATION

mi.	rd.	yd.	ft.	in.	
3	59	3	5	8	
				7	
22	99	1	0	8	*Ans.*

7 times 8 in. = 56 in. = 4 ft. 8 in. We write the 8 in. under the inches, and add the 4 ft. to the product of feet. 7 times 5 ft., plus 4 ft. (7 × 5 ft.) + 4 ft. = 39 ft. = 13 yd. exactly. We write a cipher under feet, and add the 13 yd. to the product of yards. 7 times 3 yd., plus 13 yd. (7 × 3 yd.) + 13 yd. = 34 yd. = 6 rd. 1 yd. We write the 1 yd. under yards, and add the 6 rd. to the product of rods. 7 times 59 rd., plus 6 rd. (7 × 59 rd.)

+ 6 rd. = 419 rd. = 1 mi. 99 rd. We write the 99 rd. under rods, and add the 1 mi. to the product of miles. 7 times 3 mi., plus 1 mi. (7 × 3 mi.) + 1 mi. = 22 mi., which we write under miles.

2.

mi.	rd.	ft.	in.
29	164	10	3
			15

3.

mi.	rd.	yd.
49	129	7
		16

4.

mi.	rd.	yd.	ft.
149	39	2	9
			17

5.

mi.	rd.	ft.	in.
29	164	9	5
			13

6.

rd.	yd.	ft.	in.
72	4	2	9
			18

7.

mi.	rd.	yd.	ft.	in.
24	30	3	2	11
				21

8. Multiply 349 mi. 240 rd. 9 ft. by 24.
9. Multiply 149 rd. 4 yd. 2 ft. 7 in. by 28.
10. Multiply 120 mi. 129 rd. 5 yd. 2 ft. by 42.
11. Multiply 475 mi. 65 rd. 14 ft. 9 in. by 56.
12. Multiply 319 mi. 42 rd. 13 ft. 8 in. by 67.
13. Multiply 118 rd. 5 yd. 11 in. by 94.

Note. — When the multiplier is large and can be separated into two factors, frequently it will be found convenient to multiply successively by its factors.

DIVISION OF DENOMINATE NUMBERS

312. 1. Divide 129 mi. 50 rd. 7 ft. by 8.

OPERATION

	mi.	rd.	ft.	
8)	129	50	7	
	16	46	5	*Ans.*

We write the divisor at the left of the dividend and proceed as follows: $\frac{1}{8}$ of 129 mi. = 16 mi. and 1 mi. remaining. We write the 16 in the quotient under miles, and change the 1 mi. to its equivalent, 320 rd., which we add to the 50 rd., thus making 370 rd. $\frac{1}{8}$ of 370 rd. = 46 rd. and 2 rd. remaining. We write the 46 in the quotient under rods, and change the 2 rd. to its equivalent, 33 ft., which added to 7 ft. make 40 ft. $\frac{1}{8}$ of 40 ft. = 5 ft., which we write in the quotient under feet.

2.

	rd.	yd.	ft.
9)	69	4	6

3.

	yd.	ft.	in.
11)	303	0	11

4.

	rd.	ft.	in.
12)	54	14	9

5.

	mi.	rd.	ft.	in.
7)	49	160	9	4

6.

	mi.	rd.	yd.	ft.
6)	95	0	1	8

7.

	rd.	yd.	ft.	in.
8)	191	4	2	10

313. 1. Divide 79 mi. 120 rd. 14 ft. 7 in. by 75.

OPERATION

```
     mi.  rd.  ft.  in.
75)79   120   14   7(1 mi.
   75
    4
  320
75)1400 rd.(18 rd.
   75
   650
   600
    50
   16½
 75)839 ft.(11 ft.
    75
     89
     75
     14
     12
  75)175 in.(2⅓ in.
     150
      25   1
      -- = -
      75   3
```

1 mi. 18 rd. 11 ft. $2\frac{1}{3}$ in. *Ans.*

79 mi. divided by 75 = 1 mi., and 4 mi. remaining. We write the 1 mi. in the quotient and reduce the 4 mi. to its equivalent, 1280 rd., to which we add the 120 rd. in the dividend, making 1400 rd. 1400 rd. divided by 75 = 18 rd., and 50 rd. remaining. We write the 18 rd. in the quotient and reduce the 50 rd. to its equivalent, 825 ft., to which we add the 14 ft. in the dividend, making 839 ft. 839 ft. divided by 75 = 11 ft., and a remainder of 14 ft. We write the 11 ft. in the quotient and reduce the 14 ft. to its equivalent, 168 in., to which we add the 7 in. in the dividend, making 175 in. 175 in. divided by 75 = $2\frac{1}{3}$ in. Therefore, 79 mi. 120 rd. 14 ft. 7 in. divided by 75 = 1 mi. 18 rd. 11 ft. $2\frac{1}{3}$ in.

2. Divide 1313 rd. 4 yd. 2 ft. 9 in. by 60.

3. If a man traveled 325 mi. 226 rd. 10 ft. in 26 days, what was his average distance per day?

4. It required 3 men 10 days to build a fence around a square field. Find the length of one side of the field, if they built 20 rd. 12 ft. per day.

5. If a passenger train runs from Wilkesbarre to Scranton, a distance of 18 mi. 160 rd. 8 ft., in 25 minutes, how far, on an average, does it run per minute?

314. When the divisor is a denominate number of the same kind as the dividend, we reduce both dividend and divisor to the same denomination, usually to the lowest in either, and divide as in simple whole numbers.

1. Divide 40 rd. 4 yd. 6 in. by 6 rd. 4 yd. 1 ft. 1 in.

OPERATION

40 rd. 4 yd. 6 in. = 8070 in.

6 rd. 4 yd. 1 ft. 1 in. = 1345 in.

8070 ÷ 1345 = 6 *Ans.*

2. If a man travels 3 mi. 132 rd. in 1 hour, how many hours will it take him to travel 27 mi. 96 rd.?

3. How many steps, each 2 ft. 8 in. long, will a pedestrian take in walking 25 mi.?

DENOMINATE FRACTIONS

315. A *Denominate Fraction* is one or more of the equal parts of a denominate unit. Thus, $\frac{3}{4}$ mi. and $\frac{2}{3}$ ft. are examples of denominate fractions. .2 mi. and .75 yd. are examples of denominate decimal fractions.

316. 1. Express $\frac{3}{4}$ of a rod in integers of lower denominations.

OPERATION

$\frac{3}{4}$ rd. $= \frac{3}{4}$ of $16\frac{1}{2}$ ft. $= \frac{3}{4} \times \frac{33}{2}$ ft. $= \frac{99}{8}$ ft. $= 12\frac{3}{8}$ ft.

$\frac{3}{8}$ ft. $= \frac{3}{8}$ of 12 in. $= \frac{3}{8} \times 12$ in. $= \frac{36}{8}$ in. $= 4\frac{1}{2}$ in.

$\frac{3}{4}$ rd. $= 12$ ft. $4\frac{1}{2}$ in. *Ans.*

317. 1. Express .75 rd. in integers of lower denominations.

OPERATION

$.75 \text{ rd.} = .75 \times 16\frac{1}{2} \text{ ft.} = 12.375 \text{ ft.}$

$.375 \text{ ft.} = .375 \times 12 \text{ in.} = 4.5 \text{ in.}$

$.75 \text{ rd.} = 12 \text{ ft. } 4.5 \text{ in.}$ *Ans.*

318. Express in integers of lower denominations:

1. $\frac{1}{8}$ of a mile.
2. $\frac{2}{3}$ of a mile.
3. $\frac{9}{10}$ of a mile.
4. .75 of a mile.
5. .1875 of a mile.
6. $\frac{4}{5}$ of a rod.
7. $\frac{7}{8}$ of a rod.
8. $\frac{3}{7}$ of a mile.

319. 1. Find the sum of $\frac{5}{9}$ mi. $\frac{2}{3}$ rd. .375 yd.

OPERATION

	rd.	yd.	ft.	in.
$\frac{5}{9}$ mi. =	177	4	0	10
$\frac{2}{3}$ rd. =		3	2	0
.375 yd. =			1	$1\frac{1}{2}$
	178	$2\frac{1}{2}$	0	$11\frac{1}{2}$
		$\frac{1}{2}$ =	1	6
	178	2	2	$5\frac{1}{2}$ *Ans.*

2. Add .45 mi., $\frac{5}{6}$ rd., $\frac{2}{3}$ yd., .32 ft.
3. Add $\frac{7}{9}$ mi., $\frac{3}{7}$ rd., $\frac{4}{5}$ ft.
4. Add .625 mi., .25 rd., .375 yd.

320. 1. From $\frac{3}{7}$ of a mile take $\frac{2}{3}$ of 140 rd.

OPERATION

	rd.	ft.	in.
$\frac{3}{7}$ mi. =	137	2	$4\frac{2}{7}$
$\frac{2}{3}$ of 140 rd. =	93	5	6
	43	$12\frac{1}{2}$	$10\frac{2}{7}$
		$\frac{1}{2}$ =	6
	43	13	$4\frac{2}{7}$ *Ans.*

321. 1. Find $\frac{5}{6}$ of 3 mi. 21 rd. 11 ft.

OPERATION

$$\begin{array}{lrrrl}
 & \text{mi.} & \text{rd.} & \text{ft.} & \\
 & 6)\,3 & 21 & 11 & \\ \hline
\frac{1}{6}\text{ of 3 mi. 21 rd. 11 ft.} = & & 163 & 10 & 1\text{ in.} \\
 & & & & 5 \\ \hline
 & 2 & 178 & \frac{1}{2} & 5 \\
 & & & \frac{1}{2} = & 6 \\ \hline
\frac{5}{6}\text{ of 3 mi. 21 rd. 11 ft.} = & 2 & 178 & 0 & 11\ \textit{Ans.}
\end{array}$$

322. 1. From $9\frac{2}{3}$ mi. take 6 mi. 165 rd. 9 ft.

2. Take $\frac{1}{4}$ of 17 mi. 310 rd. 8 ft. from 18.375 mi.
3. From $\frac{3}{7}$ of 42 mi. take $\frac{3}{8}$ of 16 mi. 1 rd. 9 ft.
4. Take $\frac{7}{9}$ of a rod from $\frac{7}{9}$ of a mile.
5. Take 214.375 rd. from 3.625 mi.

323. 1. Reduce 177 rd. 12 ft. 10 in. to the fraction of a mile.

OPERATION

$$177\text{ rd. }12\text{ ft. }10\text{ in.} = 35{,}200\text{ in.}$$
$$1\text{ mi.} = 63{,}360\text{ in.}$$
$$35{,}200 \div 63{,}360 = \tfrac{35200}{63360} = \tfrac{5}{9}\text{ mi. } \textit{Ans.}$$

2. Reduce 2 yd. 2 ft. 3 in. to the fraction of a rod.
3. Reduce 213 rd. 5 ft. 6 in. to the fraction of a mile.
4. Reduce 3 yd. 2 ft. 6 in. to the fraction of a mile.
5. Reduce 2 rd. 4 yd. 2 ft. to the fraction of 3 rods.
6. Reduce 90 rd. 3 yd. to the fraction of $\frac{1}{2}$ mile.
7. Reduce 147 rd. 3 ft. $3\frac{1}{2}$ in. to the fraction of a mile.
8. Reduce 125 rd. 4 yd. 2 ft. 6 in. to the fraction of a mile.
9. Reduce 128 rd. 4 yd. 1 ft. 4 in. to the fraction of 2 miles.
10. Reduce 9 mi. 10 rd. to the fraction of 72 mi. 80 rods.
11. Reduce 190 yd. 2 ft. 8 in. to the fraction of $\frac{1}{8}$ of a mile.

324. 1. Reduce 147 rd. 3 ft. 3.6 in. to the decimal of a mile.

OPERATION I

$$
\begin{array}{r|l}
12 & 3.6 \text{ in.} \\ \hline
 & .3 \text{ ft.} \\
 & 3. \\ \hline
16\frac{1}{2} & 3.3 \\ \hline
 & .2 \text{ rd.} \\
 & 147. \\ \hline
320 & 147.2 \text{ rd.} \\ \hline
 & .46 \text{ mi. } \textit{Ans.}
\end{array}
$$

$3.6 \div 12 = .3$ ft., which added to the 3 ft. $= 3.3$ ft.

$3.3 \div 16\frac{1}{2} = .2$ rd., which added to the 147 rd. $= 147.2$ rd.

$147.2 \div 320 = .46$ mi. *Ans.*

OPERATION II

147 rd. 3 ft. 3.6 in. = 29,145.6 in.

1 mi. = 63,360 in.

$29{,}145.6 \div 63{,}360 = .46$ mi. *Ans.*

2. Reduce 2 yd. 1 ft. 1.5 in. to the decimal of a rod.
3. Reduce 3 ft. 8.55 in. to the decimal of a rod.
4. Reduce 46 rd. 6 ft. 7.2 in. to the decimal of a mile.
5. Reduce 139 rd. 1 yd. 3.6 in. to the decimal of a mile.
6. Reduce 88 rd. 3 yd. 1 ft. 6.72 in. to the decimal of a mile.

325. 1. Reduce $\frac{1}{234}$ of a rod to the fraction of an inch.

OPERATION

$$\tfrac{1}{234} \text{ rd.} = \tfrac{1}{234} \times 16\tfrac{1}{2} \text{ ft.} = \tfrac{1}{234} \times \tfrac{33}{2} \text{ ft.} = \tfrac{11}{156} \text{ ft.}$$

$$\tfrac{11}{156} \text{ ft.} = \tfrac{11}{156} \times 12 \text{ in.} = \tfrac{11}{156} \times \tfrac{12}{1} \text{ in.} = \tfrac{11}{13} \text{ in. } \textit{Ans.}$$

2. Reduce $\frac{1}{264}$ of a rod to the fraction of an inch.
3. Reduce $\frac{4}{99}$ of a rod to the fraction of a foot.
4. Reduce $\frac{5}{1188}$ of a rod to the fraction of an inch.
5. Reduce $\frac{1}{6600}$ of a mile to the fraction of a foot.
6. Reduce $\frac{5}{380160}$ of a mile to the fraction of an inch.

326. 1. Reduce $\frac{3}{4}$ of a foot to the fraction of a mile.

OPERATION I

$$\tfrac{3}{4}\text{ ft.} \div 16\tfrac{1}{2}\text{ ft.} = \tfrac{3}{4} \times \tfrac{2}{33} = \tfrac{1}{22}\text{ rd.}$$
$$\tfrac{1}{22}\text{ rd.} \div 320\text{ rd.} = \tfrac{1}{22} \times \tfrac{1}{320} = \tfrac{1}{7040}\text{ mi. } \textit{Ans.}$$

OPERATION II

$$1\text{ mi.} = 5280\text{ ft.}$$
$$\frac{3}{4}\text{ ft.} \div 5280\text{ ft.} = \frac{\cancel{3}}{4} \times \frac{1}{\underset{1760}{\cancel{5280}}} = \frac{1}{7040}\text{ mi. } \textit{Ans.}$$

2. Reduce $\frac{4}{5}$ of a yard to the fraction of a mile.
3. Reduce $\frac{3}{4}$ of a foot to the fraction of a rod.
4. What part of 2 miles is $\frac{5}{12}$ of a foot?
5. Reduce $\frac{7}{8}$ of an inch to the fraction of a yard.
6. Express $\frac{2}{3}$ ft. + $\frac{3}{4}$ yd. as the fraction of a rod.
7. Reduce 3.96 in. to the decimal of a yard.
8. Express $\frac{3}{4}$ in. + $\frac{2}{3}$ ft. as the fraction of a mile.

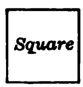
MEASURES

Surface or Square Measure

327. A *Surface* is that which has the dimensions *length* and *breadth.*

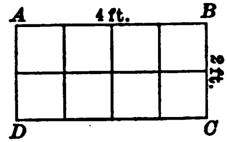
328. A *Rectangle* is a plane surface bounded by four straight lines, and having four square corners.

329. A *Square* is a rectangle whose four sides are of equal length.



330. The *Area* of a rectangular surface is the number of square units of a certain value it contains, and is expressed by the product of the two numbers representing its length and breadth.

If we suppose the rectangle *ABCD*, at the right, to be 4 ft. long and 2 ft. wide, the unit of measure will be the *Square Foot*. If the rectangle is divided by lines as represented in the illustration, it will be found to contain 2 times 4, or 8 *square feet*.



331. Since the product of the length and breadth of a rectangle equals its area, either side will equal the area divided by the other side.

332. TABLE OF SQUARE MEASURE

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$	square yards, or	= 1 square rod, or perch
$272\frac{1}{4}$	square feet	
160	square rods, or perches	= 1 acre (A.)
640	acres	= 1 square mile (sq. mi.)

NOTE. — The principles of reduction as explained in Long Measure apply to all denominate numbers.

PROBLEMS

333. 1. Reduce 18 sq. yd. 4 sq. ft. 27 sq. in. to sq. in.

2. Reduce 20 sq. rd. 17 sq. yd. 2 sq. ft. to sq. ft.

3. Reduce 8 A. 150 sq. rd. 27 sq. yd. to sq. ft.

4. Reduce 129,600 sq. in. to sq. yd.

5. How many acres are there in 212,582 sq. ft.?

6. Change $\frac{7}{12}$ of an acre to lower denominations.

7. Change .225 of an acre to lower denominations.

8. Change $\frac{7}{12}$ of a sq. mi. to lower denominations.

9. From $\frac{8}{4}$ of 9 A. take 5 A. 159 sq. rd. 30 sq. yd. 8 sq. ft.

10. How many acres are there in a rectangular piece of land 24.9 rd. long, and 18.8 rd. wide?

11. A rectangular field containing 40 acres is 128 rd. long. How many rods of fence will be required to inclose it?

12. How many square feet of boards will be required to lay a walk 5 ft. wide around the inside of a garden 120 ft. long and 100 ft. wide?

13. How many square yards of surface are contained in the walls and ceiling of a room 48 ft. long, 45 ft. wide, and 15 ft. high?

14. How many yards of ingrain carpet, 36 in. wide, will be required to cover the floor of the room described in No. 13?

NOTE.—In estimating flooring, roofing, slating, etc., the *square*, which consists of 100 sq. ft., is used. The *perch* is the same as the square rod. The *rood*, which is found now only in old land titles and surveys, is equal to 40 sq. rd. A section of land is one mile square, or 640 acres.

15. How much will it cost to slate a roof 24 ft. long, and 18 ft. wide, at $\$10\frac{1}{2}$ a square?

16. What is the width of a field that contains 64 acres, the length being 320 rd.?

17. What is the value of a piece of timber land 5 mi. long and 4 mi. wide, at $\$42.87\frac{1}{2}$ an acre?

SURVEYORS' LINEAR MEASURE

334. *Surveyors' Linear Measure* is used by surveyors and engineers in measuring distances, as roads, dimensions of land, etc.

335. TABLE

7.92 inches	= 1 link (li.)
25 links	= 1 rod
4 rods or 100 links	= 1 chain (ch.)
80 chains	= 1 mile

336. *Gunter's chain*, named after its inventor, Edmund Gunter, is 792 in. long, or 66 ft., or 4 rods. Surveyors generally use a chain or tape 100 feet long.

Surveyors' Square Measure

337. *Surveyors' Square Measure* is used by surveyors in computing the area of land.

338. TABLE

625 square links	= 1 perch, or square rod
16 perches, or 10,000 sq. li.	= 1 square chain
10 square chains	= 1 acre
640 acres	= 1 square mile
36 square miles (6 miles square)	= 1 township

PROBLEMS

339. 1. A rectangular piece of land 32 chains long and 16 chains wide contains how many acres?

2. A rectangular piece of land is 50 links long and 25 links wide. How many square rods does it contain?

3. A street 33 chains long and 50 links wide contains how many square rods?

4. How many rods are there around a four-sided field whose sides measure 35 ch. 20 li., 16 ch. 18 li., 33 ch. 12 li., 18 ch. 9 li., respectively?

5. How many acres and perches are there in 218.75 sq. ch.?

6. How many statute miles in 180 knots?

7. How many geographic miles in 8 statute miles and 366.9 ft.?

8. A vessel was sunk in $15\frac{1}{2}$ fathoms of water. How many feet deep was the water?

9. How many feet high is a horse which measures $15\frac{1}{2}$ hands high?

10. A rectangular field containing 16 acres is 10 chains wide. Find the cost of fencing it at $.95 a rod.

11. What part of 9 chains square is 27 square chains?

12. Change $\frac{5}{6}$ of a chain to rods, feet, and inches.

13. How much will it cost to pave an alley 252 ft. long and $10\frac{1}{2}$ ft. wide, at $1.75 a sq. yd.?

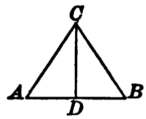
14. How much will it cost to slate a roof 32 ft. 10 in. long, each side being 18 ft. wide, at $10.75 a square (100 sq. ft.)?

15. Find the cost of plastering the sides, ceiling, and one end of a storeroom 48 ft. deep, 26 ft. wide, and 18 ft. high, at 38¢ a sq. yd., deducting wainscoting 6 ft. high.

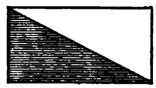
16. Find the cost of wainscoting the sides and rear of the room in Ex. 15, at 37¢ a sq. yd., the wainscoting to be 6 ft. high.

The Triangle

340. A *Triangle* is a plane surface bounded by three straight lines; as *A, B, C.* The side upon which the triangle seems to stand, as *AB,* is the *Base;* the point *C,* the *Vertex;* the perpendicular distance from the vertex to the base, as *CD,* the *Altitude.*



341. It will be seen that the area of a triangle is one half the area of a rectangle having the same base and altitude. Hence,



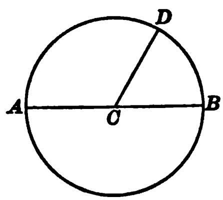
The area of a triangle equals one half the product of the base and altitude.

342. *Either the base or the altitude of a triangle equals twice the quotient obtained by dividing the area by the other dimension.*

The Circle

343. A *Circle* is a plane figure bounded by a curved line, called its *Circumference,* all points of which are equally distant from a point within called the center.

344. The *Diameter* of a circle is a straight line drawn from any point in the circumference through the center, and terminating in the circumference opposite; as *AB*.



345. The *Radius* of a circle is a straight line drawn from the center to the circumference; as *CD*. It is equal to one half the diameter.

346. An *Arc* of a circle is any part of the circumference, as *AD* and *DB*.

347. *The circumference of a circle equals the diameter multiplied by* 3.1416.

348. *The diameter of a circle equals the circumference divided by* 3.1416, *or multiplied by* .3183.

349. *The area of a circle equals the circumference multiplied by one fourth of the diameter, or the square of the radius multiplied by* 3.1416.

NOTE.—The *square* of a number is the result obtained by using the number twice as a factor. Thus, 9 is the square of 3, since $3 \times 3 = 9$; and 25 is the square of 5, since $5 \times 5 = 25$.

PROBLEMS

350. 1. What is the area of a triangle whose base is 16 ft. and altitude 10 ft.?

2. Find the area of a triangle whose base is 15 yd. and altitude 1 rd.

3. Find the area of the gable end of a house 32 ft. wide and 8 ft. high.

4. A triangular piece of land whose base is 28 rd. contains 1 A. 64 sq. rd. What is its altitude?

5. The area of a triangular piece of land whose altitude is 5 chains is 7 A. 120 sq. rd. Find the base.

6. What is the circumference of a circle whose diameter is $31\frac{1}{2}$ in.?

7. What is the radius of a circle whose circumference is 28.2744 ft.?

8. Find the area of a circular piece of land whose circumference is $\frac{1}{2}$ mi.

9. What is the area of a circular race track whose circumference is 1 mi.?

10. Find the area of the largest circle that can be inscribed in a yard square.

11. Find the area of a circle whose radius is 2640 ft.

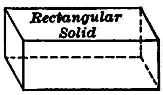
12. How many acres are there in a circular field whose diameter is 44 rd.?

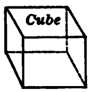
13. Find the area of a circular piece of land whose circumference is $1\frac{1}{8}$ mi.

Measures of Volume

351. A *Solid,* or *Volume,* is that which has the dimensions *length, breadth,* and *thickness,* or *height.*

352. A *Rectangular Solid* is one which is bounded by six rectangles.

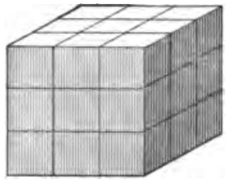




353. A *Cube* is a solid which is bounded by six equal squares.

354. The *Solidity,* or *Contents,* of a solid or volume is the quantity of space it occupies, and is expressed by the number of times it contains a cubic unit of a certain denomination used as a measure.

Let the figure in the margin represent a solid with perpendicular edges 3 ft. long, 3 ft. wide, and 3 ft. high, — a cubic yard. In one row of cubes there are 3 cubic feet; and as each layer consists of 3 rows, there are 3×3, or 9 cubic feet in one layer; and in 3 layers there are 3×9, or 27 cubic feet, which equals the product of the length, breadth, and thickness. $3 \times 3 \times 3 = 27$.



355. The number of cubic units in any six-sided solid or volume having its adjacent edges perpendicular to each other, is equal to the product of its length, breadth, and thickness.

Cubic, or Solid, Measure

356. *Cubic*, or *Solid*, *Measure* is used in measuring all things that have the dimensions length, breadth, and thickness.

Table

357.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
16 cubic feet	= 1 cord foot (cd. ft.)
8 cord feet, or 128 cubic feet	= 1 cord of wood

PROBLEMS

358. 1. How many cubic inches are there in 18 cu. yd. 13 cu. ft.?

2. Reduce 10 cu. yd. 22 cu. ft. 840 cu. in. to cubic inches.

3. Reduce 846,979 cu. in. to cubic yards.

4. Add 8 cu. yd. 25 cu. ft. 1240 cu. in.; 23 cu. yd. $16\frac{2}{3}$ cu. ft.

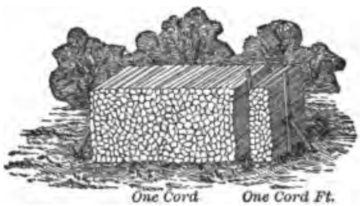
5. How many loads of earth will be removed in digging a cellar 36 ft. long, 24 ft. wide, and 8 ft. deep? (A load = 1 cu. yd.)

6. Divide 193 cu. yd. 26 cu. ft. 329 cu. in. by 11.

7. Change 21 cu. ft. 1036 cu. in. to the decimal of a cubic yard.

8. Reduce 6 cu. ft. 1296 cu. in. to the fraction of a cubic yard.

9. A stone wall 56 ft. long, 2 ft. 6 in. thick, contains 770 cu. ft. What is its height?



359. A cord of wood is usually represented by a pile 8 ft. long, 4 ft. wide, and 4 ft. high. It therefore contains $8 \times 4 \times 4$, or 128 cu. ft.

The *Cord Foot*, which is practically obsolete, is $\frac{1}{8}$ of a cord, or 16 cu. ft.; that is, it is *one foot* in length of the cord.

PROBLEMS

360. 1. How many cords are there in a pile of wood 24 ft. long, 12 ft. high, and 4 ft. wide?

The number of cubic feet in the pile $= 24 \times 12 \times 4 = 1152$. Dividing 1152 by 128, we get 9, the number of cords.

STATEMENT

$$(24 \times 12 \times 4) \div 128 = \text{answer}$$

OPERATION

$$24 \times 12 \times 4 = 1152 \text{ cu. ft. in pile}$$

$$1152 \div 128 = 9 \text{ cords}$$

2. How many cords of wood are there in a pile 32 ft. long, $9\frac{1}{2}$ ft. high, and 4 ft. wide?

STATEMENT

$$(32 \times 9\tfrac{1}{2} \times 4) \div 128 = \text{answer}$$

3. Find the value of a pile of wood 240 ft. long, 8 ft. wide, and 12 ft. high, at \$3.25 a cord.

STATEMENT

$$\$3.25 \times \overline{(240 \times 8 \times 12) \div 128} = \text{answer}$$

4. Find the value of a load of tan bark 12 ft. long, $3\frac{1}{2}$ ft. wide, and $5\frac{1}{2}$ ft. high, at \$3.50 a cord.

5. A shed 12 ft. wide and 16 ft. high contains 475 cords of tan bark. What is the length of the shed?

Board Measure

361. A *Board Foot* is a square foot one inch thick. Therefore it will readily be seen that a cubic foot contains 12 board feet. Hence, to reduce cubic feet to board feet we multiply by 12, and to reduce board feet to cubic feet we divide by 12.

362. Boards are usually quoted and sold by the thousand. A board 1 ft. wide, 1 in. thick, and 12 ft. long contains 12 board feet.

Boards less than 1 in. thick are considered the same, in commerce, as boards 1 in. thick; that is, a board $\frac{3}{4}$ or $\frac{1}{2}$ of an inch thick, 1 ft. wide, and 12 ft. long is regarded as containing 12 board feet.

But a board $1\frac{1}{4}$ in. or $1\frac{1}{2}$ in. thick contains $1\frac{1}{4}$ times or $1\frac{1}{2}$ times as many board feet as a board 1 in. thick. Thus, a board $1\frac{1}{4}$ in. thick, 1 ft. wide, and 12 ft. long contains $1\frac{1}{4} \times 1 \times 12$, or 15 board feet.

PROBLEMS

363. 1. Find the number of board feet in a board 15 ft. long and 16 in. wide.

By the first method we change the width, 16 in., to its equivalent, $1\frac{1}{3}$ ft., and then multiply the length by the width, and obtain 20, the number of board feet in the board.

By the second method we multiply the length in feet by the number expressing the width in inches, and divide the product obtained by 12. This method, which is practically the same as the first, is the one generally used.

STATEMENT

$$(16 \div 12) \times 15 = \text{answer}$$

OPERATION I

$$16 \div 12 = 1\tfrac{1}{3}\text{ ft.} = \text{width}$$
$$1\tfrac{1}{3} \times 15 = 20, \text{ board feet.}$$

OPERATION II

$$(16 \times 15) \div 12 = 20 \quad \textit{Ans.}$$

2. Find the contents of a board 16 ft. long and 15 in. wide.

3. Find the contents of a board 15 ft. long, 16 in. wide, and $1\frac{3}{4}$ in. thick.

4. How many board feet are there in 25 planks 18 ft. long, 16 in. wide, and $1\frac{1}{2}$ in. thick?

5. How many board feet are there in 30 planks 12 ft. long, 10 in. wide, and 2 in. thick?

NOTE.—When a board tapers regularly, we find the average width by taking half the sum of the two ends.

6. Find the contents of a board 20 ft. long, 18 in. wide at one end, and 10 in. wide at the other.

STATEMENT

$$\overline{(18 + 10) \div 2} \times 20 \div 12 = \text{answer}$$

OPERATION

$$(18\text{ in.} + 10\text{ in.}) \div 2 = 14\text{ in., average width}$$
$$(14 \times 20) \div 12 = 23\tfrac{1}{3}, \text{ board feet}$$

7. How many board feet are there in a joist 8 in. by $2\frac{1}{2}$ in., and 16 ft. long?

A joist $2\frac{1}{2}$ in. thick and 8 in. wide is equal to a board 1 in. thick and $2\frac{1}{2}$ times 8 in., or 20 in., wide. Hence, squared lumber may be expressed in board feet by multiplying the length in feet by the width and thickness in inches and dividing the product by 12, as shown in the operation.

OPERATION

$$(2\frac{1}{2} \times 8 \times 16) \div 12 = 26\frac{2}{3} \quad \textit{Ans.}$$

How many board feet are there in:

8. 20 joists, 3 in. by 9 in., and 18 ft. long?

9. 2 pieces of square timber, 7 in. by 7 in., and 14 ft. long?

10. A girder, 10 in. by 16 in., and 32 ft. long?

11. 32 rafters, 3 in. by 8 in., and 18 ft. long?

Excavations and Masonry

364. *Earth, sand,* and *gravel* are usually estimated by the cubic yard, or load.

365. *Stonework* is generally estimated by the perch; sometimes it is estimated by the cubic foot or cubic yard. A *Perch* of stone or of masonry contains $24\frac{3}{4}$ cubic feet. A perch of stone when laid in mortar will make about 21 cubic feet of wall.

NOTE. — Masons and bricklayers, in estimating labor, measure the walls on the outside, and make no deductions for doors, windows, and corners, unless otherwise specified by contract. But in estimating materials allowance is made for openings and corners.

PROBLEMS

366. 1. Find the cost of digging a cellar 32 ft. long, 22 ft. wide, and 6 ft. deep, at $ .36 a load.

STATEMENT

$$\$.36 \times \overline{(32 \times 22 \times 6) \div 27} = \text{answer}$$

2. What will be the cost of excavating a street 1000 ft. long, 50 ft. wide, to the depth of 18 in., at 35¢ a load?

3. How much will it cost to dig a well 6 ft. in diameter and 20 ft. deep, at 45¢ a cubic yard?

NOTE.—Multiply the area of the base by the depth to find the contents. To find area of the base, see Art. 349.

4. Find the cost of digging a cistern 10 ft. in diameter and 6 ft. deep, at 36¢ a cubic yard.

5. How much will it cost to build a stone wall 4 rd. long, 4 ft. high, and 2 ft. thick, at $6.75 a perch?

STATEMENT

$$\$6.75 \times \overline{(16\tfrac{1}{2} \times 4 \times 4 \times 2) \div 24\tfrac{3}{4}} = \text{answer}$$

6. How many perches of masonry are there in the foundation walls of a building 40 ft. long and 28 ft. wide, with a center wall running lengthwise, the walls being 6 ft. high and $2\frac{1}{2}$ ft. thick?

NOTE.—Brickwork is estimated by the thousand bricks. Bricks vary greatly in dimensions and style. In making estimates it is customary to allow 1000 bricks to make 48 cu. ft. of wall, or 21 bricks to make 1 cu. ft.

7. How many bricks are required for a partition wall 36 ft. long, 20 ft. high, and 9 in. thick?

8. Find how many bricks are required to build the outside walls of a house 32 ft. long, 24 ft. wide, 18 ft. high, and 18 in. thick, deducting for 3 doors 6 ft. 8 in. by $3\frac{1}{2}$ ft., and 10 windows 6 ft. by 4 ft. Count corners but once.

9. How many bricks are required for a wall 55 feet high, 39 feet long, and 11 in. thick?

LIQUID MEASURE

367. The standard unit of *Liquid Measure* is the *Gallon*, which contains 231 cubic inches.

TABLE

368.

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
63 gallons	= 1 hogshead (hhd.)

NOTE.—A pint of water weighs about one pound. A cubic foot of distilled water weighs about 1000 ounces Avoirdupois weight. The barrel and hogshead do not express fixed quantities; they vary in different states. The values given them in the table are generally used in estimating the capacity of cisterns and reservoirs, etc. Milk, beer, and ale used to be sold by the *beer* gallon, which contains 282 cu. in. It is now seldom used.

DRY MEASURE

369. The standard unit of *Dry Measure* is the *Bushel.* It contains 2150.42 cubic inches. Dry measure is used in measuring grain, fruit, lime, etc.

TABLE

370.

2 pints	= 1 quart
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

NOTE.—Grains, seeds, and small fruits are sold by stricken measure; that is, the measure must be *even full.* Coarse vegetables, large fruits, and bulky articles, such as potatoes, turnips, apples, lime, etc., are sold by heaped measure. In practice we say 5 stricken measures equal 4 heaped measures.

PROBLEMS

371. 1. Reduce 25 bu. 3 pk. 5 qt. 1 pt. to pints.

2. Add 9 gal. 3 qt., 18 gal. 2 qt. 1 pt., 10 gal. 1 qt., 21 gal. 2 qt.

3. From 26 gal. 2 qt., take 18 gal. 3 qt. 1 pt.

4. Divide 37 bu. 1 pk. 2 qt. 1 pt. by 9.

5. How many pint bottles will be required to hold a gallon of cider?

6. A farmer raised $321\frac{3}{4}$ bu. of wheat. He kept 64 bu. 1 pk. 3 qt. for seed, and 36.375 bu. for family use. He sold the remainder at \$.96 a bu. How much did he receive for what he sold?

7. A man feeds each of his 4 horses 10 qt. of oats daily. How long will 456 bu. 1 pk. last them?

8. Divide 9 hhd. 54 gal. 3 qt. 1 pt. by 25.

9. Add $\frac{5}{8}$ bu. 1.75 pk. $5\frac{2}{3}$ qt., $2\frac{3}{4}$ bu. 3.5 pk. $7\frac{1}{2}$ qt.

372. Since there are 2150.42 cu. in. in a bushel, and 1728 cu. in. in a cubic foot, a bushel is equal to $1\frac{1}{4}$ cu. ft., nearly.

Thus, $2150.42 \div 1728 = 1\frac{1}{4}$, *nearly.* Hence, for practical purposes, it is sufficiently accurate to say that $\frac{5}{4}$ of the number of bushels will represent the number of cubic feet, and $\frac{4}{5}$ the number of cubic feet will equal the number of bushels.

373. As 5 stricken measures equal 4 heaped measures, *nearly*, we may say that $\frac{5}{4}$ of the number of heaped measures equal the number of stricken measures, and $\frac{4}{5}$ of the number of stricken measures equal the number of heaped measures.

374. The number of gallons liquid measure may be found by multiplying the number of cubic feet by 7.48, since a cubic foot contains about 7.48 gallons ($1728 \div 231 = 7.48 +$).

NOTE. — **The following problems are to be solved according to the preceding suggestions.**

PROBLEMS

375. 1. How many bushels of shelled corn will a rectangular box hold, 6 ft. long, 4 ft. wide, and 6 ft. deep?

2. How many bushels of corn in the ear will the box described in Example 1 hold?

3. How many cubic feet are there in a bin that will hold 376 bu. of potatoes?

STATEMENT

$$376 \times \frac{5}{4} \times \frac{5}{4} = \text{answer}$$

4. How many gallons of water will a round cistern hold, 5 ft. in diameter and 10 ft. deep?

STATEMENT

$$2\tfrac{1}{2} \times 2\tfrac{1}{2} \times 3.1416 \times 10 \times 7.48 = \text{answer}$$

NOTE.—To find the contents of a cylinder, multiply the square of the radius (one half of the diameter) by 3.1416, and the product thus obtained by the altitude.

5. How many bushels of oats will a wagon box hold, 11 ft. long, 3 ft. wide, and 2 ft. deep? How many bushels of apples will it hold?

6. How many cubic feet are there in a tank that will hold 5 hogsheads of water?

7. A bin that will contain 260 bu. of wheat is 5 ft. wide and 5 ft. deep. What is its length?

8. A corn crib 20 ft. long, 5 ft. wide, and 10 ft. high is filled with corn in the ear. What is the corn worth, when shelled, at 85¢ a bushel, allowing 2 bushels of *ears* to make 1 bushel of *shelled corn?*

9. A well 4 ft. in diameter and 36 ft. deep is $\frac{1}{6}$ full of water. How many gallons of water are in the well?

10. A man constructed a tank which he found held 897.6 gallons of water. The bottom of the tank was 4 ft. by 5 ft. Find its depth.

Troy Weight

376. *Troy Weight* is used in weighing jewels and the precious metals, as gold and silver. The unit of weight is the *Pound*, which contains 5760 grains.

TABLE

377.

24 grains (gr.)	=	1 pennyweight (pwt.)
20 pennyweights	=	1 ounce (oz.)
12 ounces	=	1 pound (lb.)

Note.—The carat is a unit of 4 imaginary grains employed in rating diamonds and precious stones, as the ruby, topaz, emerald, etc. The term is also used to express the fineness of gold. Thus, gold 18 carats fine consists of 18 parts pure gold, alloyed with 6 parts of some other metal, the whole mass being divided into 24 equal parts.

PROBLEMS

378. 1. Reduce 8 oz. 17 pwt. 23 gr. to grains.

2. Reduce 14 lb. 7 oz. 12 pwt. to grains.

3. How many ounces, pennyweights, and grains in 84,793 gr.?

4. How many pounds, ounces, pennyweights, and grains in 96,843 gr.?

5. If 4 silver water pitchers weigh 29 lb. 5 oz. 17 pwt. 8 gr., find their average weight.

6. When 18 pwt. of silver are worth $1.26, what is the value of 3 lb. 9 oz. 17 pwt. 8 gr. of silver?

7. Add $\frac{4}{5}$ of a pound, $\frac{7}{8}$ of an ounce, and $3\frac{3}{4}$ pwt.

8. Reduce 9 oz. 12 pwt. to the fraction of a pound.

9. From $\frac{5}{9}$ of a pound take 4 oz. .875 pwt.

10. Reduce 2 oz. 15 pwt. 12 gr. to the decimal of a pound.

Avoirdupois Weight

379. *Avoirdupois Weight* is used in weighing nearly everything except gold, silver, and jewels. The unit of weight is the *Pound*. It contains 7000 Troy grains.

380. Table

16 ounces	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight	= 1 ton (T.)

Note.—At the United States customhouses, in invoices of imported goods, and in the wholesale trade of iron and coal, the ton of 2240 lb. is generally used.

381. The following table will show the number of avoirdupois pounds in a bushel of the principal kinds of farm products, as fixed by law. The weight of a bushel varies slightly in a few states, but the weights given in the table are those adopted by a majority of the states.

382.

COMMODITIES	WEIGHTS	COMMODITIES	WEIGHTS
Barley	48 lb.	Oats.	32 lb.
Beans	60 lb.	Potatoes	60 lb.
Clover seed	60 lb.	Rye	56 lb.
Corn in the ear . . .	70 lb.	Timothy seed . . .	45 lb.
Corn shelled	56 lb.	Wheat	60 lb.

383. Following are a few denominations in common use:

56 lb. of butter	= 1 firkin	100 lb. dry fish	= 1 quintal
84 lb. of butter	= 1 tub	196 lb. flour	= 1 barrel
100 lb. of grain	= 1 cental	200 lb. pork or beef	= 1 barrel

PROBLEMS

384. 1. How many ounces in 18 cwt. 46 lb. 9 oz.?

2. How many pounds in 38 T. 39 cwt. 83 lb.?

3. How many ounces in 2 T. 21 cwt. $84\frac{1}{2}$ lb.?

4. Reduce 8964 oz. to higher denominations.

5. Reduce 18,694 lb. to higher denominations.

6. Change $\frac{5}{6}$ T. to integers of lower denominations.

7. Find the sum of $3\frac{7}{15}$ T. $17\frac{4}{5}$ cwt. $68\frac{7}{8}$ lb.

8. From $3\frac{5}{8}$ T. take 2 T. 17 cwt. $25\frac{3}{4}$ lb. 15 oz.

9. Express .8658 T. in integers of lower denominations.

APOTHECARIES' WEIGHT

385. *Apothecaries' Weight* is used by physicians and apothecaries in prescribing and preparing dry medicines. The unit is the *Pound,* which contains 5760 gr., like the Troy pound. Medicines are bought and sold by avoirdupois weight.

TABLE

386.

20 grains	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (℔)

APOTHECARIES' FLUID MEASURE

387. *Apothecaries' Fluid Measure* is used by physicians and apothecaries in prescribing and preparing liquid medicines.

TABLE

388.

60 minims (m.)	= 1 fluidrachm (fʒ)
8 fluidrachms	= 1 fluidounce (f℥)
16 fluidounces	= 1 pint (O.) (Octavus)
8 pints	= 1 gallon (Cong.) (Congius)

PROBLEMS

389. 1. Find the sum of 24 ℔ 9 ℥ 6 ʒ 18 gr. and 19 ℔ 5 ʒ 2 ℈ 15 gr.

2. Reduce $\frac{3}{5}$ ℔ to integers of lower denominations.

3. Reduce 5 ℥ 2 ʒ 2 ℈ to the decimal of a pound.

4. Reduce 9 Cong. 7 O. 8 f℥ 4 fʒ to minims.

Reduce 938,462 m. to higher denominations.

TIME

390. The unit of *Time* is the *Day*.

TABLE

391.

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 common year (yr.)
366 days	= 1 leap year
100 years	= 1 century (cen.)

392. The year is divided into 12 periods, called calendar months. The following table will show the names of the calendar months and the number of days in each:

393. TABLE

Names of Months	Days in Each	Names of Months	Days in Each
1. January	31	7. July	31
2. February	28 or 29	8. August	31
3. March	31	9. September	30
4. April	30	10. October	31
5. May	31	11. November	30
6. June	30	12. December	31

394. Four weeks constitute what is called a *lunar month.* 13 lunar months and 1 day make a common year, 365 da.

395. The year is also divided into periods of 3 months, each constituting what is called a season. December, January, and February form the winter season; March, April, and May, spring; June, July, and August, summer; and September, October, and November, fall, or autumn.

By thoroughly committing the following stanza, the number of days in each month may be readily remembered:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except February alone,
Which has but twenty-eight in fine,
Till leap year gives it twenty-nine."

NOTE.—In business transactions it is customary to consider 30 days as a month, and 12 months a year.

THE CALENDAR

396. A solar year is the exact period of the revolution of the earth around the sun. It consists of 365 da. 5 hr. 48 min. 46 sec., which is nearly $365\frac{1}{4}$ da.

NOTE. — When we consider 365 days as a year, the time lost on the calendar in one year is not quite 6 hours, and in 4 years about 45 minutes less than a full day. Hence we add 1 day to February every fourth year. But by so doing we gain on the calendar in 100 years 25 times 45 minutes, or about 5 hours less than a day. We rectify this by making most centennial years common instead of leap years; but the 5 hours less than a day amounts in 400 years to nearly a full day lost on the calendar in that time, which is rectified by counting every 400th year a leap year.

Hence, years divisible by 4, except centennial years, are leap years; and centennial years divisible by 400 are also leap years.

PROBLEMS

397. 1. Reduce 46 da. 16 hr. 22 min. to minutes.

2. How many seconds in 2 yr. 21 hr. 58 minutes?

3. In 38,649 hr. how many days?

4. Change .26 of a common year to integers of lower denominations.

5. Change 45 da. 18 hr. to the decimal of a leap year.

6. To $\frac{1}{6}$ of a common year add $\frac{5}{8}$ of a leap year.

7. From 3 yr. 5 da. 18 hr. take 166 da. 14 hr. 32 min.

8. Divide 38 wk. 5 da. 18 hr. 21 min. 46 sec. by 7.

9. What decimal of 9 da. 4 hr. 14 min. 16 sec. is 1 da. 3 hr. 31 min. 47 sec.?

10. Find the sum of $\frac{3}{8}$ of a common year, $\frac{1}{6}$ of a week, $\frac{3}{8}$ of a day, $\frac{5}{8}$ of an hour, and $.33\frac{1}{3}$ of a minute.

11. How many solar years equal 2921 da. 22 hr. 30 min. 8 sec.?

12. A note dated April 10, 1890, was paid Jan. 3, 1896, with interest. Find the time the note was on interest.

When the period is long, the method of compound subtraction is generally employed to find the time. 30 days is considered a month.

OPERATION

yr.	mo.	da.
1896	1	3
1890	4	10
5	8	23

13. George Washington was born on Feb. 22, 1732, and died on Dec. 14, 1799. How old was he at the time of his death?

14. Gen. Robert E. Lee was born Jan. 19, 1807. What was his age when the battle of Gettysburg was fought, July 1, 1863?

15. A note dated June 8, 1898, was paid 9 mo. 3 da. after date. Find the date of payment.

NOTE. — When the period is short, the time is usually found by counting the actual number of days, including the last date, but not the first. Banks include both dates.

16. Find the number of days from June 8, 1898, to Jan. 5, 1899.

SUGGESTION. — In June there are 30 − 8, or 22 days remaining; in July there are 31 days; in August, 31; September, 30; October, 31; November, 30; December, 31; and 5 in January.

17. Find the number of days from Oct. 30, 1891, to April 20, 1892.

18. A coal miner worked full time in May, the first day of the month being Friday, and half time in June. How many days did he work in both months?

COUNTING

398.

12 units	=	1 dozen (doz.)
12 dozen	=	1 gross (gro.)
12 gross	=	1 great gross (grt. gro.)
20 units	=	1 score (sc.)

PAPER

399.

24 sheets	=	1 quire (qr.)
20 quires (480 sheets)	=	1 ream (rm.)
2 reams	=	1 bundle (bun.)
5 bundles	=	1 bale (B.)

Books

400. In printing books the terms *folio, quarto, octavo,* etc., are used to denote the number of leaves formed by folding a sheet of paper.

A folio book (fol.) is formed of sheets folded in 2 leaves (4 pages).
A quarto book (4to) is formed of sheets folded in 4 leaves (8 pages).
An octavo book (8vo) is formed of sheets folded in 8 leaves (16 pages).
A duodecimo book (12mo) is formed of sheets folded in 12 leaves (24 pages).
A 16mo book is formed of sheets folded in 16 leaves (32 pages).

PROBLEMS

401. 1. How much paper will be required to make 2000 copies of a 12mo book of 240 pages?

2. Find the number of pages in an octavo book containing 20 sheets.

3. A hardware dealer bought 30 kegs of nails for \$ 96.25. He sold at wholesale $\frac{8}{15}$ of them at an advance of $37\frac{1}{2}$¢ per keg, and the remainder he sold at retail. His entire gain was \$ 30.50. At what price per pound did he retail?

4. What decimal of 51 bales 2 bundles 11 quires 16 sheets is 6 bales 2 bundles 6 quires 11 sheets?

5. Find the sum of $2\frac{3}{4}$ doz. 3 score and 10, $\frac{5}{6}$ of a gross, and $1\frac{5}{8}$ great gross.

6. A stationer bought 3 reams of legal cap for \$ 7.20; 5 gross of penholders for \$ 4.75; 10 gross of writing pads for \$ 72. He retailed the legal cap at 2¢ a sheet, the penholders at 1¢ each, and the writing pads at 10¢ each. Find the entire gain.

7. Puckey Bros. bought 25 gross of Spencerian engrossing steel pens for \$ 18. If they sell 3 pens for 5¢, how much will they gain, allowing 2 dozen as worthless?

United States Money

402. *Money* is the measure of value of articles bought, sold, or exchanged.

It consists of two kinds, specie or coin, and paper money.

403. *Specie* or *Coin* consists of metal, such as gold, silver, nickel, and bronze, stamped, and authorized under the government to be used as money.

404. The gold coins of the United States are the double-eagle, eagle, half-eagle, and quarter-eagle.

405. The silver coins are the dollar, half-dollar, quarter-dollar, and dime.

406. The nickel coin is the five-cent piece.

407. The bronze coin is the cent.

408. Paper money, issued for convenience in business under authority of government, consists of printed promises to pay the bearer a stated amount on demand. National banknotes, gold certificates, silver certificates, United States notes, and treasury notes are the kinds of paper money now in circulation.

English Money

409. *English Money,* sometimes called *Sterling Money,* is the legal currency of Great Britain. The unit is the *Pound Sterling.*

Table

410.

4 farthings (far.)	= 1 penny	(*d.*)
12 pence	= 1 shilling	(*s.*)
20 shillings	= 1 pound	(£)

Note.—There is no coin known as the *pound sterling.* It is represented by a gold coin called a sovereign, which is equal in value to 20 shillings sterling, and $4.8665 in United States coin. The *guinea,* a gold coin equal in value to 21 shillings sterling, is not coined now. The *crown* is a silver coin equal in value to 5 shillings sterling. The *florin* is a silver coin equal in value to 2 shillings sterling.

411. The following table shows the estimated value, in United States money, of the monetary unit of the most important nations of the world:

COUNTRY	DENOMINATION	VALUE
Austria-Hungary	Crown	$ .203
Belgium	Franc	.193
Brazil	Milreis	.546
Canada	Dollar	1.00
Chile	Peso	.912
Denmark	Crown	.268
France	Franc	.193
German Empire	Mark	.238
Great Britain	Pound Sterling	4.8665
Italy	Lira	.193
Japan	Yen	.997
Mexico	Dollar { Gold	.983
	Silver	.528
Russia	Ruble	.772
Sweden	Crown	.268
Switzerland	Franc	.193
Spain	Peseta	.193

PROBLEMS

412. 1. What is the sum of 2.5 eagles, $.37\frac{1}{2}$ of a dollar, $\frac{3}{4}$ of a dime, .75 of a half-eagle, $\frac{1}{2}$ of a quarter-eagle, $\frac{5}{8}$ of a dollar, $\frac{3}{8}$ of a dime, and .6 of a cent?

2. How many pence in £75 16 *s.* 9 *d.*?

3. Reduce £84 18 *s.* 7 *d.* 2 far. to farthings.

4. Express 117,526 far. in integers of higher denominations.

5. Express £8 10 *s.* 5 *d.* in pounds.

6. Add £ .375 .27 *s.* 1.1 *d.*

7. Change 9 *s.* 4 *d.* 3.2 far. to the decimal of a pound.

8. From £10 6 *s.* 4 *d.* take £2 13 *s.* 8 *d.* 3 far.

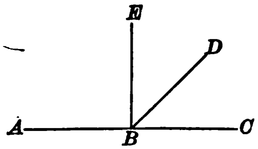
9. Find the cost of 5 umbrellas at £2 5 *s.* 6 *d.* each.

CIRCULAR MEASURE

413. *Circular Measure* is used in measuring angles, in determining latitude and longitude, etc.

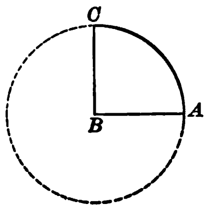
NOTE. — For definition of circle, circumference, diameter, etc., see Arts. 343–346.

414. When two straight lines are drawn from the same point, the opening between the lines is called an angle. Thus, *ABE*, *EBD*, and *DBC* are angles. Their vertex is *B*.



415. For the purpose of measuring angles the circumference is supposed to be divided into 360 equal parts, called degrees. The degree is the unit of circular measure.

416. To measure an angle its vertex is made the center of a circle, and the arc included between its sides is the "size" or measure of the angle. Thus, the angle *ABC* is measured by the arc *AC*. As the arc *AC* is $\frac{1}{4}$ of the circumference, the angle *ABC* is $\frac{1}{4}$ of 360 degrees, or 90 degrees.



417. An angle of 90 degrees is called a *Right Angle*. Angles less than a right angle are called *Acute Angles;* angles greater than a right angle, *Obtuse Angles.*

418. The *degree* does not express a fixed length. It is $\frac{1}{360}$ of the circumference of a circle; it varies, therefore, according to the size of the circle. If a *great circle* (the equator or a meridian circle) is divided into 360 equal parts, one of these equal parts expresses the length of a degree of latitude or longitude at the equator.

NOTE. — Practically, it is sufficiently correct to say that a degree of latitude or longitude at the equator equals 69.16 statute miles, although a degree of latitude at the equator is somewhat less.

TABLE

419.

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
360 degrees	=	1 circumference (C.)

NOTE. — The term *sign* is sometimes used to express 30°, from the fact that the *ancients* divided the zodiac into 12 parts of 30° each, and represented each part by an arbitrary sign.

To distinguish these from minutes and seconds of *time*, the phrase *of arc* is employed. Thus, 30″ is read, 30 seconds of arc.

PROBLEMS

420. 1. Reduce 81° 45′ 28″ to seconds.

2. Change 84,672″ to higher denominations.

3. Find the sum of 24° 32′ 46″, 64° 55′ 38″, 75° 36′ 22″.

4. From $\frac{1}{4}$ of 98° 25′ 24″ take 9° 59″.

5. Find the sum of 72.325°, 4.28′, 29.5″.

6. What part of a quadrant ($\frac{1}{4}$ of the circumference) is $\frac{4}{5}$ of a minute?

7. Reduce .236 of 70° to integers of lower denominations.

8. Find the length, in statute miles, of 54′ on the equator.

LATITUDE AND LONGITUDE

421. The position of a place on the earth's surface is indicated by its latitude and longitude.

422. The latitude of a place on the earth's surface is its distance north or south of the equator, reckoned in degrees, minutes, and seconds. Places north of the equator are said to be in north latitude; places south of the equator, in south latitude. *Latitude* is reckoned from the equator toward the

poles. Hence all places on the equator have no latitude. Latitude cannot exceed 90°, the latitude of the poles. At the equator the length of a degree of latitude is about 68.7 statute miles, but as we approach the poles, the length of a degree increases, owing to the flattening of the earth.

Halfway between the equator and the poles the length of a degree is about 69.1 statute miles, and at the poles, 69.4 statute miles.

423. The longitude of a place on the earth's surface is its distance east or west of a given meridian, reckoned in degrees, minutes, and seconds. The given meridian is called the *Prime Meridian.* Nearly all countries have adopted the meridian passing through Greenwich, near London, as the standard from which to reckon longitude. Sometimes, however, longitude is reckoned from the capital of the country, as from Washington or Paris.

Places not exceeding 180° east of the standard meridian are said to be in east longitude; places not exceeding 180° west of it, are said to be in west longitude.

Degrees of longitude vary in length from about 69.16 statute miles at the equator to nothing at the poles. The length of a degree of longitude, at latitude 20°, is about 65 statute miles; at 30°, about 60 miles; at 40°, about 53 miles; at 50°, about 44.4 miles; at 60°, about 34.5 miles; and at 80°, about 12 miles.

PROBLEMS

424. 1. The latitude of Washington is 38° 53′ 20″ north, and of Richmond 37° 32′ 17″ north. Find the difference in latitude.

2. The latitude of Savannah is 32° 1′ north, and of Cincinnati 39° 6′ north. What is the difference in latitude?

3. The latitude of New Orleans is 29° 57′ north, and of Quito 13′ south. Find the difference in latitude.

4. The latitude of New York is 40° 42′ 43″ north, and of Rio Janeiro 22° 54′ south. Find the difference in latitude.

5. Boston is 71° 3′ 58″, and Washington 77° 0′ 28″ west of Greenwich. What is the difference in longitude?

6. New York is 74° 0′ 3″, and Omaha 96°, west of Greenwich. Find the difference in longitude.

7. The longitude of Chicago is 10° 33′ 41″ west of Washington, and that of Vienna 16° 23′ east of Greenwich. Find the difference in longitude between Chicago and Vienna.

8. The longitude of St. Paul, Minn., is 93° 5′ west, and of Marseilles 5° 22′ east. What is the difference in longitude?

9. The longitude of San Francisco is 122° 23′ 54″ west. What is the difference in longitude between New York and San Francisco?

10. The difference in longitude between St. Louis and St. Petersburg is 120° 30′ 19″. What is the longitude of St. Petersburg, that of St. Louis being 90° 12′ 23″ west?

LONGITUDE AND TIME

425. Since the earth turns upon its axis from west to east once in 24 hours, the sun appears to revolve from east to west around the earth in the same time. Therefore a circumference (360°) is described by the apparent revolution of the sun around the earth in 24 hours.

Since the sun appears to travel through 360° of longitude in 24 hours, in 1 hour it travels through $\frac{1}{24}$ of 360°, or 15° of longitude; in 1 minute it travels through $\frac{1}{60}$ of 15°, or 15′; and in 1 second, through $\frac{1}{60}$ of 15′, or 15″ of longitude. Therefore we have the following table:

15°	of longitude	correspond	to	1 hour	of	time,
15′	"	"	"	1 minute	"	"
15″	"	"	"	1 second	"	"

All places east of a certain point have later time, all places west earlier time. Thus, when it is 10 o'clock A.M. in Philadelphia, it is 11 o'clock A.M. at a point 15° east of Philadelphia; 12 o'clock 30° east; 1 o'clock P.M. 45° east, etc. Again, when it is 10 o'clock A.M. in Philadelphia, it is 9 o'clock A.M. 15° west; 8 o'clock 30° west; 7 o'clock 45° west, etc. Therefore, when the difference in time between two places is known, and the exact time of *one* of them, the exact time of the other may be found by adding the time given to the difference, if the place where the time sought is east of the other, and subtracting it if west.

PROBLEMS

426. 1. The difference in longitude between two places is 35° 4′. What is the difference in time?

OPERATION

$$15\overline{)35°\quad 4'}$$
$$\text{2 hr. 20 min. 16 sec.}$$

Since a difference of 15° of longitude corresponds to a difference of 1 hour in time, 15′ of longitude to 1 minute in time, and 15″ of longitude to 1 second in time, it is evident that $\frac{1}{15}$ of the difference in longitude will give a corresponding difference in time. By dividing the difference in longitude by 15, the difference in time is found to be 2 hr. 20 min. 16 sec.

2. Boston is 71° 3′ 58″, and Omaha 96°, west. Find the difference in time.

3. The longitude of New York is 74° 3″, and of St. Paul, Minn., 93° 5′, west. Find the difference in time.

4. Vienna is 16° 23′, and Marseilles 5° 22′, east. Find the difference in time.

5. The longitude of St. Petersburg is 30° 17′ 56″ east. When it is noon at Vienna, what time is it at St. Petersburg?

6. What is the difference in time between New York and Marseilles?

SUGGESTION. — Add to find difference in longitude.

7. The longitude of Paris is 2° 20′ 9″ east. What change must a person make in his watch in going from St. Petersburg to Paris?

8. The longitude of San Francisco is about 122° 23′ 54″ west. When it is 10.45 A.M. at Omaha, what time is it at San Francisco?

9. Chicago is 87° 34′ 9″ west. When it is 10.30 P.M. July 24, at Chicago, what time is it at Paris?

10. The longitude of New Orleans is 90° west. When it is 1.30 A.M. Friday, at New Orleans, what time is it at San Francisco?

11. Rome is 12° 28′ 26″ east, and Washington 77° 28″ west. What time is it at Rome when it is 7 min. $4\frac{2}{5}$ sec. past 6 o'clock P.M. July 26, at Washington?

PROBLEMS

427. 1. The difference in time between Philadelphia and St. Louis is 1 hr. 21 sec. What is the difference in longitude?

Since a difference of 1 hour in time corresponds to a difference of 15° in longitude, 1 minute in time to 15′ in longitude, and 1 second in time to 15″ in longitude, it is evident that the difference in time multiplied by 15 will give a corresponding difference in longitude. By multiplying the difference in time by 15, we find the difference in longitude to be 15° 5′ 15″.

OPERATION

hr.	min.	sec.
1	0	21
		15
15°	5′	15″

2. The difference in time between two places is 1 hr. 22 min. 45 sec. What is the difference in longitude?

3. A traveler on reaching 108° 24′ 28″ west longitude found his watch 2 hr. 5 min. 36 sec. too fast. What was the longitude of his starting point, and in what direction was he traveling?

4. The difference in time between two places is 45 min. 30 sec.; the longitude of the one having the faster time is 75° 10′ west. What is the longitude of the other place?

5. The longitude of New York is 74° 3″ west. When it is 4 o'clock P.M. at New York, it is about 3.18 P.M. at Cincinnati. What is the longitude of Cincinnati?

6. The difference in time between Cape of Good Hope, longitude 18° 28′ east, and New Orleans is 7 hr. 13 min. 22 sec. Find the longitude of New Orleans.

7. When it is midnight, Aug. 6, at London, longitude 5′ west, it is 50 min. $13\frac{1}{15}$ sec. A.M. Aug. 7, at Rome. What is the longitude of Rome?

8. When it is 4.30 P.M. at Berlin, it is 52 min. $9\frac{1}{3}$ sec. past 10 A.M. at Boston, longitude 71° 3′ 58″ west. What is the longitude of Berlin?

9. When it is 4.30 P.M. at Boston, what time is it at Berlin?

10. In going from Detroit, longitude 82° 58′ west, to Philadelphia, I find it necessary to set my watch forward 31 min. $15\frac{2}{3}$ sec. What is the longitude of Philadelphia?

11. I left Boston, longitude 71° 3′ 58″ west, at 9.35 A.M., and when I reached St. Louis it was 10 P.M. by St. Louis time, but by my watch it was 16 min. 34 sec. past 11 P.M. Find the longitude of St. Louis.

STANDARD TIME

428. *Standard Time* is the true time of some standard meridian. It is used by nearly all railroads, cities, and towns in the United States.

NOTE.—The time considered in Art. 426 is the true local time, and should not be confounded with the Standard Time of the United States.

For the convenience of railroads, especially, the United States is divided by particular meridians into four sections, each 15° of longitude, known as the Eastern, Central, Mountain, and Pacific sections. The meridians selected are the 75th, 90th, 105th, and 120th west of Greenwich.

Each section extends $7\frac{1}{2}°$ on each side of the standard meridian. For example, the Eastern section embraces all the territory extending $7\frac{1}{2}°$ on each side of the standard meridian of this section; that is, the 75th meridian.

The time of the Eastern section is known as Eastern standard time. It is the true local time of the 75th meridian west of Greenwich, and is 5 hours slower than Greenwich time. All places within the Eastern section have the clock time of the meridian 75° west of Greenwich. For example, when it is 4 o'clock P.M. at longitude 75° W. it is 4 o'clock P.M. at all other places within this section.

The time of the 90th meridian is known as the Central standard time. Central standard time is 6 hours slower than Greenwich time, and 1 hour slower than Eastern standard time. It is the clock time of the meridian 90° west of Greenwich.

Mountain standard time is the clock time of the meridian of 105° west of Greenwich.

Western, or Pacific standard time is the true local, or clock time of the meridian 120° west from Greenwich.

NOTE. — All confusion and inconvenience arising from different local times is avoided by the Standard method, for the reason that the difference of time between two places not in the same section is always one, two, or three hours.

QUESTIONS AND PROBLEMS

429. 1. How many degrees on each side of the 105th meridian does the Mountain section extend?

2. How many hours slower is Mountain standard time than Greenwich time? How many hours slower than Eastern standard time? How many hours slower than Central standard time?

3. How far on each side of the 120th meridian does the Pacific section extend?

4. How many hours slower is Pacific standard time than Greenwich time? Than Eastern standard time? Than Mountain standard time?

5. Boston is in the Eastern section; Cincinnati, in the Central; Denver, in the Mountain; and San Francisco, in the

Pacific. When it is 10 o'clock A.M. standard time in Boston, what is the standard time in Cincinnati? In Denver? In San Francisco? When it is 12 o'clock M. standard time in Denver, what is the standard time in Boston? In Cincinnati? In San Francisco?

6. Write the names of six cities that are within the Eastern section; five within the Central section; two within the Mountain section; and two within the Pacific section.

7. When it is 15 min. past 3 o'clock P.M. in Cincinnati, standard time, what is the standard time in Denver? In San Francisco? In New York?

NOTE. — Knowing the distance in degrees of any given place from its standard meridian, and the local time of that place, local time may be changed into standard time by subtracting from the local time 4 minutes of time for every degree of longitude, or 1 minute of time for 15′ of longitude, when the place is east of its standard meridian, or by adding to the local time 4 minutes of time for every degree of longitude, or 1 minute of time for 15′ of longitude when the place is west of its standard meridian.

For illustration, we shall assume that standard time coincides with local time at Philadelphia. Now, when it is 9 o'clock at Philadelphia, it is 9 o'clock at a point 5 degrees east, and also 9 o'clock at a point 5 degrees west of Philadelphia, according to the standard method, while in reality it is 20 minutes past 9 o'clock 5 degrees east, and 20 minutes before 9 o'clock 5 degrees west.

8. New York is about 74° west longitude. Find the difference between standard time and the actual time of that city.

9. What is the difference between the standard time and the true local time of Chicago, longitude 87° 35′ west?

10. How much time must be added to the true local time of Washington, longitude 77° 3′ west, to express the standard time of that city?

REVIEW PROBLEMS

430. **1.** A butcher purchased 2 T. 8 cwt. of beef at $190 per ton, and retailed it at an average of $12\frac{1}{2}$¢ per pound. How much did he gain?

2. A land speculator bought 80 building lots, each 50 ft. by 80 ft. How many acres do they contain?

3. Find the cost of 345 bbl. of pork at \$ 4.87½ per hundredweight.

4. How many bushels in 9676.89 cu. in.?

5. Find the difference, in cubic inches, between 5 qt. dry measure and 5 qt. liquid measure.

6. If a man can walk 14.8 miles in 4 hr., how far can he walk in 6 days of 10 hr. each?

7. Reduce 150 A. 6 sq. ch. 9 P. 116 sq. li. to square links.

8. How many steps will a man take in walking 20 miles, allowing 2 ft. 6 in. to each step?

9. Reduce $\frac{4}{7}$ of a pound Troy to lower denominations.

10. How deep must a cistern be to contain 200 barrels of water, if it is 12 ft. square?

11. Find the cost of 288 fence boards, each 16 ft. long and 8 in. wide, at \$ 12 per M.

12. How much is a piece of land 12 miles square worth at \$ 12.50 per acre?

13. How many yards 1¼ yd. wide will be required to line 7½ yd. of cloth ¾ yd. wide?

14. A farmer has a bin 9 ft. long, 5 ft. wide, and 6 ft. deep, which is $\frac{3}{4}$ full. How much is the wheat worth at \$ .87½ per bushel? (See Art. 372.)

15. What decimal of 9 gal. is 7 gal. 1 qt. 2 gi.?

16. Find the cost of 25 lb. 14 oz. of quinine, when \$ 192.50 is paid for 5 lb. 8 oz.

17. A lot containing ¼ of an acre is 90 ft. wide. How deep is it?

18. A tank 10 ft. long and 7 ft. wide contains $15\frac{5}{9}$ cu. yd. of water. Find its depth.

19. What is the value of a pile of wood 60 ft. long, 8 ft. wide, and 8 ft. high, at $ 4.75 a cord?

20. Find the number of board feet in 25 joists, each $2\frac{1}{2}$ in. by 10 in., and 18 ft. long.

21. How many bottles, each holding $1\frac{1}{2}$ pt., will be required to contain 3 barrels of vinegar?

22. A druggist bought a pound of quinine, and sold from it 8 ℥ 4 ʒ 2 ℈ 16 gr. How many 2-grain pills will the remainder make?

23. How much will it cost to plaster the sides and ceiling of a room 56 ft. by 38 ft., and 16 ft. high, at 22 ¢ a square yard, allowing $\frac{1}{4}$ of the perpendicular walls for windows and doors?

24. At $ 5.75 per M, find the cost of the shingles that will cover a double roof 32 ft. by 18 ft., allowing 1000 shingles to cover a square (100 sq. ft.).

25. Find how many acres in a rectangular piece of land 4 ch. 30 li. by 8 ch. 75 li.

26. A ship was driven 2° 18′ out of her direct course by a storm. How many miles was that?

27. A cellar 42 ft. long and 24 ft. wide is flooded to the depth of 8 in. How many barrels of water are there in the cellar?

28. Reduce 4 oz. 19 pwt. to the decimal of a pound.

29. A six-inch cube is put into a cubical box measuring 8 in. each way. How many cubic inches are unoccupied?

30. I have a lot 90 ft. long and 75 ft. wide, which is surrounded by a tight board fence 6 ft. 3 in. high. Find the cost of painting both sides of the fence at $12\frac{1}{2}$ ¢ a square yard.

31. If a man walks 3 mi. 146 rd. 3 yd. 2.88 in. in 1 hr. 15 min., how long will it take him to walk 2 degrees, near the equator?

PRACTICAL MENSURATION

Measurement of Land

431. Government lands are divided into *Townships*, 6 miles square, by parallels and meridians. Townships are divided into *Sections*, one mile square, 640 acres. Each section is subdivided into *half-sections*, 320 acres; *quarter-sections*, 160 acres; *half-quarter-sections*, 80 acres; and *quarter-quarter-sections*, 40 acres.

Township

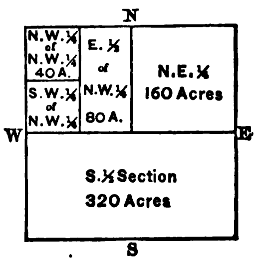
N

W / E

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

S

Section



PROBLEMS

432. 1. How many acres are there in a tract of land 1 mi. long and 160 rd. wide?

2. Find the number of acres in a tract 40 ch. long and 1980 ft. wide.

3. How many acres are there in a piece of land 10 chains square?

4. How many acres are there in a rectangular field $\frac{1}{2}$ mi. long and 880 yd. wide?

5. What is the distance in chains around a quarter-section of land? What is the distance in rods?

6. A rectangular piece of land 60 ch. 60 li. long contains 345 A. 67.2 sq. rd. How many chains wide is it?

7. Find how many quarter-sections there are in a piece of land 4 mi. long and 2 mi. wide.

8. From a tract of woodland there was sold a piece 16.25 ch. long and 13.75 ch. wide. How many acres were there in the tract, if there remained 19 A. 10 sq. rd.?

9. How many rods around a half-quarter-section of land?

10. A rectangular field contains 25 A. 14 sq. rd. It is 22 ch. 50 li. long. What is the distance around the field in rods?

Lathing and Plastering

433. Plastering-laths are usually 4 ft. long, $1\frac{1}{2}$ in. wide, and $\frac{3}{8}$ in. thick. They are generally put up in bundles of 100 laths each. After allowing for waste, it is estimated that a bundle will cover 6 sq. yd.

Note. — *Plastering* is estimated by the square yard. In some localities it is not customary to make any deduction for windows, doors, etc., while in others as much as one half is sometimes allowed for openings. In estimating materials, allowance is made for openings.

PROBLEMS

434. 1. How many bundles of laths will cover both sides of a partition 36 ft. long and 18 ft. high?

STATEMENT

$$\overline{(36 \times 18 \times 2) \div 9} \div 6 = \text{answer}$$

2. Find how many bundles of laths will be required to cover the walls and ceiling of a room 45 ft. long, 27 ft. wide, and 18 ft. high, allowing 200 sq. ft. for windows and doors.

3. How many bundles of laths will be required for a room 63 ft. long, 45 ft. wide, and 18 ft. high, deducting 4 doors 12 ft. by 6 ft., and 8 windows 10 ft. by 4 ft. each?

4. At 25¢ a bundle, what will be the cost of the laths required to cover the ceiling and walls of 4 rooms each 12 ft. by 16 ft., and 9 ft. high, deducting from each room 2 windows 8 ft. by 4 ft., and 1 door 6 ft. 8 in. by 3 ft. 6 in.?

5. In Ex. 1, find the cost of plastering, at 16¢ a sq. yd.

6. In Ex. 2, what will be the cost of lathing and plastering if the laths cost 25¢ a bundle, and the plastering 18¢ a sq. yd., making no allowance for windows and doors in plastering?

7. In Ex. 3, find the cost of lathing and plastering, if the laths cost 22¢ a bundle, and the plastering 18¢ a square yard, making no allowance for windows and doors in plastering.

8. In Ex. 4, find the cost of plastering, at 20¢ a sq. yd., deducting for windows and doors.

Painting, Papering, and Carpeting

435. *Painting* and *kalsomining* are usually estimated by the square foot or square yard.

The wall paper commonly used is 18 in. wide, and put up in single rolls 24 ft. long or in double rolls 48 ft. long. The exact cost of the paper required for a room can be ascertained only by counting the number of rolls actually used, reckoning any part of a roll used as a whole one, after the work has been completed.

In making estimates, paper hangers ascertain the approximate number of rolls that will be required by dividing the

exact number of square feet to be covered by the number of square feet in a double roll.

1. Find the approximate number of double rolls of paper required for a room 12 ft. by 16 ft. and 9 ft. high, with 2 windows 3 ft. by 6 ft. and one door 4 ft. by 8 ft.

OPERATION

$\overline{(16\times 2+12\times 2)}\times 9+(16\times 12)=696$ sq. ft. in walls and ceiling

$(3\times 6\times 2)+(4\times 8)=$ 68 sq. ft. in windows and door

696 sq. ft. − 68 sq. ft. = 628 sq. ft. to be covered

$48\times 1\frac{1}{2}=$ 72 sq. ft. in a double roll

$628\div 72=8\frac{13}{18}$, or 9 double rolls *Ans.*

436. *Carpeting* is sold by the yard. Brussels carpet is 27 in. wide; ingrain, 36 in. In order to ascertain the number of yards needed for a given room, first decide which way the strips shall run, lengthwise or across the room, then, allowing for waste in matching, multiply the length of one strip in yards by the number of strips needed, and the product will be the number of yards required.

PROBLEMS

437. 1. How many yards of carpet 27 in. wide will be required to cover a floor 27 ft. long, 20 ft. wide, the strips to run lengthwise?

OPERATION

27 in. $=2\frac{1}{4}$ ft.; 20 ft. $\div 2\frac{1}{4}$ ft. $=8+$;

$\therefore$ 9 = number strips needed

27 ft. = 9 yd.; 9 yd. × 9 = 81 yd. *Ans.*

2. How much will it cost to whiten the walls of a schoolroom 24 ft. wide, 36 ft. long, and 14 ft. high, at 5¢ per sq. yd., deducting 20 sq. yd. for openings?

3. A frame house is 48 ft. long, 26 ft. wide, and 22 ft. high. How much will it cost to paint the outside, at $.12\frac{1}{2}$ per sq. ft., the gable ends being 6 ft. high? (To find area of gable ends, see rule for finding area of triangle, Art. 341.)

4. How many double rolls of paper would be required to cover the walls of the room described in Ex. 3, Art. 434?

5. Find the cost of the paper, at 40¢ a double roll, that would be required to cover the walls of the 4 rooms described in Ex. 4, Art. 434.

6. Find the entire cost of papering the following rooms, at 60¢ a double roll, bordering 10¢ a yard:

16 ft. by 18 ft., and 9 ft. high
12 ft. 8 in. by 14 ft., and 9 ft. high
14 ft. 6 in. by 16 ft. 9 in., and 9 ft. high
13 ft. 9 in. by 15 ft. 4 in., and 9 ft. high

In each room there are 3 windows 4 ft. by 8 ft., and 1 door 2 ft. 8 in. by 6 ft. 8 in. (No allowance is made for bordering.)

7. How many yards of Brussels carpet are required for a room $20\frac{1}{4}$ ft. wide and 25 ft. long, the strips to run lengthwise?

8. Find the number of yards of ingrain carpet required to cover a room 15 ft. by 8 ft. 3 in., the strips to run lengthwise.

9. A room is 17 ft. 6 in. by 20 ft. Find the total cost of carpeting it with Brussels carpet, at $1.40 per yard, surrounded by a border 2 ft. wide, mitered at the corners, at $1.25 a yard, the strips to run crosswise.

10. A room 22 ft. 6 in. long, and 17 ft. wide is to be carpeted with ingrain carpet, the strips to run the way of the room that will require the least to be turned under. Which way should the strips run, and how many yards will be required?

11. Find the entire cost of carpeting the following rooms:

25 ft. 9 in. long by $18\frac{3}{4}$ ft. wide, with Brussels
12 ft. 8 in. long by 9 ft. wide, with ingrain
18 ft. 4 in. long by 15 ft. wide, with ingrain
27 ft. 9 in. long by 21 ft. wide, with Brussels

The strips are to run the way of the rooms that will not require any to be turned under. The carpet of the *first* and *fourth* rooms is to be surrounded by a carpet border 18 in. wide, mitered at the corners. The cost per yard of the Brussels is $1.40; ingrain, $.85; the border, $.85.

Measurement of Wood and Lumber

438. *Wood* cut 4 ft. long is called cord wood. A pile of such wood 4 ft. high and 8 ft. long (128 cu. ft.) is called a *Cord.* (Art. 359.)

The term *Lumber*, as used here, includes all kinds of hewed or sawed materials.

PROBLEMS

439. **1.** How many cords of wood are in a pile 120 ft. long, 8 ft. high, and 4 ft. wide?

STATEMENT

$$(120 \times 8 \times 4) \div 128 = \text{answer}$$

2. How many cords of wood can be piled into a shed 32 ft. long, 16 ft. wide, and 12 ft. high?

3. How high must wood, 4 ft. long, be piled upon a sled 12 ft. long, to make 2 cords?

4. A shed 32 ft. long, 24 ft. wide, and 14 ft. high is filled with tan bark. What is the value of the bark, at $8.25 per cord?

5. The cost of a pile of wood 4 ft. wide and 8 ft. high, at $3.25 a cord, is $29.25. What is the length of the pile?

6. How many board feet are there in 32 boards 16 ft. long, 10 in. wide, and 1 in. thick? (Art. 361.)

7. Find the cost of the 2-in. planks required to floor a bridge 240 ft. long, and 20 ft. wide, at $18 per M. (Art. 362.)

8. How many board feet are there in the following bill of lumber:

14 rafters, 20 ft. long, 6 in. wide, and $2\frac{1}{2}$ in. thick;
14 joists, 36 ft. long, 12 in. wide, and $3\frac{1}{2}$ in. thick;
2 girders, 40 ft. long, 12 in. wide, and 10 in. thick.

9. A baseball ground 640 ft. square is to be inclosed by a tight inch-board fence 10 ft. high. The posts are to be 8 ft. apart, and the boards nailed in an upright position, at the bottom, middle, and top, to rails 2 by 4 in. Find the cost of the material, the boards costing $13 per M, the rails $14 per M, and the posts $20 per C.

10. Find the number of board feet in the 2 by 4 studding required for the walls of a house 24 ft. long, 16 ft. wide, and 18 ft. high, if the studs are placed 16 in. apart, and doubled at the corners.

Roofing

440. *Shingling* and *slating* are usually estimated by the square (100 sq. ft.); *tinning* by the square foot. The most common lengths of shingles are 16 in. and 18 in. They are considered to average 4 in. in width, and are generally laid 4 in., $4\frac{1}{2}$ in., or 5 in. to the weather. For convenience in handling, and in order to meet the demands of trade, shingles are put up in bunches of 250 each. As a rule, dealers do not sell less than one bunch. In making estimates carpenters ordinarily reckon 1000 shingles to the square.

PROBLEMS

441. 1. It took 25 bunches of shingles to lay the roof of a tobacco shed. How many thousands were used?

2. How many bunches of shingles will be required to lay a double roof 32 ft. long, the rafters being 14 ft. long?

3. Find the cost of the shingles required to cover a double roof 48 ft. long, the length of the rafters being 22 ft., at $ 3.50 per bunch, if the shingles are laid 4 in. to the weather.

4. What will be the cost of slating a roof 50 ft. 8 in. long, at $ 12.75 per square, if each side of the roof is 20 ft. wide?

5. Find the cost of tinning a porch roof 32 ft. long and 7 ft. 9 in. wide, at 7 ¢ a square foot.

6. Find the exact number of shingles required to lay a double roof 36 ft. long, the distance from the eaves to the ridge being 18 ft., if the shingles are laid 5 in. to the weather, and the first course along the eaves doubled. (Each shingle to average 4 in. in width.)

7. Mr. Johnson wishes to cover his stable, the distance from the eaves to the ridge being 14 ft., and the length 26 ft. Find the difference in cost between a tin roof, at $ 6 a square, and a shingle roof, at $ 4.75 per thousand, allowing each shingle to cover 20 sq. in.

8. What will be the difference in cost between a gravel roof, at $ .55 a square yard, and a tin roof, at 8 ¢ a square foot, if the roof is 45 ft. long and 16 ft. wide?

Masonry

442. A *Perch* of stone is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. thick, and 1 ft. high; it contains $16\frac{1}{2} \times 1\frac{1}{2} \times 1$, or $24\frac{3}{4}$ cu. ft. A perch of stone will make about 21 cu. ft. of wall. Stone is sometimes estimated by the cord, 128 cu. ft. (See Art. 365.)

Brickwork is ordinarily estimated by the thousand bricks. The size and style of bricks vary greatly. The most common size is $8\frac{1}{2}$ in. long, 4 in. wide, and $2\frac{1}{2}$ in. thick. In making estimates it will be sufficiently accurate to reckon

21 bricks to the cubic foot. In estimating material, deductions are made for corners, windows, doors, etc., but in estimating labor no deductions are made unless agreed upon by contract.

PROBLEMS

443. 1. How many perches of masonry are in a stone wall 42 ft. long, 6 ft. high, and $2\frac{1}{2}$ ft. thick?

2. How much will the stone, lime, and sand cost for a wall 36 ft. long, 9 ft. high, and 2 ft. thick, if the stone costs $1.25 a perch, the sand 75¢ a load, and the lime 25¢ a bushel, allowing a perch of stone to make 21 cu. ft. of wall?

NOTE. — A load of sand and 3 bu. of lime will lay about 4 perches of stone. Stone masons reckon a fractional part of a perch as a whole one.

3. Find the cost of the materials required for the walls described in Ex. 1, at rates specified in Ex. 2.

4. How much will it cost, at 75¢ a perch, to lay the foundation walls of a house 48 ft. long, 25 ft. wide, and 8 ft. high, with a center wall running crosswise, if the outside walls are 2 ft. thick to the height of 6 ft., and 20 in. thick the remaining 2 ft., the center wall being 7 ft. high and 20 in. thick? (Use outside measurements.)

5. Find the cost of the materials required for the preceding example, if the stone costs $$2.12\frac{1}{2}$ a perch, the lime 30¢ a bushel, and the sand 90¢ a load.

NOTE. — Allow 21 cu. ft. of wall to equal 1 perch of stone.

6. How many bricks will be required to build a wall 30 ft. long and 6 ft. high, the wall being 2 bricks, or 9 in., thick?

NOTE. — Allow 15 bricks to 1 sq. ft. of wall.

7. I intend to build a two-story brick house 48 ft. long and 30 ft. wide. How many bricks will be required to build the walls of the first story 12 ft. high, and 3 bricks, or 13 in.,

thick? Deduct 8 windows 8 ft. by 4 ft., and 2 doors 8 ft. 10 in. by 4 ft.

NOTE. — Allow 21 bricks for 1 sq. ft. of wall, and count corners but once.

8. How many bricks will be required to build the walls of the second story in Ex. 7, 10 ft. high, and 2 bricks, or 9 in., thick. Allow for 8 windows 8 ft. by 4 ft.

NOTE. — Reckon 15 bricks to 1 sq. ft. of wall, and count corners but once.

CAPACITY OF BINS, CISTERNS, ETC.

Stricken Measure

444. To find the exact number of stricken bushels a bin will hold, we divide the contents of the bin expressed in cubic inches by 2150.42. Since a bushel is equal to $1\frac{1}{4}$ cu. ft., nearly (Art. 372), it will be approximately correct to divide the contents of the bin expressed in cubic feet by $1\frac{1}{4}$.

PROBLEMS

445. 1. Find the number of bushels in a bin 8 ft. long, 6 ft. wide, and 4 ft. deep.

OPERATION

$(8 \times 6 \times 4 \times 1728) \div 2150.42 = 154.28+$, exact number bushels
$(8 \times 6 \times 4) \div 1\frac{1}{4} = 153.6$, approximate number bushels

For ordinary purposes the approximate result is sufficient.

2. Find the exact number of bushels contained in a bin 15 ft. long, 10 ft. wide, and 6 ft. deep.

3. Find the exact number of bushels contained in a bin 20 ft. long, $8\frac{1}{2}$ ft. wide, and 6 ft. deep.

4. Find the approximate number of bushels in a bin 12 ft. long, 6 ft. wide, and $4\frac{1}{2}$ ft. deep.

5. Find the approximate number of bushels in a bin $18\frac{1}{2}$ ft. long, 9 ft. wide, and $8\frac{1}{2}$ ft. deep.

446. To reduce a given number of bushels to cubic feet, multiply 2150.42 by the number of bushels, and divide the product by 1728. Since the bushel contains $1\frac{1}{4}$ cu. ft. of space nearly, an approximate result may be obtained by multiplying the number of bushels by $1\frac{1}{4}$.

PROBLEMS

447. 1. Find the number of cubic feet in a bin that will hold 200 bu.

OPERATION

$(2150.42 \times 200) \div 1728 = 248.89+$, exact number cubic feet
$200 \times 1\frac{1}{4} = 250$, approximate number cubic feet

2. Find exact number of cu. ft. in a bin holding 250 bu.

3. Find exact number of cu. ft. in a bin holding 1324 bu.

4. Find the exact number of cubic feet in a rectangular box that will contain $64\frac{1}{4}$ bu.

5. Find the approximate number of cubic feet in a box that will hold $12\frac{1}{2}$ bu.

6. Find the approximate number of cubic feet of space 440 bu. of grain will occupy.

7. What is the difference between the exact and approximate number of cubic feet in the space occupied by 450 bu.?

Heaped Measure

448. To find the number of heaped bushels that will be contained in a given space, divide the space expressed in cubic inches by 2747.7, the number of cubic inches in a heaped bushel.

Since the space occupied by a heaped bushel is 1.6 cu. ft., nearly $(2747.7 \div 1728 = 1.59+$ cu. ft.), an approximate result may be obtained by dividing the given space expressed in cubic feet by 1.6.

PROBLEMS

449. 1. How many bushels of apples will a box hold that is 6 ft. square and 4 ft. deep?

OPERATION

$(6 \times 6 \times 4 \times 1728) \div 2747.7 = 90.56$, exact number bushels
$(6 \times 6 \times 4) \div 1.6 = 90$, approximate number bushels

2. How many bushels of potatoes will a bin hold that is 12 ft. long, 8 ft. wide, and 6 ft. deep?

3. How many bushels of turnips will a wagon box hold that is 11 ft. long, 3 ft. wide, and $2\frac{1}{2}$ ft. deep?

4. Find how many bushels of unshelled corn will fill a corn crib 20 ft. long, 4 ft. wide, and 8 ft. high.

5. Find the approximate number of bushels of potatoes that can be put into a rectangular box 4 ft. long, 3 ft. wide, and $3\frac{1}{2}$ ft. deep.

450. To express a given number of heaped bushels in cubic feet, multiply 2747.7 by the number of bushels, and divide the product by 1728. Since the space occupied by a heaped bushel is 1.6 cu. ft., nearly, an approximate result may be obtained by multiplying the number of heaped bushels by 1.6.

PROBLEMS

451. 1. Find the number of cubic feet in a bin that will hold 75 bushels of parsnips.

OPERATION

$(2747.7 \times 75) \div 1728 = 119.25+$, exact number cubic feet
$75 \times 1.6 = 120$, approximate number cubic feet

2. Find the exact number of cubic feet of space required to hold 125 bu. of onions.

3. Find the exact number of cubic feet of space that will contain 225 heaped bushels.

4. Find the exact number of cubic feet of space that will contain 300 heaped bushels.

5. Find the approximate number of cubic feet of space that will contain 50 bushels of lime.

6. Find the difference between the exact and approximate number of cubic feet of space required to contain 385 bu. of corn in the ear.

Liquid Measure

452. In liquid measure the standard unit is the *Gallon*, which contains 231 cubic inches. The dry gallon, stricken measure, contains 268.8 cubic inches.

453. To find the exact capacity of a cistern or vessel in gallons, divide the contents expressed in cubic inches by 231. An approximate result may be obtained by dividing the contents expressed in cubic feet by .1336, as a gallon equals about .1336 cubic feet.

PROBLEMS

454. 1. Find the number of gallons of water contained in a tank 6 ft. long, 4 ft. wide, and 3 ft. deep.

OPERATION

$(6 \times 4 \times 3 \times 1728) \div 231 = 538.6$, nearly, number gallons exact method

$(6 \times 4 \times 3) \div .1336 = 538.9+$, approximate result

2. How many gallons of water will a cistern 6 ft. square and 8 ft. deep hold?

3. From a rectangular cistern 10 ft. long, 8 ft. wide, and 6 ft. deep 100 barrels of water were drawn. How many gallons remained in the cistern, if it was full at first?

4. How many hogsheads of water can be contained in a tank 8 ft. by 7 ft., and 9 ft. deep?

5. Find the approximate number of gallons of water in a well $3\frac{1}{2}$ ft. in diameter and 30 ft. deep, if the well is $\frac{1}{4}$ full.

6. Find the approximate number of gallons of water a reservoir 500 ft. in diameter and 10 ft. deep will hold.

455. To find the contents, in cubic feet, of a given number of gallons, multiply 231 by the given number of gallons and divide the product by 1728.

An approximate result in cubic feet may be obtained by multiplying .1336 by the given number of gallons.

PROBLEMS

456. 1. How many cubic feet of space will be required to contain 425 gallons?

OPERATION

$(231 \times 425) \div 1728 = 56.81$, number cubic feet by exact method

$.1336 \times 425 = 56.78$, approximate result in cubic feet.

2. How many cubic feet of space will be required to contain 46 barrels?

3. Find the approximate number of cubic feet in a cistern that will contain 36 hogsheads.

NOTE.—To find the approximate number of barrels a cistern will hold, divide the contents expressed in cubic feet by 4.21, the number of cubic feet, nearly, in a barrel.

4. How many barrels will a cistern hold that is 10 ft. long, 8 ft. wide, and 12 ft. deep?

STATEMENT

$$(10 \times 8 \times 12) \div 4.21 = \text{answer}$$

5. Find the number of barrels in a cylindrical cistern 8 ft. in diameter and 10 ft. deep.

6. How many barrels of water will be contained in a tank 6 ft. 6 in. square and 8 ft. deep?

REVIEW PROBLEMS

457. 1. At 15¢ a square yard, how much will it cost to sod a piece of ground $82\frac{1}{2}$ ft. by 132 ft.?

2. Find the cost of 36 joists 18 ft. long, 12 in. wide, and $2\frac{1}{2}$ in. thick, at $ 1.30 per hundred feet board measure.

3. A field 8.25 ch. wide contains 8 A. 7 P. Find its length.

4. What will be the cost of the carpet for a stair of 22 steps 10 in. wide, each rising 8 in., at 75¢ a yard, allowing 22 in. extra for projection of steps?

5. A floor 22 ft. by $24\frac{1}{2}$ ft. is to be covered with Brussels carpet, surrounded by a border 2 ft. wide, the strips of carpet to run lengthwise. What will be the cost, if the carpet is worth $ 1.35 and the border $87\frac{1}{2}$¢ a yard?

6. Find the cost, at 25¢ a yard, of fencing a rectangular field containing 24 A., if its width is 60 yards.

7. It will require 1200 ft. of siding for a house if it is put on 4 in. to the weather. How much less will it require if it is laid 5 in. to the weather?

8. What will be the cost of the flooring required for two rooms 18 ft. by 22 ft. and 24 ft. by 30 ft., respectively, at $ 24 per M, the flooring to be $1\frac{1}{4}$ in. thick, adding $\frac{1}{5}$ for waste?

9. At 25¢ a double roll and 5¢ a yard for border, find the cost of the paper and border for the walls and ceiling of 2 rooms, each 24 ft. by 16 ft. and 10 ft. high, with 3 windows 3 ft. by 6 ft. and 1 door $2\frac{1}{2}$ ft. by 6 ft. 8 in., reckoning border to be placed around walls and ceiling.

10. Find the cost of the rafters, at $13 per M, for the double roof of a house 42 ft. long, if they are placed 2 ft. apart, and are $2\frac{1}{2}$ in. by 5 in., and $16\frac{1}{2}$ ft. long.

11. Find the number of posts required for a fence around a piece of ground 875 ft. by 910 ft., if the posts are to be set 7 ft. apart.

12. Find the number of 2×4 scantlings 14 ft. long, that will be required for the stringers in Ex. 11, one stringer being placed at the bottom and one at the top.

13. Find the number of boards 12 in. wide and 14 ft. long, that will be required for the base in Ex. 11.

14. Find the number of 3-inch pales, placed 2 in. apart, that will be required in Ex. 11.

15. Find the cost of the material required for the fence in Ex. 11, if the posts are worth $10 per hundred, the 2×4 scantling $11 per thousand feet, the base boards $14 per thousand feet, and the pales $30 per thousand.

16. How many cubic feet of space will be required to contain 2500 bu. of corn in the ear? (Find the approximate result.)

17. How deep must a cistern 6 ft. by 8 ft. be to contain 3000 gallons? (Find the approximate result.)

18. A bin 8 ft. wide and 6 ft. deep contains 384 bu. of apples. Find its length. (Approximate result is required.)

19. How many cubic feet of space will be required to contain 300 barrels of water? (Approximate result is required.)

20. A tank 10 ft. wide and 12 ft. long contains 45.6 barrels of water. Find its depth. (Approximate result is required.)

THE METRIC SYSTEM

458. The *Metric System* is a system of weights and measures based upon the decimal plan of notation. It originated in France about 1795. It has been adopted, or its use is recommended, wholly or in part, by nearly all civilized countries. It is called the *metric system* because the *meter* is taken as the fundamental unit. The length of the meter is about the one ten-millionth part of the distance on the earth from the equator to the north pole, or 39.37 inches.

459. One of the commendable features of the *metric system* is its simplicity. When the unit of measure has been decided upon, all denominations ascend and descend regularly in a tenfold ratio; that is, they are formed by multiplying or by dividing by 10. For example, if we take the meter as the unit of measure, the denominations above the meter are formed by prefixing the words *deka, hecto, kilo,* and *myria* to the primary unit: thus, dekameter, meaning 10 meters; hectometer, meaning 100 meters; kilometer, meaning 1000 meters; and myriameter, meaning 10,000 meters.

The denominations below the meter are formed by prefixing the abbreviations deci, centi, and milli to the unit: thus, decimeter, or $\frac{1}{10}$ of a meter; centimeter, or $\frac{1}{100}$ of a meter, and millimeter, or $\frac{1}{1000}$ of a meter.

NOTE. — Deka, hekto, kilo, and myria are Greek numerals, meaning respectively 10, 100, 1000, and 10,000. Deci, centi, and milli are Latin abbreviations, meaning respectively $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$.

Measures of Length

460. The unit of *Length* is the *Meter*. It is about 39.37 in. or 3.28 ft. in length.

Table

461.

10 millimeters (mm)	=	1 centimeter (cm)
10 centimeters	=	1 decimeter (dm)
10 decimeters	=	1 *meter* (m)
10 meters	=	1 dekameter (Dm)
10 dekameters	=	1 hektometer (Hm)
10 hektometers	=	1 kilometer (Km)
10 kilometers	=	1 myriameter (Mm)

Equivalents

1 centimeter = .3937079 in.	1 dekameter = 32.80899 ft.
1 decimeter = 3.937079 in.	1 hektometer = 19.92781 rd.
1 meter = 39.37079 in.	1 kilometer = .621382 mi.

1 myriameter = 6.21328 mi.

462. Very long distances are measured by the kilometer; very short distances by the millimeter, and intermediate distances by the meter.

463. The change from one denomination to another of equal value is effected by changing the position of the decimal point. To change from a lower to a higher denomination, we move the decimal point to the left. For example, 1456.7 meters = 145.67 dekameters = 14.567 hektometers = 1.4567 kilometers.

To change from a higher to a lower denomination, we move the decimal point to the right. For example, 86.456 meters = 864.56 decimeters = 8645.6 centimeters = 86456 millimeters.

464. Metric numbers involving integers and decimals are read just as integral and decimal denominate numbers are read. Thus, 12.35^{m} is read twelve and thirty-five hundredths meters.

465. Figures used to represent decimal measures may be written in terms of any denomination. Thus, 7^{m} 4^{dm} 8^{cm} 5^{mm} may be written as 7.485^{m}, or 74.85^{dm}, or 748.5^{cm}, or 7485^{mm}.

PROBLEMS

466. 1. How many decimeters in 4 meters? In $5\frac{1}{2}$ meters?

2. How many centimeters in 3 meters and 5 decimeters?

3. How many dekameters in 456 meters?

4. How many decimeters in 3869 millimeters?

5. How many dekameters in $3\frac{1}{2}$ kilometers?

6. How many meters in 5^{Dm} 2^{m} 4^{dm} 3^{cm}?

7. How much will 5^{m} 8^{dm} of ribbon cost at 20¢ a meter?

8. How many centimeters in 3^{m} 4^{dm}?

9. If 4^{Dm} 3^{m} of silk cost $86, find the cost of 1 meter.

10. Find the cost 2^{Dm} 3^{m} 5^{dm} of cassimere at 50¢ a meter.

11. Reduce 9^{m} 8^{cm} 7^{mm} to millimeters.

12. Reduce 8^{Dm} 7^{m} to centimeters.

13. Express 1463.24^{Dm} as kilometers; as meters.

14. Write 3684.9^{dm} as dekameters; as centimeters.

15. Change 8^{Km} 4^{Hm} 8^{m} 7^{dm} to meters.

16. Add 7.5^{Dm} 5^{dm} 4054^{mm} and 12^{m}. Answer in meters.

17. Find the difference between 28.46^{m} and 216^{dm}. Answer in meters.

18. Multiply 92.5^{cm} by 24. Answer in meters.

19. A merchant bought 4 pieces of dress goods, each piece containing 5^{Dm} 4^{m} 5^{dm}, at 80¢ a meter. Find the cost.

20. A man paid $179.20 for 8^{Dm} 9^{m} 6^{dm} of cloth. How much was that per meter?

Measures of Surface

467. The unit of *Surface Measure* is the *Square Meter.* It is a square each side of which is a meter in length.

468. Let us suppose that Fig. 1 represents a square meter, and Fig. 2 a square decimeter.

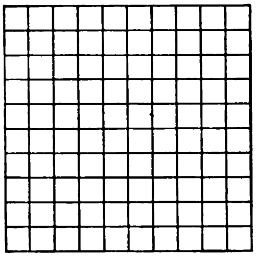


Fig. 1.

Fig. 2.

Since the meter equals 10 decimeters, it is evident that each side of Fig. 1 is 10 decimeters in length, and it follows that the square meter contains 10×10, or 100 square decimeters. Again, since each decimeter equals 10 centimeters, a square decimeter equals 10×10, or 100 square centimeters. Also, as the centimeter equals 10 millimeters, the square centimeter equals 10×10, or 100 square millimeters.

Hence the following table:

469. Table

100 square millimeters (sq mm)	= 1 square centimeter (sq cm)
100 square centimeters	= 1 square decimeter (sq dm)
100 square decimeters	= 1 square meter (sq m)
100 square meters	= 1 square dekameter (sq Dm)
100 square dekameters	= 1 square hektometer (sq Hm)
100 square hektometers	= 1 square kilometer (sq Km)

Equivalents

1 sq. centimeter	= .155+ sq. in.	1 sq. dekameter	= 119.6034 sq. yd.
1 sq. decimeter	= 15.5+ sq. in.	1 sq. hektometer	= 2.47114 A.
1 sq. meter	= 1.196+ sq. yd.	1 sq. kilometer	= 247.114 A.
			= .3861 sq. mi.

470. In measuring land, only three of the preceding denominations are used. These are the square meter, called a *Centare*

(ca), the square dekameter, callĕd an *Are* (a), and the square hektometer, called the *Hektare* (Ha).

NOTE. — Since it takes 100 of one denomination to make one of the next higher in measures of surface, the decimal point must be moved two places for every interval, in changing from one denomination to another.

PROBLEMS

471. 1. How many hektares are there in 38,649 ares?

2. How many square centimeters are there in 89,645 square decimeters?

3. Express $22.14^{\text{sq Dm}}$ as square meters; as square centimeters.

4. Express $89,634^{\text{sq m}}$ as square millimeters; as square dekameters.

5. Find the cost of 14^{Ha} 25^{a} of land at $ 56 per hectare.

6. How many hektares are there in $53,684^{\text{a}}$? How many ares in 38.645^{ca}?

7. How many square dekameters are there in $89,645^{\text{sq m}}$?

8. How many centares are there in a floor 22.3^{m} long and 18.8^{m} wide?

9. A man bought 6800 ares of woodland at $ 5 an are, and sold it at $ 550 a hektare. Find the gain.

10. A rectangular piece of land containing 1028.31^{ca} is 45.3^{m} long. What is its width?

11. How many square yards are there in 427.5 ares?

12. How many ares are there in 1080 square yards?

13. How many acres are there in 1516.8 ares?

14. How many ares are there in 90 acres?

15. I have a room 10.6 meters long and 8.4 meters wide. How much carpet a meter wide will be required to cover the floor?

MEASURES OF VOLUME

472. In *Solid* or *Cubic Measure* the *Cubic Meter* is regarded as the unit.

473. Let us suppose that Fig. 1 represents a cubic meter, and Fig. 2 a cubic decimeter.

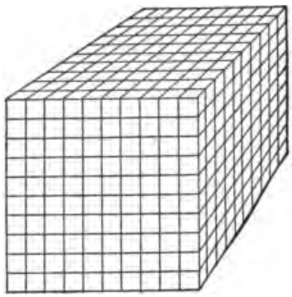


FIG. 1.

FIG. 2.

Since the meter equals 10 decimeters, it is evident that each edge of Fig. 1 is 10 decimeters in length, and it follows that the cubic meter contains 10 × 10 × 10, or 1000 cubic decimeters. Since each decimeter equals 10 centimeters, the cubic decimeter equals 10 × 10 × 10, or 1000 centimeters. Also, as the centimeter equals 10 millimeters, the cubic centimeter equals 10 × 10 × 10, or 1000 cubic millimeters.

Hence the following table:

474. TABLE

1000 cubic millimeters (cu mm)	=1 cubic centimeter (cu cm)
1000 cubic centimeters	=1 cubic decimeter (cu dm)
1000 cubic decimeters	=1 *cubic meter* (cu m)

EQUIVALENTS

1 cu. centimeter	= .061+ cu. in.
1 cu. decimeter	= 61.026+ cu. in.
1 cu. meter	= 35.316+ cu. ft., or 1.308 cu. yd.

475. The denominations above the cubic meter are not generally used. In expressing the contents of all ordinary solids the cubic meter is employed. In measuring wood, and lumber in piles, the cubic decimeter, cubic meter, and cubic dekameter are employed.

For the sake of brevity the cubic meter is generally called a *stere* (st); one tenth of a cubic meter equals a *decistere* (dst), and 10 cubic meters equal a *dekastere* (Dst).

TABLE OF WOOD MEASURE

476.

10 decisteres (dst)	= 1 stere (st)
10 steres	= 1 dekastere (Dst)

EQUIVALENTS

1 decistere	= 3.531+ cu. ft.
1 stere	= 35.316+ cu. ft.
1 dekastere	= 13.079+ cu. yd.

NOTE. — Since it takes 1000 of one denomination to make one of the next higher in cubic measure, the decimal point must be moved *three* places for every interval in changing from one denomination to another.

PROBLEMS

477. 1. How many cubic millimeters are there in $8\frac{1}{2}^{\text{cu cm}}$? In $4\frac{1}{2}^{\text{cu dm}}$?

2. How many cubic meters are there in $95{,}600^{\text{cu dm}}$?

3. Write $421.372^{\text{cu m}}$ as cubic decimeters; $75{,}006^{\text{cu dm}}$ as steres.

4. Write $136.84^{\text{cu m}}$ as cubic dekameters; as cubic decimeters.

5. Reduce 7^{st} 850^{dst} to cubic meters.

6. Find the cost of 14 steres 6 decisteres of wood at $1.87 a stere.

7. If 7^{Dst} 5^{st} of oak plank cost $110.625, how much will 3^{Dst} 7^{st} 5^{dst} cost?

8. If it costs $45.12 to remove 112^{st} 8^{dst} of earth, how much will it cost to remove $21\frac{3}{5}^{\text{st}}$?

MEASURES OF CAPACITY

478. The unit of *Capacity* is the *Liter*. It is used for both liquid and dry measure. It is equal to a cubical vessel whose edge is a decimeter, or 3.937079 in.

TABLE

479.

10 milliliters (ml)	=	1 centiliter (cl)
10 centiliters	=	1 deciliter (dl)
10 deciliters	=	1 *liter* (l)
10 liters	=	1 dekaliter (Dl)
10 dekaliters	=	1 hektoliter (Hl)
10 hektoliters	=	1 kiloliter (Kl)

EQUIVALENTS

DENOMINATIONS	DRY MEASURE	LIQUID MEASURE
1 centiliter	= .6102 cu. in.	= .338 fluid oz.
1 deciliter	= 6.1022 cu. in.	= .845 gill
1 *liter*	= .908 qt.	= 1.0567 qt.
1 dekaliter	= 9.08 qt.	= 2.6417 gal.
1 hektoliter	= 2.8372+ bu.	= 26.417 gal.
1 kiloliter	= 28.372+ bu.	= 264.17 gal.

480. The liter is used chiefly in measuring liquids, and the hektoliter in measuring dry substances.

PROBLEMS

481. 1. Change 7.36^{l} to deciliters; to milliliters.

2. Express $9^{Dl}\ 7^{l}\ 2^{cl}$ as liters; as centiliters.

3. Write 5386^{cl} as liters; as hektoliters.

4. Change 72.3467^{Kl} to liters; to deciliters; to milliliters.

5. Add 7.3^{l}, 5^{dl}, 29^{cl}, and 274^{ml}.

6. Multiply 6.48^{cl} by 36. Divide 6.48^{cl} by 8.

7. Find the cost of 7 dekaliters 4 liters of currants, at 12¢ a liter.

8. Find the sum of 3 hektoliters 4 dekaliters 7 liters, 5 hektoliters 4 liters, 9 dekaliters 4 liters.

9. Find the cost of 5 hektoliters 3 dekaliters of corn, at $1.80 a hektoliter.

10. At 62¢ a liter, find the cost of 9 liters 8 deciliters of molasses.

482. The unit of weight is the *Gram*. It equals 15.432 Troy grains.

TABLE

483.

10 milligrams (mg)	= 1 centigram (cg)
10 centigrams	= 1 decigram (dg)
10 decigrams	= 1 *gram* (g)
10 grams	= 1 dekagram (Dg)
10 dekagrams	= 1 hektogram (Hg)
10 hektograms	= 1 kilogram (Kg)
10 kilograms	= 1 myriagram (Mg)
10 myriagrams	= 1 quintal (Q)
10 quintals	= 1 tonneau, or ton (T)

EQUIVALENTS

1 centigram	= .1543+ Troy gr.	1 hektogram	= 3.5274+ oz. Av.
1 decigram	= 1.5432+ Troy gr.	1 kilogram or kilo	= { 2.6792 lb. Troy 2.2046+ lb. Av.
1 gram	= 15.432+ Troy gr.	1 myriagram	= 22.046+ lb. Av.
1 dekagram	= .3527+ oz. Av.	1 quintal	= 220.46+ lb. Av.

1 tonneau, or ton = 2204.62+ lb. Av., or 1.1023+ tons.

484. The gram and its subdivisions are used for weighing very small articles, letters, medicines, gold, silver, and for philosophical experiments.

For ordinary business purposes the kilogram, or kilo, is probably more frequently employed than any other denomination. It is about $2\frac{1}{5}$ lb. Av.

The quintal and the tonneau are principally used for weighing heavy articles, such as iron, hay, etc.

The quintal = 100 kilos, or about 220.46 lb. Av. The tonneau = 1000 kilos, or about 2204.6 lb.

The hektogram is very frequently written simply *hecto*.

PROBLEMS

485. 1. Express $296{,}431^{cg}$ as grams; as hektograms; as kilograms.

2. Express 76.34^{Kg} as tons; as hektograms; as grams.

3. Write $7^{Kg}\ 5^{Dg}\ 7^{g}\ 5^{cg}$ and 4^{mg} as grams.

4. How many grams in 560 grains?

5. How many grams in 5 lb. Av.?

6. How many pounds Av. in 536 kilos?

7. Find the sum of $7^{Dg}\ 6^{g}\ 8^{dg}$, $3^{Dg}\ 5^{dg}$, $8^{g}\ 6^{cg}\ 4^{dg}$. Answer in grams.

8. Multiply 36.7 grams by 65. Answer in hektograms.

9. At 8¢ a gram, what would be the cost of 25 grams 8 decigrams of quinine?

10. How many letters each weighing 3.5^{g} will be required to weigh 1.75^{Kg}?

486. Value of Denominate Numbers in Terms of Metric Measures

1 inch	= 2.540 centimeters	1 cu. yd.	= .7645 cu. meters
1 foot	= 3.048 decimeters	1 cord	= 3.624 steres
1 yard	= .9144 meters	1 liquid qt.	= .9463 liters
1 rod	= .5029 dekameters	1 gallon	= .3785 dekaliters
1 mile	= 1.6093 kilometers	1 dry qt.	= 1.101 liters
1 sq. in.	= 6.452 sq. centimeters	1 peck	= .881 dekaliters
1 sq. ft.	= 9.2903 sq. decimeters	1 bushel	= 3.524 dekaliters
1 sq. yd.	= .8361 sq. meters	1 ounce Av.	= 28.35 grams
1 sq. rd.	= 25.293 sq. meters	1 pound Av.	= .4536 kilograms
1 acre	= .4047 hektares	1 T. (2000 lb.)	= .9072 metric tons
1 sq. mi.	= 2.590 sq. kilometers	1 grain Troy	= .0648 grams
1 cu. in.	= 16.387 cu. centimeters	1 ounce Troy	= 31.1035 grams
1 cu. ft.	= 28.317 cu. decimeters	1 pound Troy	= .3732 kilograms

A centare (sq. meter)	= 1.196+ sq. yd.
An are (sq. dekameter)	= 119.6034 sq. yd.
A hektare (sq. hektometer)	= 2.4714 A.
A stere (cu. meter)	= .2759 cord = 35.316+ cu. ft.
A decistere ($\frac{1}{10}$ of a cu. meter)	= 3.5316+ cu. ft.
A dekastere (10 cu. meters)	= 2.759 cords

A cubic centimeter of water weighs a gram
A cubic meter of water weighs a ton
A liter of water weighs a kilogram (1000 grams)

PROBLEMS

487. 1. Reckoning the meter to equal 39.37 inches, find the length of 5 kilometers.

2. $.265^{m} + 763^{cm} + 3846^{mm} + .3865^{Hm} =$ how many meters?

3. $.836^{Hm} - 4693^{cm} =$ how many meters?

4. $.0914^{Km} \times 4 =$ how many meters?

5. $76{,}048^{mm} \div 194 =$ how many meters?

6. A boy rode on his bicycle 2568^{m} in 5 min. At the same rate how far would he ride in $1\frac{1}{2}$ hours? Answer in kilometers.

7. Change $726{,}438^{sq\,m}$ into ares; into hektares.

8. How many centares in 8 hektares?

9. Find the cost of 36.8^{a} of land when $850 is paid for 4.25^{Ha}.

10. Find the weight, in kilograms, of the water that may be contained in a cistern 2.25^{m} long, 1.75^{m} broad, and 1.5^{m} deep.

11. How many hektoliters of wheat will be contained in a bin 8.5^{m} long, 5.2^{m} wide, and 4.75^{m} deep? (1 Hl. = $\frac{1}{10}$ cu. m.)

12. Reduce $45\frac{1}{2}$ yd. to meters; $3\frac{1}{2}$ miles to Kilometers.

13. How much is a pile of wood 10 ft. long, 4 ft. wide, and 5 ft. high worth, at $1.25 a stere?

14. A man divided 5^{Hl} of wheat equally among 20 persons. How much did each person receive? Answer in liters.

PERCENTAGE

(SEE ART. 264)

PROBLEMS

488. 1. 36% of 1600 is 144% of what number?

2. What number diminished by 50% of itself equals 42.15?

3. What number increased by 34.4% of itself equals 3816.96?

4. The percentage is $22\frac{2}{9}$ and the rate $88\frac{8}{9}$%. What is the base?

5. Mt. Monadnock is 3186 ft. high. What per cent of a mile high is that?

6. A's money is \$145 more than B's. How much money has each, if $66\frac{2}{3}$% of A's equals 75% of B's?

7. A man rents his farm, valued at \$8125, for \$712.50 per year. After paying \$225 for taxes, insurance, etc., what per cent does he realize on the value of his farm?

8. A speculator in stocks sold a part of his investments for the year for \$4698, which was at a loss of 13%; the remainder he sold for \$4452, which was at a loss of $12\frac{1}{2}$%. Find the amount of his investments for the year.

9. What per cent of 271 bu. 3 pk. 4 qt. is 150% of 3 bu. 2 pk. 4 qt.?

10. A, B, and C divided a certain number of bushels of grain among themselves. A received $42\frac{1}{2}\%$, B $35\frac{3}{4}\%$, and C the remainder. How many bushels were divided if C received 87 bushels?

11. Mr. Stevens sold 20 tons of hay for \$288, and thereby lost 20%. Had he sold at \$$22\frac{1}{2}$ a ton, what would have been the gain per cent?

12. A real estate speculator bought a house and lot. A year later he was offered 20% more than he paid. At the end of the second year he sold the property for 10% more than his first offer, and received \$4488. Find the cost.

13. A dealer asked 20% more for shingles than cost, but was afterward obliged to sell them for 90% of his asking price. If he received \$7.29 a thousand, what was the cost?

14. If 25 tons of coal are bought by the *long ton*, and retailed by the *short ton*, how much is the gain, the buying and selling price being the same in each case, \$2.25?

15. A piano was sold for \$414, which was 15% more than cost. Had it been sold for \$450, what would have been the gain per cent?

16. What is the gain per cent on goods bought 20% below market price and sold 20% above market price?

17. A piece of land 80 rd. long and 20 rd. wide is what per cent of 50 acres?

18. 75 lb. equal $12\frac{1}{2}\%$ of how many hundredweight?

19. A farmer raised 1200 bu. of potatoes in 1898, which was $33\frac{1}{3}\%$ more than he raised in 1897. Find the number of bushels he raised in 1897.

20. 4260 feet equal $41\frac{2}{3}\%$ of how many miles?

21. A man paid 35% of his money for 140 acres of land. At the same rate what per cent of his money would 210 acres cost?

22. $16\frac{2}{3}\%$ of a man's age added to $12\frac{1}{2}\%$ of his age equals 14 years. Find his age.

23. I bought goods 25% below market price, and sold them 5% above market price. Find the cost if the profit realized was $270.

24. By selling apples $15\frac{1}{2}\%$ above cost I make $$.46\frac{1}{2}$ a barrel. How much must I advance the price in order to realize a profit of $33\frac{1}{3}\%$?

25. Cord wood was sold so that 60% of the selling price equaled $85\frac{5}{7}\%$ of the cost. What was the gain per cent?

26. A school which has been increased by 325% of its original number of pupils now numbers 1275 pupils. What was the original number?

27. Two horses were sold for $198 each. On one there was a gain of 10% and on the other a loss of 10%. Was there a total gain or a loss, and how much?

COMMISSION

489. A *Commission Merchant* is a person who buys and sells merchandise for another.

490. An *Agent* is a person authorized to transact business for another. He buys, sells, and rents property; collects money, lends money, etc.

491. *Commission* is a percentage charged by commission merchants and agents for transacting business. It is reckoned on the amount invested, sold, or collected.

492. The *Net Proceeds* is the sum left after the commission and other expenses have been deducted.

NOTE.—The person sending goods to be sold on commission is called the *Consignor*, or *Shipper*; the goods sent, a *Consignment*; and the person to whom the goods are sent, the *Consignee*.

PROBLEMS

NOTE. — Statements only of the following problems are required. Pupils may be required to solve the problems by analysis also, at the discretion of the teacher.

493. 1. A commission merchant sold a car load of potatoes for $270. What was his commission at $3\frac{1}{2}\%$?

2. A real estate agent sold a house and lot for $6250, charging $2\frac{1}{2}\%$. Find his commission.

3. A flour merchant sold 860 barrels of flour, at $6.75 a barrel. What was his commission at $2\frac{1}{2}\%$, and what was the amount paid over to the consignor?

4. An agent collects 80% of a debt of $3680. What is his commission at $4\frac{1}{2}\%$, and how much does he pay over to his employer?

STATEMENT

$$(\$3680 \times .80) \times .04\frac{1}{2} = \text{commission}$$
$$(\$3680 \times .80) - \text{com.} = \text{amount paid over}$$

5. When the commission is $192.30, and the rate 5%, what is the amount of the sales?

STATEMENT

$$\$192.30 \div .05 = \text{answer}$$

Since the commission, $192.30, equals the amount of the sales *multiplied* by the rate expressed decimally, it is evident that the commission *divided* by the rate will give the sum on which the commission is reckoned.

6. A commission of $154.20 was charged by a commission merchant for selling apples, his rate being $3\frac{1}{3}\%$. Find the amount of the sales.

7. My agent charged me $$42.31\frac{1}{2}$ for collecting a bill. What was the amount of the bill, his rate of commission being 5%?

8. A New York commission merchant sold a consignment of pork for a Chicago dealer, and remitted $440.37 proceeds

to the consignor. The rate of commission being $5\frac{1}{2}\%$, how much did the pork bring?

STATEMENT

$$\$\,440.37 \div .94\tfrac{1}{2} = \text{answer}$$

Since the commission merchant retained $5\frac{1}{2}\%$, or $.05\frac{1}{2}$ of the selling price as his commission, he remitted to his employer $100\% - 5\frac{1}{2}\%$, or $94\frac{1}{2}\%$, or $.94\frac{1}{2}$ of the selling price. Hence \$440.37 must equal $.94\frac{1}{2}$ of the selling price.

9. A western dealer shipped 320 head of cattle to Philadelphia to be sold on a commission of 8%. The sale netted him \$13,248. Find the average selling price per head.

10. Mr. A collected a claim for me, charging 10% commission. If he paid me \$238.77, find the amount of the claim.

11. I paid an agent \$41.40 for collecting a claim of \$720. What was the rate of commission?

STATEMENT

$$\$\,41.40 \div \$\,720 = \text{answer}$$

Since the commission, \$41.40, is obtained by *multiplying* the claim, \$720, by the rate expressed decimally, it is evident that the rate equals \$41.40 divided by \$720, which gives $.05\frac{3}{4}$, or $5\frac{3}{4}\%$.

12. If \$63 is charged for collecting \$2520, what is the rate?

13. When \$124.80 commission is paid for selling \$3840 worth of corn, what is the rate?

14. How many barrels of flour, at \$5 a barrel, can be bought for \$1734, after deducting a commission of 2% for buying?

OPERATION

$$\$\,1734 \div 1.02 = \$\,1700 = \text{sum invested}$$
$$\$\,1700 \div \$\,5 = 340 = \text{number of barrels}$$

For every \$1.02 there is \$1 invested in flour, the \$.02 being the commission for buying. Hence \$1734 equals 1.02 times the sum invested.

STATEMENT

$$(\$\,1734 \div 1.02) \div \$\,5 = \text{answer}$$

15. I sent my agent in Chicago $13,376 to invest in western land, at $40 an acre, after taking out his commission of $4\frac{1}{2}\%$. Find the number of acres and his commission.

16. How many pounds of sugar, at 5¢ a pound, can be bought for 17,250 lb. of hay, at $16 a ton, after deducting a commission of 5% for selling and 5% for buying?

STATEMENT

$$\$16 \times (17250 \div 2000) \times .95 = \text{proceeds from sale of hay}$$

$$(\text{Proceeds} \div 1.05) \div \$.05 = \text{answer}$$

17. A commission merchant sold a consignment of cotton for $2500. After deducting his commission of 3%, and $54.80 storage and freight, what amount was paid to the consignor?

18. An attorney collected 65% of a debt, charging 5% commission for collecting. His commission was $11.70. What was the sum collected, and what the full amount of the debt?

19. An agent collected 80% of a firm's outstanding claims, charging $4\frac{1}{2}\%$ for collecting. If the amount paid over to the firm was $2674, what was the sum collected, and the full amount of the firm's outstanding claims?

(To be solved by analysis)

20. My agent bought $325 worth of produce for me. After paying the commission and $8.25 freight, the produce cost me $\$347.87\frac{1}{2}$. What was the rate of commission?

21. I shipped my commission merchant in Baltimore 140 barrels of apples, with instructions to sell them so that I might realize $2.55 per barrel, after paying a commission of 15%. Find the commission.

22. I sold goods which cost $1250 for $1750. What per cent profit did my employer make, if I charged him $\$87\frac{1}{2}$ commission, $25 freight, and $\$12\frac{1}{2}$ cartage? Find my rate of commission.

23. An agent sold 80 tubs of butter, 56 lb. each, at 20¢ a pound. After deducting 5 % for selling, $ 46.80 shipping expenses, and $1\frac{1}{2}$ % for buying, he invested the proceeds in sugar at 4¢ a pound. How many pounds did he buy?

24. An agent sold cord wood at $ 3.75 a cord, on 10 % commission. How many cords did he sell if he had $ $129.80\frac{10}{13}$ to invest in lime, after deducting his commission of 4 %?

25. I sold flour on a commission of 10 %, and invested the proceeds in shingles, reserving 5 % for buying. If my commission for buying was $ 70, for what sum did I sell the flour?

TRADE DISCOUNT

494. Sometimes a certain percentage is deducted from the amount of a bill, or of a debt, for cash payment, or for other reasons. It is not an unusual thing for manufacturers, "firms," or "houses" to deduct two or more percentages from the nominal, or list price of goods. These successive deductions are known in business as *Commercial*, or *Trade Discounts*. They result usually from fluctuations in price, occasioned by competition and the condition of the market.

When two or more discounts are taken off, the first is taken from the list, or gross price; the second from the remainder; the third from the second remainder, etc.

PROBLEMS

495. **1.** A man bought a bill of goods amounting to $ 600, at 20, 10, and 5 % off. What was the net cost?

OPERATION

$\$600 \times .80 = \$480 = 20\ \%$ off
$\$480 \times .90 = \$432 = 20$ and $10\ \%$ off
$\$432 \times .95 = \$410.40 = 20, 10,$ and $5\ \%$ off, or net cost

STATEMENT

$$\$600 \times .80 \times .90 \times .95 = \text{answer}$$

2. Find a single discount equal to 20, 10, and 5% off.

$$1.00 - (.80 \times .90 \times .95) = .316 = 31\tfrac{3}{5}\% \quad \textit{Ans.}$$

3. What is the difference between 35% off and 20, 10, and 5% off?

$$35\% - 31\tfrac{3}{5}\% = 3\tfrac{2}{5}\% \quad \textit{Ans.}$$

(Solve by analysis)

4. What is the difference between 40% off and 4 tens off? Between 30% off and 15 and 15% off?

5. What is the difference between 15 and 10% off, and 10 and 15% off?

6. What single discount is equivalent to 80, 10, and 5% off?

7. A man sold a bill of goods amounting to $840, at a discount of $12\frac{1}{2}\%$ for cash. What was the discount?

8. I bought pocket knives, listed at $1.50 each, at 50 and 8% off. How much did they cost me?

9. I purchased 150 yd. of carpet, the list price being 80¢ a yard, at 15 and 10% off, an additional discount of 5% being allowed me for cash. How much did the carpet cost me per yard?

10. My agent bought a bill of hardware for me at 30 and 4% off the list price, and charged me 5% for buying. My entire bill was $529.20. What would have been the amount of my bill had the goods been purchased at list price?

11. I bought shovels at a discount of 20 and $12\frac{1}{2}\%$, the list price being $1.25 apiece. How many shovels were bought if the discount amounted to $112.50?

12. The amount of discounts at 10 and 5% off is $43.50. Find the gross amount of the bill.

13. An agent sold a consignment of wheat for $460, on a commission of $3\frac{1}{2}\%$. After deducting his commission of 5% for buying, he invested the proceeds in prints at 15¢ a yard. If he bought at a discount of 10 and 6%, how many yards did he buy?

TAXES

496. A *Tax* is a sum of money assessed on persons, property, certain incomes, productions, etc., to be used for public purposes.

497. In some states there is levied what is called a *Poll*, or *Capitation Tax.* This tax varies in different states, and is assessed on the person of each male citizen over 21 years of age.

498. A *Property Tax* is a tax on property, and is a certain percentage of its assessed value.

499. Property is divided into two classes, Real Estate and Personal Property.

500. *Real Estate* consists of immovable property, such as lands and houses. *Personal Property* consists of movable property, such as furniture, utensils, merchandise, cattle, etc.

PROBLEMS

(Solve by analysis)

501. 1. If my property is valued at $3600, and the tax rate is $1\frac{1}{2}\%$, how much is my tax?

2. Find Mr. Wilson's tax, the assessed value of whose real estate is $13,680, and personal property $7360, at the rate of $5\frac{1}{2}$ mills on the dollar.

3. The assessed value of a man's property is $30,000. If he pays $3\frac{1}{2}$ mills city tax, 13 mills school tax, $\frac{1}{2}$ mill poor tax, and $1.50 poll tax, what is the amount of his taxes?

4. I own real estate worth \$ 6800, personal property worth \$ 3150, and have \$ 4360 on interest. I pay $4\frac{3}{4}\%$ on $\frac{1}{4}$ of the value of the real estate, 5% on $\frac{1}{3}$ of the value of the personal property, and 3 mills on the money at interest. How much tax do I pay?

5. At the rate of 9 mills on the dollar, what must be the assessed value of property to yield \$ 38,520 tax?

6. The assessed value of the property of a town is \$ 192,000. What must be the rate in order to raise \$ 1536 tax?

7. A school district needs \$ 12,000 for building purposes, \$ 7500 for salary of teachers, and \$ 2500 for other expenses. The state appropriates \$ 1500, and the rest is to be raised on taxable property valued at \$ 4,100,000. What is the tax rate?

8. What sum must be assessed, at 9 mills on the dollar, in order to raise \$ 726.75 tax, after allowing 5% for collecting?

9. In a town containing 840 voters, a tax of \$ 3680 is to be levied for various purposes. If each voter pays \$ 1.25 poll tax, and the remainder is raised by a tax of 3 mills on the dollar, what is the value of the property?

10. A county needs \$ 5600 to build a bridge. Allowing 3% for collecting, what amount of property must be taxed, at 4 mills on the dollar, to raise the requisite sum?

INSURANCE

502. *Insurance* is a contract by which one party, usually a company, agrees to indemnify another against loss or damage.

503. There are several kinds of insurance, as Fire, Life, Accident, Stock, Marine, etc.

504. An *Insurance Policy* is a certificate of insurance made by the insurance company with the party insured.

505. The *Premium* is a sum of money paid for the insurance.

PROBLEMS

506. 1. What is the premium for insuring a barn valued at \$ 3500, at $1\frac{3}{4}\%$?

2. How much will it cost to insure a house for \$ 3000, at $\frac{5}{8}\%$, and the furniture for \$ 1600, at $\frac{3}{4}\%$?

3. A merchant has goods worth \$ 4800 insured for $\frac{3}{4}$ of their value, at $1\frac{3}{4}\%$. What is his premium?

4. What will be the cost of insuring 800 barrels of flour worth \$ 6.50 a barrel, at $\frac{7}{8}\%$?

5. If I paid \$ 43 for insuring property valued at \$ 8600, what was the rate?

6. The premium for insuring a house for $\frac{3}{4}$ of its value, at 3%, is \$ 108. What is the value of the house?

7. For how much must a consignment of tea, valued at \$ 33,775, be insured, at $3\frac{1}{2}\%$, in order to include the premium in case of loss?

STATEMENT

$$\$\,33{,}775 \div (1.00 - .03\tfrac{1}{2}) = \text{answer}$$

8. An insurance policy is drawn for \$ 33,000, which includes the value of the property and the premium. If the rate is 10%, what is the value of the property?

9. In Ex. 4, at what price per barrel must the flour be sold to gain 10%?

10. I imported 26 gal. of olive oil, at \$ 1.60 a gallon. I had it insured at $2\frac{3}{4}\%$. If I sell for \$ 53.43, what per cent will I gain?

11. A company agrees to insure property for enough more than its value to cover the premium, at the rate of 52¢ per \$ 100. Find the amount of the policy if the property is worth \$ 44,326.

SIMPLE INTEREST

PROBLEMS

(See Art. 286)

507. 1. Find the interest of \$ 4000 for 3 yr. 9 mo. at 6%.

$$3 \text{ yr. } 9 \text{ mo.} = 3\tfrac{3}{4} \text{ yr.}$$
$$\$\,4000 \times .06 = \$\,240, \text{ interest for 1 yr.}$$
$$\$\,240 \times 3\tfrac{3}{4} = \$\,900 \quad \textit{Ans.}$$

STATEMENT

$$\$\,4000 \times .06 \times 3\tfrac{3}{4} = \text{answer}$$

2. What is the interest of \$ 375 for 5 yr. 7 mo. at 6% ?

3. What is the interest of \$ 766.25 for 3 yr. 8 mo. at $5\frac{1}{2}$% ?

4. What is the interest of \$ 648 for 5 yr. 10 mo. 15 da. at 6% ?

OPERATION

$$5 \text{ yr. } 10 \text{ mo. } 15 \text{ da} = 70.5 \text{ mo.}$$
$$\$\,648 \times .06 = \$\,38.88 = \text{int. for 1 yr.}$$
$$\$\,38.88 \div 12 = \$\,3.24 = \text{int. for 1 mo.}$$
$$\$\,3.24 \times 70.5 = \$\,228.42 \quad \textit{Ans.}$$

STATEMENT

$$\overline{(\$\,648 \times .06) \div 12} \times 70.5 = \text{answer}$$

5. What is the interest of \$ 2040 for 3 yr. 6 mo. 9 da. at 5% ?

6. What is the interest of \$ 3928 for 2 yr. 4 mo. 5 da. at 7% ?

7. What is the interest of \$ 4128.50 for 4 yr. 8 mo. 6 da. at 8% ?

8. Find the interest of \$ 560 for 90 da. at 6%.

OPERATION

$$90 \text{ da.} = \tfrac{90}{360}, \text{ or } \tfrac{1}{4} \text{ of a year}$$
$$\$\,560 \times .06 \times \tfrac{1}{4} = \$\,8.40 \quad \textit{Ans.}$$

9. Find the interest of \$ 2140 for 80 da. at 6%.

10. Find the interest of \$ 7562.20 for 210 da. at $5\frac{1}{2}$%.

11. Find the interest of \$ 9684.26 for 185 da. at 4%.

12. Find the interest of \$ 650 from Sept. 20, 1893, to March 5, 1896, at 6%.

	yr.	mo.	da.
	1896	3	5
	1893	9	20
Time =	2	5	15

Time reduced = 29.5 mo.

STATEMENT

$$\overline{(\$\,650 \times .06) \div 12} \times 29.5 = \text{answer}$$

Make statements similar to the preceding one of the following:

13. Find the interest of \$ 2452 from Oct. 4, 1898, to May 6, 1899, at 5%.

14. Find the interest of \$ 684.84 from Feb. 5, 1897, to March 15, 1898, at 5%.

15. Find the interest of \$ 7658 from Aug. 1, 1897, to April 1, 1899, at $4\frac{1}{2}$%.

16. Find the amount of \$ 576.85 from April 1, 1892, to July 3, 1896, at 7%.

Time = $51\frac{1}{15}$ mo.

STATEMENT

$$\overline{(\$\,576.85 \times .07) \div 12} \times 51\tfrac{1}{15} + (\$\,576.85) = \text{answer}$$

17. Find the amount of \$ 3604.20 from June 4, 1897, to Jan. 3, 1899, at $5\frac{1}{2}$%.

18. Find the amount of \$ 7368.41 from Sept. 15, 1898, to Feb. 7, 1899, at $6\frac{1}{2}$%.

SIX PER CENT METHOD

PROBLEMS

508. 1. Find the interest of \$480 for 3 yr. 6 mo. 15 da., at 6%.

OPERATION

The interest of \$1 for 1 yr., at 6%, is \$.06; for 1 mo. it is $\frac{1}{12}$ of \$.06, which is \$.00$\frac{1}{2}$; and for 1 da. it is $\frac{1}{30}$ of \$.00$\frac{1}{2}$, which is \$.00$\frac{1}{60}$, or \$.000$\frac{1}{6}$.

$$\begin{aligned}
&\text{Int. of \$1 for 3 yr.} = 3 \times \$.06 = \$.18\\
&\text{Int. of \$1 for 6 mo.} = 6 \times \$.00\tfrac{1}{2} = \$.03\\
&\text{Int. of \$1 for 15 da.} = 15 \times \$.000\tfrac{1}{6} = \$.0025\\
&\text{Int. of \$1 for 3 yr. 6 mo. 15 da.} = \$.2125\\
&\text{Int. of \$480} = 480 \times \$.2125 = \$102 \quad \textit{Ans.}
\end{aligned}$$

As the interest of \$1 for 1 mo. is $\frac{1}{2}$ of a cent, and for 1 da. $\frac{1}{6}$ of a mill, another method is to reduce the time to months and days, and take $\frac{1}{2}$ of the number of months as cents, and $\frac{1}{6}$ of the number of days as mills.

2. Find the interest of \$500 for 4 yr. 8 mo. 18 da., at 6%.

OPERATION

$$\begin{aligned}
&\text{Time reduced} = 56 \text{ mo. } 18 \text{ da.}\\
&\text{Int. of \$1 for 56 mo.} = \tfrac{1}{2} \text{ of } 56, \text{ or } \$.28\\
&\text{Int. of \$1 for 18 da.} = \tfrac{1}{6} \text{ of } 18, \text{ or } \$.003\\
&\text{Int. of \$1 for 4 yr. 8 mo. 18 da.} = \$.283\\
&\text{Int. of \$500} = 500 \times \$.283 = \$141.50 \quad \textit{Ans.}
\end{aligned}$$

3. What is the interest of \$270 for 2 yr. 4 mo. 24 da., at 6%?

4. What is the interest of \$645 for 5 yr. 8 mo. 28 da., at 6%?

5. What is the interest of \$3860 for 6 yr. 3 mo. 27 da., at 6%?

6. What is the interest of $4670 from April 1, 1894, to June 8, 1896, at 6% ?

At 6%, $\frac{6}{100}$ of the principal equals the interest for 1 yr., and for 1 mo. the interest equals $\frac{1}{12}$ of $\frac{6}{100}$, or $\frac{1}{200}$ of the principal. *Therefore, the interest for* 1 *mo., at* 6%, may be obtained by dividing the principal by 200, which is the same as dividing the principal by 2 and moving the decimal point two places to the left.

7. Find the interest of $832 for 3 yr. 7 mo. 18 da., at 6%.

Time reduced = 43.6 mo.

By dividing $832 by 2 and moving the decimal point *two* places to the left, we have $4.16 as the interest for 1 mo.

$4.16 × 43.6 = $181.376 *Ans.*

8. Find the interest of $462.36 for 7 yr. 9 mo. 21 da., at 6%.

9. Find the interest of $792.25 for 2 yr. 11 mo. 27 da., at 6%.

10. Find the interest of $3846.31 for 9 yr. 2 mo. 8 da., at 6%.

Since the interest for 1 mo., or 30 da., at 6% equals $\frac{1}{200}$ of the principal, for 1 da. it will equal $\frac{1}{30}$ of $\frac{1}{200}$, or $\frac{1}{6000}$ of the principal. *Therefore, the interest for* 1 *da., at* 6%, may be obtained by dividing the principal by 6000, or, which is the same, by dividing the principal by 6, and moving the decimal point *three* places to the left.

11. Find the interest of $6960 for 126 da., at 6%.

By dividing $6960 by 6, and moving the decimal point *three* places to the left, we have $1.16, the interest for 1 da.

Hence $1.16 × 126 = $146.16 *Ans.*

12. Find the interest of $4387 for 38 da., at 6%.

13. Find the interest of $368.42 for 65 da., at 6%.

14. Find the interest of $8460.31 for 186 da., at 6%.

The *six per cent* method may be employed to find the interest at any other per cent. When the rate is 5%, take $\frac{5}{6}$ of the interest at 6%; when 7%, take $\frac{7}{6}$ of the interest at 6%, etc.

15. Find the interest of $462.36 for 4 yr. 5 mo. 27 da., at 8%.

OPERATION

Time = 53.9 mo.

$$\$462.36 \div 200 = \$2.311 = \text{int. for 1 mo. at } 6\%$$
$$\$2.311 \times 53.9 \times \tfrac{8}{6} = \$166.08 \quad \textit{Ans.}$$

Find the interest of:

16. $362.41 for 8 mo., at 6%.

17. $724.36 for 7 mo. 21 da., at 5%.

18. $642.37 for 3 yr. 4 mo. 11 da., at $4\frac{1}{2}$%.

19. 3946.21\frac{1}{2}$ for 4 yr. 125 da., at $5\frac{1}{2}$%.

20. 7643.01\frac{1}{4}$ for 266 da., at $7\frac{1}{2}$%.

21. What is the amount of $3624.61 for 2 yr. 3 mo. 24 da., at 7%?

Time = 27.8 mo.

STATEMENT

$$\overline{(\$3624.61 \div 200) \times 27.8 \times \tfrac{7}{6}} + \$3624.61 = \text{answer}$$

22. What is the amount of $468.42 for 7 yr. 6 mo. 17 da., at $5\frac{1}{2}$%?

23. What is the amount of $5642.10 for 4 yr. 78 da., at 1% a month?

24. What is the amount of $7624.65 for 287 da., at $1\frac{1}{2}$% a month?

25. What is the amount of $9864.32, from Jan. 9, 1894, to Nov. 16, 1896, at $3\frac{1}{2}$%?

26. What is the amount of $1738.46, from March 23, 1893, to Dec. 4, 1896, at .5% a month?

PROBLEMS

509. 1. Find the interest of $1326.28 for 7 yr. 3 mo. 6 da., at 5%.

Time = 87.2 mo.

STATEMENT

$$(\$1326.28 \div 200) \times 87.2 \times \tfrac{5}{6} = \text{answer}$$

2. Find the interest of $362.48 for 2 yr. 11 mo. 23 da., at $4\frac{1}{2}$%.

SUGGESTION. — At $4\frac{1}{2}$%, the int. is $\frac{4\frac{1}{2}}{6}$, or $\frac{3}{4}$ of the int. at 6%.

3. Find the interest of $6721.39 for 8 yr. 9 mo. 16 da., at $5\frac{1}{2}$%.

4. Find the interest of $1722.78, from April 1, 1895, to June 3, 1899, at $6\frac{1}{4}$%.

5. Find the amount of $3846.30, from May 29, 1896, to Sept. 14, 1899, at $3\frac{3}{4}$%.

6. Find the interest of $9648.54 for 66 da., at 7%.

STATEMENT

$$(\$9648.54 \div 6000) \times 66 \times \tfrac{7}{6} = \text{answer}$$

7. Find the interest of $3847.52 for 136 da., at 3%.

8. Find the interest of $7643.61 for 125 da., at $7\frac{1}{2}$%.

9. Find the amount of $$9438.74\frac{1}{2}$ for 216 da., at $3\frac{3}{4}$%.

STATEMENT

$$\overline{(\$9438.74\tfrac{1}{2} \div 6000) \times 216 \times \tfrac{5}{8}} + \$9438.74\tfrac{1}{2} = \text{answer}$$

10. Find the amount of $$3684.37\frac{3}{4}$ for 162 da., at $4\frac{1}{4}$%.

EXACT METHOD

510. In computing interest for a period of time less than a year, the United States government takes the exact number of days, and reckons the interest for that part of a year which the given number of days is of 365 days.

PROBLEMS

511. 1. Find the exact interest of \$400 for 135 da., at 5%.

STATEMENT

$$\$400 \times .05 \times \tfrac{135}{365} = \text{answer}$$

2. Find the exact interest of \$860 for 129 da., at 6%.

3. Find the exact interest of \$385.40 for 245 da., at $5\frac{1}{2}$%.

4. Find the exact interest of \$2000, from March 23 to July 8, at 6%.

Time = 107 da.

STATEMENT

$$\$2000 \times .06 \times \tfrac{107}{365} = \text{answer}$$

5. Find the interest of \$3860 from Jan. 1, 1899, to Aug. 9, 1899, at $4\frac{1}{2}$%.

6. Find the amount of \$3786 from April 1 to Oct. 17, at 7%.

7. Find the interest of \$5370 from Dec. 29, 1898, to Sept. 15, 1899, at $3\frac{1}{2}$%.

8. Find the interest of \$4575 from April 25, 1895, to Sept. 15, 1896, at 6%.

Time from April 25, 1895, to April 25, 1896 = 1 yr.
Time from April 25, 1896, to Sept. 15, 1896 = 143 da.
Time = $1\frac{143}{365}$ yr.

STATEMENT

$$\$4575 \times .06 \times 1\tfrac{143}{365} = \text{answer}$$

9. What is the amount of \$1750.96 from July 27, 1895, to Nov. 20, 1896, at 6%?

10. What is the amount of \$8362.50 from Feb. 26, 1895, to June 18, 1896, at 4%?

11. What is the interest, at $5\frac{1}{2}$%, of \$3862.25 from May 12, 1894, to Nov. 23, 1896?

Common Method

512. By the *Common Method* 360 da. are considered a year, in computing interest for a period of time less than a year. Hence common interest is $\frac{5}{365}$, or $\frac{1}{73}$ greater than exact interest.

PROBLEMS

(Make statements only)

513. 1. Mr. Siebel loaned \$364.40 June 3, 1895. How much will be due him May 24, 1896, at 6%?

Time = 11.7 mo.

STATEMENT

$$\overline{(\$364.40 \times .06) \div 12 \times 11.7} + \$364.40 = \text{answer}$$

2. Find the interest due July 14, 1899, on a note for \$2160, at 6%, dated April 1, 1896.

3. A note dated Aug. 3, 1897, for \$2756 was paid Nov. 25, 1899. What was the amount due at $5\frac{1}{2}$%?

4. If I borrow \$3460, at 2% a month, how much do I owe at the end of 9 mo. 27 da.?

5. A man borrowed \$4380 Sept. 13, 1896, and paid it Dec. 24, 1899. How much did he have to pay, interest at $4\frac{1}{2}$%?

(Use six per cent method for the following)

6. A note for \$4500, dated March 16, 1896, was paid July 27, 1896, with interest at 6%. Find the amount.

Time = $4\frac{11}{30}$ mo.

STATEMENT

$$\overline{(\$4500 \div 200) \times 4\tfrac{11}{30}} + \$4500 = \text{answer}$$

7. How much interest was due on a note for \$865.36, dated Jan. 15, 1897, and paid April 8, 1898, at 6%?

8. Find the amount of $746.52 for 2 yr. 136 da., at 8%.

Time = $28.5\frac{1}{3}$ mo.

STATEMENT

$$\overline{(\$746.52 \div 200) \times 28.5\tfrac{1}{3} \times \tfrac{8}{6}} + \$746.52 = \text{answer}$$

9. What is the interest on a note for $8392 for 3 yr. 214 da., at ¾% a month?

10. What is the amount of $36, borrowed Jan. 3, 1896, and paid June 29, 1896, at 1¼% a month?

514. 1. What principal will give an interest of $83.25 in 3 yr. 9 mo., at 6%?

OPERATION

Time = 45 mo.

$1.00 ÷ 200 = $.005 = int. of $1.00 for 1 mo. at 6%
$.005 × 45 = $.225 = int. of $1.00 for 45 mo. at 6%
$83.25 ÷ .225 = $370 *Ans.*

2. What principal will give an interest of $43.74 in 4 yr. 6 mo., at 6%?

3. What principal will give an interest of $494.73 in 5 yr. 9 mo., at 6%?

4. What principal will give an interest of $2110.23 in 4 yr. 3 mo., at 5½%?

SUGGESTION. — At 5½%, the interest = $\frac{5\frac{1}{2}}{6}$, or $\frac{11}{12}$ of the interest at 6%.

Time = 51 mo.

STATEMENT

$$\$2110.23 \div \overline{(\$1.00 \div 200) \times 51 \times \tfrac{11}{12}} = \text{answer}$$

5. What principal will give an interest of $250.60 in 7 yr. 8 mo., at 4½%?

6. What principal will give an interest of $225.21 in 3 yr. 8 mo. 21 da., at 7%?

7. What principal will give an interest of \$47.56, at $4\frac{1}{2}\%$, from Jan. 3, 1894, to April 6, 1896?

8. What principal, at 8%, will give an interest of \$250.305 from June 3, 1892, to July 15, 1896?

9. What principal will amount to \$446.50 in 3 yr. 6 mo., at 5%?

Time = 42 mo.

STATEMENT

$$\$446.50 \div (\$1.00 + \overline{\$1.00 \div 200 \times 42 \times \tfrac{5}{6}}) = \text{answer}$$

10. What principal, at $3\frac{1}{2}\%$, will amount to $\$922.26\frac{2}{3}$ in 4 yr. 4 mo. 12 da.?

11. What principal, at 7%, will amount to \$1498.20 from March 9, 1893, to Sept. 27, 1896?

515. 1. At what rate will \$400 give an interest of \$64 in 4 yr.?

OPERATION

$$\$400 \times .01 \times 4 = \$16 = \text{int. for 4 yr. at } 1\%$$
$$\$64 \div \$16 = 4. \quad \text{Therefore the rate is } 4\%$$

If \$400 at 1% for 4 yr. gives an interest of \$16, to give an interest of \$64 it will require as many per cent as \$16 is contained times in \$64, or 4. Therefore the rate is 4%.

STATEMENT

$$\$64 \div (\$400 \times .01 \times 4) = \text{answer}$$

2. At what rate will \$250 give an interest of \$33.75 in 1 yr. 6 mo.?

3. At what rate will \$500 give an interest of \$105 in 2 yr.?

4. At what rate will \$145 amount to \$174.435 in 3 yr. 4 mo. 18 da.? Time = 40.6 mo.

STATEMENT

$$(\$174.435 - \$145) \div \overline{(\$145 \times .01) \div 12} \times 40.6 = \text{answer}$$

5. At what rate will \$26.50 amount to \$40.624½ in 8 yr. 10 mo. 18 da.?

6. At what rate will \$9750 amount to \$12,986.45⅝ in 6 yr. 7 mo. 20 da.?

516. 1. In what time will \$468 gain \$98.28 at 6%?

OPERATION

$$\$468 \times .06 = \$28.08, \text{ int. for 1 yr. at } 6\%$$
$$\$98.28 \div \$28.08 = 3\tfrac{1}{2}. \quad 3\tfrac{1}{2}\text{ yr., or 3 yr. 6 mo. } \textit{Ans.}$$

The interest of \$468 for 1 yr. at 6% is \$28.08, and to produce an interest of \$98.28 it will require as many years as \$28.08 is contained times in \$98.28, or 3½. Therefore the time is 3½ yr., or 3 yr. 6 mo.

STATEMENT

$$\$98.28 \div (\$468 \times .06) = \text{answer}$$

In what time will:

2. \$1500 gain \$105 at 6%?
3. \$720 gain \$88.20 at 7%?
4. \$1880 gain \$216.20 at 6%?
5. \$648 gain \$228.42 at 6%?
6. \$175 amount to \$214.81¼ at 7%?
7. \$560 amount to \$673.40 at 6%?

PROBLEMS

(Make statements only)

517. 1. Find the interest of \$3820 for 2 yr. 9 mo. 6 da., at 5%. Time = 33.2 mo.

STATEMENT I

$$\overline{(\$3820 \times .05) \div 12} \times 33.2 = \text{answer}$$

STATEMENT II

$$\overline{(\$3820 \div 200) \times 33.2} \times \tfrac{5}{6} = \text{answer}$$

NOTE. — Pupils should be encouraged to state examples in as many different ways as possible.

2. Find the interest of \$726.30 from Feb. 7, 1894, to April 9, 1899, at 3½%.

3. How much interest will be due at the end of 123 da. on a note for $1763, at 5%?

STATEMENT

$$\overline{(\$1763 \div 6000) \times 123} \times \tfrac{5}{6} = \text{answer}$$

4. A man wishes to place such a sum of money on interest at $4\frac{3}{4}\%$ as will yield him $427.50 annually. Find the required sum.

5. What amount is due on a note for $425, dated July 23, 1896, and paid April 1, 1899, at $\frac{1}{2}\%$ a month?

6. Find the interest of $5052 for 88 da., at 7%.

7. Find the amount of $2760 for 126 da., at 5%.

8. Find the exact interest of $292 for 318 da., at 5%.

9. Find the exact interest of $3862 from Aug. 24, 1893, to Jan. 3, 1899, at $3\frac{1}{2}\%$.

10. Find the amount due on a note for $462.80, dated Sept. 21, and paid Dec. 18, at $4\frac{1}{2}\%$, exact interest.

(To be worked and explained)

11. What principal, at 5%, will yield an interest of $159.82 in 3 yr. 8 mo. 18 da.?

12. In what time will $1446.33 gain $\frac{5}{8}$ of itself at 7%?

13. The interest of $\frac{1}{2}$ of a certain principal for $4\frac{1}{2}$ yr. equals $12\frac{1}{2}\%$ of the whole principal. What is the rate of interest?

14. At what rate will $3750 gain $156.25 in 5 mo.?

15. A certain sum of money invested at 6% for 11 yr. amounted to $1162. In how many years, at the same rate, would it amount to $1234.45?

16. What sum of money invested at 6% for 1 yr. 4 mo. will amount to the interest of $2000 for 6 yr. 6 mo. at 4%?

17. In what time will any sum of money double itself at $4\frac{1}{2}\%$?

18. What sum of money, at $4\frac{1}{2}\%$, will yield a semiannual income of $ 388.678?

19. What sum of money will give an amount of $ $4947.05\frac{1}{4}$ in 2 yr. 8 mo 9 da., at 5%?

20. A friend loaned me $ 463.50, at 6%, which I kept till it amounted to $ 472.77. How long did I keep it?

NOTES

518. A *Note* is a written or printed promise to pay a certain sum of money on demand, or at a specified time.

519. To every note there must be two parties, the maker and the payee. The party who promises to pay is called the *Maker;* the party to whom the money is to be paid is called the *Payee.*

520. The *Face* of a note is the amount written in the body of the note, and usually in figures at the top or bottom.

521. The *Indorser* of a note is the party who, by putting his name on the back of the note, becomes responsible for its payment.

522. A note is *Negotiable* — that is, it can be transferred from one party to another — when it is drawn payable to bearer, or to the order of the payee.

523. In some states *three days,* called *Days of Grace,* are allowed for the payment of a note after the expiration of the time named in the note.

Days of grace have been abolished by statute in Alaska, California, Colorado, Connecticut, District of Columbia, Florida, Idaho, Illinois, Maine, Maryland, Massachusetts, Montana, New Hampshire, New Jersey, New York, North Dakota, Ohio, Oregon, Pennsylvania, Utah, Vermont, and Wisconsin.

524. If a note contains the words *with interest,* it bears interest from the date of the note; if not, it bears interest from the date of maturity until it is paid. Though usually inserted, the words *value received* are not essential, consideration being always implied.

525. Following are the principal kinds of notes:

A Demand Note

A *Demand Note* is one in which no time is specified for payment, the note being due when demand of payment is made.

1. FORM OF DEMAND NOTE

175\frac{35}{100}$. HARRISBURG, PA., April 11, 1898.

On demand, I promise to pay Wilson Connor, or order, One Hundred Seventy-five and $\frac{35}{100}$ Dollars, value received.

S. E. SWEET.

Time Notes

A *Time Note* is one which runs for a specified time.

2. TIME NOTE — INDIVIDUAL *

349\frac{45}{100}$. DOVER, DEL., Sept. 19, 1898.

Ninety days after date, for value received, I promise to pay John McClurg, or order, Three Hundred Forty-nine and $\frac{45}{100}$ Dollars, with interest at 5%.

H. J. FASSETT.

Find the amount due on the preceding note at maturity. Allow three days of grace.

A *Joint Note* is one made by two or more parties who are jointly responsible for its payment.

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* An individual note is one for whose payment the maker alone is responsible unless it is indorsed by others.

3. FORM OF JOINT NOTE

3060\frac{00}{100}$. PROVIDENCE, R.I., May 28, 1898.

Sixty days after date, we promise to pay Samuel G. Downs, or order, Three Thousand and Sixty Dollars, with interest at $5\frac{1}{2}\%$. Value received.

WARREN STRAW,
THOMAS RIDDLE.

Find the amount due at maturity. Allow three days of grace.

A *Joint and Several Note* is one made by two or more parties who are jointly and severally responsible for its payment.

4. FORM OF JOINT AND SEVERAL NOTE

463\frac{87}{100}$. PLAINFIELD, N.J., Aug. 23, 1898.

Three months after date, for value received, we jointly and severally promise to pay Jacob Payne, or order, Four Hundred Sixty-three and $\frac{87}{100}$ Dollars, with interest at 5%.

HORACE GROVE,
WILLIAM WALLACE,
HARRISON BELL.

Find the value of the preceding note at maturity. No grace.

A *Bankable Note* is one which is to be paid at a bank.

5. FORM OF BANKABLE NOTE

$640. BALTIMORE, MD., June 18, 1898.

Thirty days after date, for value received, we promise to pay Samuel Liggitt, or order, Six Hundred and Forty Dollars, at the First National Bank, with interest at 6%.

VOORHIS & MURRAY.

How much will be due on this note Aug. 10, 1898? No grace.

A *Surety Note* is one made payable to the order of the surety, who must indorse the note on the back to the order of the creditor.

6. FORM OF SURETY NOTE

$ 965$\frac{50}{100}$. RICHMOND, VA., Oct. 3, 1898.

Four months after date, for value received, I promise to pay Edgar Alexander, or order, Nine Hundred Sixty-five and $\frac{50}{100}$ Dollars, with interest at 5%. HORACE HOWE.

Edgar Alexander should direct the note to be paid to the order of the creditor, *H. Singerly*, by writing across the back of the note the following: "Pay to the order of H. Singerly," and signing his name.

Find the value of the preceding note at maturity. Allow grace.

7. A 60-day note for $ 890, dated March 16, without interest, was paid June 21. What was the amount due at 6%? No grace.

8. A 90-day note for $ 960, dated April 5, with interest at 6% after May 10, was paid at maturity. What sum was paid? No grace.

9. A 4-month note for $ 742, dated Oct. 21, 1897, with interest at 6%, was paid Feb. 24, 1898. Find the sum paid. No grace.

PARTIAL PAYMENTS

526. When partial payments of notes and other interest-bearing obligations are made, the amount and time of each payment are written on the back of the obligation as receipts. Such entries on the back of the obligation are called *Indorsements*.

PROBLEMS

527. 1.

$ 500$\frac{00}{100}$. HARRISBURG, PA., Sept. 26, 1895.

Three years after date, for value received, I promise to pay John Edie, or order, Five Hundred Dollars, with interest at 6%. JONAS LONG.

On this note the following payments were indorsed: Jan. 14, 1896, \$ 25; March 29, 1896, \$ 40; April 25, 1897, \$ 8; Oct. 16, 1897, \$ 12. How much was due Nov. 13, 1898?

OPERATION

Face of note	\$ 500.00
Int. on \$ 500 from Sept. 26, 1895, to Jan. 14, 1896 (3 mo. 18 da.)	
$\left\{\begin{array}{rrr} 1896 & 1 & 14 \\ 1895 & 9 & 26 \\ \hline & 3 & 18 \end{array}\right\}$	9.00
Amt. due at 1st payment, Jan. 14, 1896	509.00
First payment to be deducted	25.00
Bal. due Jan. 14, 1896 (1st new principal)	484.00
Int. on \$ 484 from Jan. 14, 1896, to March 29, 1896 (2 mo. 15 da.)	6.05
Amt. due March 29, 1896	490.05
Second payment to be deducted	40.00
Bal. due March 29, 1896 (2d new principal)	450.05
Int. on \$ 450.05 from March 29, 1896, to April 25, 1897 (1 yr. 26 da.)	28.95
Third payment, \$ 8, being less than the interest due, is not deducted.	
Int. on \$ 450.05 from Apr. 25, 1897, to Oct. 16, 1897 (5 mo. 21 da.)	12.83
The sum of the 3d and 4th payments, \$ 20, being less than the interest due, is not deducted.	
Int. on \$ 450.05 from Oct. 16, 1897, to Nov. 13, 1898 (1 yr. 27 da.)	29.03
Amt. due Nov. 13, 1898	520.86
Sum of 3d and 4th payments to be deducted	20.00
Bal. due Nov. 13, 1898	\$ 500.86

When partial payments have been made on interest-bearing obligations running longer than a year, the method employed in the preceding example for computing interest is the one adopted by the United States courts, and is, therefore, known as the *United States Rule.* It is based upon the principle that neither payment nor interest shall bear interest. It may be seen in the preceding example that the 3d payment (\$ 8) is less than the interest (\$ 28.95) due, and is not deducted as in the case of the 1st and 2d payments.

Now, if we add the $28.95 to the 2d new principal ($450.05), and then deduct the 3d payment, as we did in the case of the 1st and 2d payments, there will remain, as 3d new principal, $471. If we now compute interest on $471 up to the next payment, we shall be computing *interest* on *interest*, and therefore violating the principle upon which this rule is founded.

When a payment is less than the interest due, we take no account of it until such time as the payments taken together equal or exceed the interest due. We then proceed as we did in the case of the 1st and 2d payments. Accordingly, in the preceding example, the sum of the 3d and 4th payments ($20), being less than the interest due ($41.78) at the time the 4th payment was made, is not deducted. We next find the amount due at the date of settlement, and from this amount deduct the sum of the last two payments.

2.

$8000. DENVER, COL., Oct. 3, 1896.

Three months after date, for value received, I promise to pay J. N. Schott, or order, Eight Thousand Dollars, with interest at 6%. T. S. HAMMER.

On this note the following payments were indorsed: Jan. 1, 1897, $200; March 5, 1897, $100; Sept. 25, 1897, $150; Nov. 23, 1897, $400; Dec. 25, 1897, $250. How much was due Jan. 1, 1898?

3. A note for $5600, dated April 1, 1896, had the following payments indorsed on it: July 1, 1896, $200; Nov. 1, 1896, $75; Jan. 1, 1897, $125; Aug. 12, 1897, $600. How much was due March 31, 1898, interest 6%?

4. A note for $9500, dated Sept. 12, 1893, had indorsed on it the following payments: Nov. 15, 1893, $400; April 1, 1894, $190; Oct. 3, 1897, $450; April 25, 1898, $140. How much was due July 1, 1898, interest 6%?

5. A note for $7500, dated Aug. 26, 1897, had the following payments indorsed on it: Oct. 14, 1897, $250; Jan. 1, 1898, $60; June 30, 1898, $80; Sept. 25, 1898, $650. How much was due Dec. 1, 1898, interest 6%?

6. A note for $9000, dated May 10, 1897, had indorsed on it the following payments: July 15, 1897, $80; Oct. 21, 1897, $50; Feb. 14, 1898, $100; April 1, 1898, $1000. How much was due June 26, 1898, interest 6%?

Commercial Rule

528. When notes and accounts bearing interest are settled within a year, the following method of computing interest is usually employed.

PROBLEMS

529. 1. I had a note for $2000, dated April 1, 1895, on which the following payments were indorsed: June 25, $450; Aug. 16, $800; Nov. 10, $600; Jan. 1, 1896, $150. How much was due April 1, 1896, interest 6%?

OPERATION

Amount of $2000 for 1 yr. at 6%		$2120.00
Amt. of 1st payt. to time of settlement (9 mo. 6 da.)	$470.70	
Amt. of 2d payt. to time of settlement (7 mo. 15 da.)	830.00	
Amt. of 3d payt. to time of settlement (4 mo. 21 da.)	614.10	
Amt. of 4th payt. to time of settlement (3 mo.) . .	152.25	2067.05
Balance due April 1, 1896		$52.95

2. A note for $884, dated Jan. 15, 1897, had indorsed on it the following payments: Feb. 25, $100; May 10, $288; July 25, $260; Sept. 19, $150. Find the balance due Jan. 15, 1898, interest 6%.

3. A note for $6840, dated May 15, 1897, bears indorsements as follows: July 21, $300; Oct. 10, $500; Feb. 24, 1898, $200. Find the balance due May 15, 1898, interest 6%.

4. A note for $1440, dated March 25, 1897, bears indorsements as follows: May 12, $360; June 8, $600; Sept. 18, $380. Find the balance due March 25, 1898, interest 6%.

BANK DISCOUNT

530. A *Bank* is an institution incorporated chiefly for the purpose of loaning money, and receiving deposits. National Banks, under the direction of the national government, furnish a paper circulation.

To obtain money from a bank it is customary for a man to present a note, either made or indorsed by himself, payable at the expiration of some definitely fixed period of time, usually 30 da., 60 da., or 90 da.

The bank will give him for it a sum equal to the face of the note less the interest for the time named in the note, or in some states for 3 da. more than that time.

Thus, if a man presents a note for $600 at a bank for discount, for 30 da., at 6%, in a state where days of grace are allowed, the bank will deduct the interest for 33 da. at 6%, which is $3.30, and return to him the difference, or $596.70. The $3.30 is called Bank Discount, and $596.70 the Proceeds or Avails.

531. *Bank Discount* is simple interest computed on the face of the note from the day it is discounted to the time of its maturity. Bank discount is always paid in advance.

532. The *Proceeds* or *Avails* of a note is the sum left after the bank discount has been deducted.

533. *Days of Grace* are three days allowed by law in some states for the payment of a note after it becomes due. When no grace is allowed a note is legally due, or at maturity, at the end of the time named in the note; but when grace is allowed it is legally due on the last day of grace. (See Art. 523.)

534. When a note matures on a Sunday or on a legal holiday, in most states it must be paid on the preceding day.

535. If a note is not paid when it becomes legally due, a formal declaration, called a *Protest*, is made by a notary public, giving legal notice to the maker and indorsers that the note has not been paid.

536. When the time of a note is expressed in days, the day of maturity, in states where grace is allowed, is ascertained by counting forward from the date of the note three days more than the number of days named in the note. Thus, a note for 60 da., dated Nov. 14, matures 63 da. after Nov. 14, or Jan. 16. When the time is expressed in months, the day of maturity is ascertained by counting forward the number of calendar months named in the note, and adding three days of grace. Thus, a note for 4 mo., dated Aug. 30, matures 4 months and 3 days after Aug. 30, or Jan. 2.

537. The *Term of Discount* is the time for which the note is discounted. It is reckoned from the day the note is discounted to the day of maturity. In some states the day of discount and the day of maturity are both counted. Hence, a 30-day note in these states would be discounted for 31 or 34 days.

Note. — **In this book the day of maturity alone is reckoned, and grace is allowed in all cases except where "no grace" is specified.**

PROBLEMS

538. Find date of maturity, term of disc., disc., and proceeds.

1.

$300. SHREWSBURY, PA., Nov. 15, 1894.

Sixty days after date, for value received, I promise to pay Warren Graver, or order, Three Hundred Dollars.

WILLIS BONNER.

Discounted Dec. 1, at 6%. No grace.

OPERATION

Date of maturity, Jan. 14, 1895. Term of discount, 44 da.

$$\$300 \times .06 \times \tfrac{44}{360} = \$2.20, \text{ discount}$$

$$\$300 - \$2.20 = \text{proceeds}$$

The date of maturity, Jan. 14, is found by counting forward 60 da. from Nov. 15. Deducting from 60 da. the 15 da. remaining in November, we have 45 da. left. Deducting 31 da., December, from 45 da., we have left 14 da., which is the date of maturity.

The term of discount, 44 da., is found by counting the number of days from Dec. 1 to, and including, Jan. 14. Counting 30 da. in December, and 14 da. in January, we have 44 da., the term of discount.

2.

$3600. DETROIT, MICH., Oct. 5, 1898.

Sixty days after date, for value received, I promise to pay Hiram Crocker, or order, Thirty-six Hundred Dollars.

JOHN ABNER.

Discounted Oct. 16, at 6%.

3.

$980. NEW COLUMBUS, PA., May 10, 1898.

Ninety days after date, for value received, I promise to pay Edward Wilton, or order, Nine Hundred Eighty Dollars.

ISAAC NEWHOUSE.

Discounted Dec. 5, at 6%. No grace.

4.

$ $386\frac{50}{100}$. ELMIRA, N.Y., April 1, 1898.

Ninety days after date, for value received, I promise to pay Jason Branning, or order, Three Hundred Eighty-six and $\frac{50}{100}$ Dollars.

THOMAS GUTHRIE.

Discounted April 10, at $5\frac{1}{2}$%. No grace.

5.

$570. PROVIDENCE, R.I., May 10, 1898.

Four months after date, for value received, I promise to pay John Markey, or order, Five Hundred Seventy Dollars.

EDWARD LANIUS.

Discounted June 1, at 7%.

OPERATION

Due Sept. 13. Term of discount, 104 da.

$$\$570 \times .07 \times \tfrac{104}{360} = \$11.526, \text{ discount}$$

$$\$570 - \$11.526 = \$558.474, \text{ proceeds}$$

The date of maturity is found by counting forward 4 calendar months from May 10, and adding 3 da. of grace. The term of discount is found by counting the number of days from the day of discount to, and including, the day of maturity, as in the preceding examples.

6.

$790. GLEN ROCK, PA., June 20, 1898.

Five months after date, for value received, I promise to pay Nathan Bowers, or order, Seven Hundred Ninety Dollars.

HUGH DAY.

Discounted Sept. 5, at 8%. No grace.

7.

$\$386\tfrac{40}{100}$. NEW ORLEANS, LA., Aug. 8, 1898.

Six months after date, for value received, I promise to pay William Clapham, or order, Three Hundred Eighty-six and $\tfrac{40}{100}$ Dollars.

JOHN GROVE.

Discounted Sept. 20, at 6%.

NOTE.—When a note bears interest, the discount is reckoned on the amount of the note at maturity.

8.

$800. YORK, PA., Nov. 23, 1898.

Three months after date, for value received, I promise to pay Frank Hopper, or order, Eight Hundred Dollars, with interest at 6%.

JOHN HANSON.

Discounted Dec. 25, at 6%. No grace.

OPERATION

Time, 3 mo.

$\overline{(\$800 \times .06) \div 12} \times 3 + \$800 = \$812$, amt. of note at maturity.

Due Feb. 23. Term of discount, 60 da.

$$\$812 \times .06 \times \tfrac{60}{360} = \$8.12, \text{ discount}$$
$$\$812 - \$8.12 = \$803.88, \text{ proceeds}$$

9.

$690. DAYTON, OHIO, Dec. 18, 1898.

Four months after date, for value received, I promise to pay Charles Dodson, or order, Six Hundred Ninety Dollars, with interest at 6%.

JOHN VOIGHT.

Discounted Jan. 1, 1899, at 6%.

10. A note for $640, dated March 21, for 3 mo., was discounted March 25, at 6%. Find the discount.

Due June 24. Term of discount, 91 da.

STATEMENT

$$\$640 \times .06 \times \tfrac{91}{360} = \text{discount}$$

11. A note for $765, dated Feb. 3, for 4 mo., was discounted the same day it was drawn, at 5%. Find the proceeds.

Due June 7. Term of discount, 4.1 mo.

STATEMENT

$$\$765 - \overline{(\$765 \times .05) \div 12} \times 4.1 = \text{proceeds}$$

Make statements of the following similar to the 10th and 11th above.

12. A note for $3860, dated July 5, for 90 da., was discounted 7 da. after date, at $5\frac{1}{2}\%$. Find the discount.

13. A note for $8365, dated Aug. 10, for 4 mo., was discounted Sept. 1, at 6%. Find the proceeds.

14. A note for $9000, dated March 23, for 60 da., was discounted 3 da. after it was drawn, at 5%. Find the proceeds.

15. A note for $386.60, dated Jan. 1, for 6 mo., was discounted April 1, at 6%. Find the proceeds.

16. A note for $500, dated June 14, for 120 da., was discounted June 17, at 8%. Find the proceeds.

17. A note for $300, dated Oct. 12, for 60 da., with interest at 6%, was discounted Oct. 15, at 6%. Find the proceeds.

Due Dec. 14. Term of discount, 60 da.

STATEMENT

$$(\$300 \times .06 \times \tfrac{63}{360}) + \$300 = \text{amount}$$
$$= \text{sum to be discounted}$$
$$\text{Amount} - (\text{amount} \times .06 \times \tfrac{60}{360}) = \text{proceeds}$$

18. A note for $875.60, dated Sept. 7, for 90 da., with interest at 5%, was discounted Sept. 20, at 6%. Find the proceeds. (Statement.)

19. A note for $960.50, dated Dec. 9, 1898, for 3 mo., with interest at $4\frac{1}{2}\%$, was discounted Dec. 24, at 6%. Find the proceeds. (Statement.)

20. A note for $680.75, dated Oct. 29, 1898, for 4 mo., with interest at 6%, was discounted Nov. 10, at 8%. Find the proceeds. (Statement.)

21. A note for $7640, dated Oct. 30, 1894, for 4 mo., with interest at 6%, was discounted Oct. 31, 1894, at $6\frac{1}{2}\%$. Find the proceeds. (Statement.)

(Solve by analysis)

22. For what sum must a 60-date note, without interest, be drawn, so that when discounted at bank at 6% it will yield \$573.90 proceeds?

OPERATION

$\$1.00 \times .06 \times \frac{63}{360} = \$.0105$, discount on \$1 for 63 da.

$\$1.00 - \$.0105 = \$.9895$, proceeds of \$1 for 63 da.

$\$593.70 \div .9895 = \$600$ *Ans.*

23. Find the face of a 30-day note that will yield \$1200 when discounted at 6%.

24. Find the face of a 60-day note that will yield \$800 when discounted at 8%.

25. Find the face of a 3-month note that will yield \$630.46 when discounted at $7\frac{1}{2}$%.

26. Find the face of a 6-month note that will yield \$1384 when discounted at 8%.

27. For what sum must a note payable in 6 mo. 15 da. be drawn so that if discounted at 8% it will give \$6520 proceeds?

28. For what sum must a 90-day note be drawn so that when discounted at 8% it will yield \$825 proceeds?

29. A merchant bought goods for \$1000 cash and sold them immediately for \$1140, receiving in payment a 6-month note, without interest, which he had discounted at bank at 8%. What was his gain?

30. I owe \$1350.52, which I wish to pay. To raise the amount I have a 30-day note for \$960 discounted at 6%, and the remainder I borrow from bank on a 60-day note discounted at 8%. Find the face of the second note.

31. If the discount on a 60-day note at 6% is \$127.26, what is the face?

32. For what sum must an interest-bearing note for 60 da. at 6% be drawn so that when discounted at bank at 5% it will yield $1600 proceeds?

TRUE DISCOUNT

539. The *Present Worth* of a sum of money due at some future time, without interest, is that sum which, if put at interest for the given time, will yield an amount equal to the given sum. Thus, the present worth of $318 due 1 yr. hence, without interest, is $300, money being worth 6%, because $300 put at interest for 1 yr. at 6% will amount to $318.

540. *True Discount* is the difference between the amount of the debt bearing no interest and its present worth. In the preceding example the difference between $318 and $300, or $18, is the discount.

PROBLEMS

541. 1. What is the present worth of $553.80, due 5 yr. hence, without interest, when money is worth 6%?

OPERATION

($1.00 × .06 × 5) + $1.00 = $1.30 = amt. of $1 for 5 yr., at 6%.
$553.80 ÷ 1.30 = $426, present worth.

The amt. of $1 for 5 yr. at 6% is $1.30; hence the present worth of $1.30 is $1, and the present worth of $553.80 is as many dollars as $1.30 is contained times in $553.80, or $426.

2. What is the present worth of a debt of $2688, payable in 2 yr., without interest, when money is worth 6%?

3. What is the present worth of $1500, due in 1 yr. 6 mo. without interest, money being worth 6%?

4. What is the present worth of $257.50, due in 2 yr. 6 mo., without interest, money being worth 9%?

5. What is the true discount of $1280.36, due in 9 mo., without interest, when money is worth $4\frac{1}{2}$%?

6. What is the difference between the true discount and the interest of $3860.40 for 8 mo., money being worth $4\frac{1}{2}\%$?

7. Which is more profitable, to buy flour at $$4\frac{1}{2}$ a barrel on 4 mo., or at $5 a barrel on 6 mo., money being worth 6%?

8. I bought $600 worth of furniture, receiving a discount of 10% for cash, and sold it at list price on 2 mo. time. What was my gain, money being worth 6%?

9. I bought goods to the amount of $3000 on 60 da. If I am offered 2% discount for cash, shall I gain or lose, if I accept the offer, money being worth 6%?

10. A merchant bought goods at three different times, as follows: $3860 worth on 3 mo., $580 worth on 2 mo.; $975 worth on 4 mo. How much ready cash will be required to pay the bills, discounting at 6%?

COMPOUND INTEREST

542. If I borrow $1000 for a year, at 6%, at the end of the year I owe $1000, plus the interest ($60) for 1 year, or $1060. If I desire to keep the money longer, and do not pay the interest due at the end of the first year, I must pay interest on $1060. The interest for the second year would therefore be $63.60. If I desire to keep the money still longer, and do not pay the interest due at the end of the second year, I must pay interest on $1123.60, etc. This plan of computing interest is based on the assumption that if interest is not paid when due, it should bear interest until paid. The plan of adding the interest to the principal, and thus forming a new principal, at regular intervals of time, is called *compounding interest.*

543. *Compound Interest,* therefore, is interest computed on both principal and interest, if the interest is not paid when due; while *Simple Interest* is computed on the principal alone.

PROBLEMS

544. 1. What is the compound interest of $600 for 3 yr. at 6% ?

OPERATION I

$600	= principal
.06	
36.00	= int. due at the end of first year
$600.	
$636.	= 2d prin., or amt. due at the end of first year
.06	
38.16	= int. due at the end of second year
$636.	
$674.16	= 3d prin., or amt. due at the end of second year
.06	
40.4496	= int. due at the end of third year
$674.16	
$714.6096	= 4th prin., or amt. due at the end of third year
$600.	= 1st prin. to be deducted.
$114.6096	= compound interest

OPERATION II

$600	
1.06	
3600	
600	
$636.00	= amt. due at the end of first year
1.06	
3816	
636	
$674.16	= amt. due at the end of second year
1.06	
404496	
67416	
$714.6096	= amt. due at the end of third year
$600	
$114.6096	= compound interest

STATEMENT I

$$(\$600 \times 1.06 \times 1.06 \times 1.06) - \$600 = \text{answer}$$

STATEMENT II

$$\$600 \times (1.06)^3 - \$600 = \text{answer}$$

The work may be very much abbreviated by getting from the Compound Interest Table on page 238 the amount of $1 for the given time and rate. Thus, the amount of $1 for 3 yr. at 6% is $1.191016; the amount of $600 is 600 times $1.191016, or $714.6096.

STATEMENT III

$$(\$1.191016 \times 600) - \$600 = \text{answer}$$

NOTE. — When the interest is payable semiannually or quarterly, we find the interest for a half year or a quarter of a year, to which we add the principal, and thus form a new principal every such period.

2. What is the compound interest of $560 for 3 yr. at 6%?

3. What is the compound interest of $850 for 4 yr. at 6%?

4. What is the compound interest of $3680 for 4 yr. at 4%?

5. Find the amount of $4760 for 2 yr. compounded semiannually at 8%.

SUGGESTION. — Multiply the principal by one half the annual rate to find the interest for a half year.

6. Find the amount of $4780 for 2 yr. 6 mo. compounded semiannually at 4%.

7. Find the interest of $880 for 1 yr. 6 mo. compounded quarterly at 8%.

SUGGESTION. — Multiply the principal by one fourth of the annual rate to find the interest for a quarter of a year.

8. Find the compound interest of $960 for 1 yr. 4 mo. 21 da. at 4%, interest payable quarterly.

NOTE. — If the time consists of years, months, and days, we find the amount for the entire number of years, and then the amount of the last amount for the remaining time.

545. COMPOUND INTEREST TABLE

Showing the amount of $1, at 2, 2½, 3, 3½, 4, 4½, 5, 6, 7, 8, 9, and 10% compound interest, from 1 to 25 years.

Yr.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.
1	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000	1.0450 0000
2	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000	1.0920 2500
3	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400	1.1411 6612
4	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856	1.1925 1860
5	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290	1.2461 8194
6	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902	1.3022 6012
7	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178	1.3608 6183
8	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905	1.4221 0061
9	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181	1.4860 9514
10	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428	1.5529 6942
11	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406	1.6228 5305
12	1.2682 4179	1.3448 8882	1.4257 6089	1.5110 6866	1.6010 3222	1.6958 8143
13	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351	1.7721 9610
14	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645	1 8519 4492
15	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351	1.9352 8244
16	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8604	1.8729 8125	2.0223 7015
17	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050	2.1133 7681
18	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652	2.2084 7877
19	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918	2.3078 6031
20	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314	2.4117 1402
21	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 6807	2.5202 4116
22	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879	2.6336 5201
23	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555	2.7521 7635
24	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417	2.8760 1383
25	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633	3.0054 3446

Yr.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000
2	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000
3	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000
4	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000
5	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100
6	1.3400 956	1.4185 191	1.5007 304	1.5868 743	1.6771 001	1.7715 610
7	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171
8	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888
9	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477
10	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425
11	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167
12	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284
13	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712
14	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983
15	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482
16	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730
17	2.2920 183	2 6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703
18	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173
19	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390
20	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000
21	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499
22	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749
23	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024
24	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327
25	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059

9. Find the compound interest of $2000 for 1 yr. 5 mo. 18 da. at 8%, interest payable quarterly.

10. What principal will amount to $476.4064 in 3 yr. at 6%?

SUGGESTION. — The amount of $1.00 at 6% compound interest for 3 yr. = $1.191016. Hence the required principal = $476.4064 ÷ 1.191016, or $400 *Ans.*

11. What principal will amount to $327.8181 in 1 yr. 6 mo. at 6%, interest compounded semiannually?

ANNUAL INTEREST

546. *Annual Interest* is simple interest computed on the principal, and on each year's unpaid interest from the time it is due until settlement.

NOTE. — *Annual interest* is recognized by some states as reasonable and just when the obligation contains the words "with interest payable annually."

PROBLEMS

547. 1. Find the amount due on a note for $800, at 6%, for 3 yr. 8 mo. 21 da., with interest payable annually, if no interest was paid until settlement.

OPERATION

Annual interest = $800 × .06 = $48.

Amt. of $800, at 6%, for 3 yr. 8 mo. 21 da.,	$978.80
Int. of $48, at 6%, for 2 yr. 8 mo. 21 da.,	7.848
Int. of $48, at 6%, for 1 yr. 8 mo. 21 da.,	4.968
Int. of $48, at 6%, for 8 mo. 21 da.,	2.088
Amt. due at settlement,	$993.704

The above operation may be shortened by finding the interest of $48 for the sum of these periods:

yr.	mo.	da.	
2	8	21	
1	8	21	
	8	21	
5	2	3	= aggregated time

Amt. of \$800, at 6%, for 3 yr. 8 mo. 21 da.,	\$978.80
Int. of \$48, at 6%, for 5 yr. 2 mo. 3 da.,	14.904
Amt. due at settlement,	\$993.704

STATEMENT

44.7 mo. = time prin. bears int.

62.1 mo. = aggregated time of unpaid int.

$$\overline{(\$800 \times .06) \div 12} \times 44.7 + \$800 + \overline{(\$48 \times .06) \div 12} \times 62.1 = \text{answer}$$

2. Find the amount due on a note for \$3000, at 6%, annual interest, for 3 yr. 6 mo., if no interest has been paid.

3. At 6%, annual interest, find the amount due on a note for \$3650, if it ran 2 yr. 6 mo. 6 da., and no interest was paid till settlement.

4. Find the amount due on a note for \$600, dated March 4, 1894, with annual interest, at 6%, if it was paid July 20, 1898, no interest being paid till settlement.

5. Find the amount due April 20, 1898, on a note for \$860, at 5%, annual interest, dated Nov. 27, 1894, if the interest was paid for the first two years.

6. A note for \$2460, dated Jan. 8, 1894, at 5% annual interest, was paid Jan. 16, 1898. Find the amount due, the interest being regularly paid for 3 years.

7. What is the difference between the annual interest and the compound interest of \$8500 for 3 yr. 6 mo. at 6%?

PROBLEMS

(Analyze)

548. 1. What principal will give an amount of \$1005.196 in 2 yr. 8 mo. 27 da., at 6%?

2. A man placed \$650 at simple interest for his son when he was 12 yr. 6 mo. 21 da. old. How much will he receive when he becomes 21 yr. old, reckoning interest at 6%?

3. At 4%, compounded semiannually, what principal will give an amount of \$1475.50 in 2 yr. 6 mo.?

4. Find the amount of \$1750.96 from Dec. 19, 1892, to Feb. 29, 1896, at 6%, reckoning interest by the exact method.

5. At what rate will a principal, at simple interest, triple itself in 12 yr.?

6. A speculator bought 800 barrels of flour at \$5.25 a barrel, and sold it immediately at \$5.30, receiving in payment a 60-day note, which he had discounted at the bank at 8%. Did he gain or lose, and how much?

7.

\$2250. ST. PAUL, NOV. 26, 1896.

Two years after date, for value received, I promise to pay Ernest Detrick, or order, Twenty-two Hundred and Fifty Dollars, with interest at 5%.

AMOS MARKEY

Indorsements: March 26, 1897, \$700.
July 7, 1897, \$22.50.
March 13, 1898, \$290.

How much was due Nov. 26, 1898?

8. A note for \$157.40, dated May 10, 1896, and bearing interest at 8%, was paid July 9, 1898. What was the amount paid?

9. A offered B \$4080 cash for a building lot, or \$4584 to be paid as follows: \$2120 in 9 mo., and \$2464 in 1 yr. 6 mo. without interest. If B accepted the latter offer, did he gain or lose, and how much, money being worth 6%?

10. How much is due Jan. 1, 1898, on a note for \$2000, dated June 16, 1883, interest payable annually at 6%, if the annual interest was paid up for the first two years?

11. In what time will a principal gain three times itself at 5%?

STOCKS AND BONDS

549. The term *Stock* is used to denote the capital of incorporated companies. The capital invested is usually divided into shares of $100 each. Sometimes each share represents more than $100 of capital, and frequently less.

550. The *Par Value* of a stock is its original value. The *Market Value* is what it will sell for.

When stock sells for its original, or face value, it is said to be at par; when it sells for more than its original value, it is above par, or at a premium; and when it sells for less, it is below par, or at a discount.

Stock is usually quoted at a certain per cent of its original value. When the price of stock is quoted at 105, it is above par, or at a premium of 5%; that is, $100 worth of stock sells for $105; or $1 of stock sells for $1.05. When the price of stock is 95, it is below par, or at a discount of 5%; that is, $100 worth of stock sells for $95; or $1 of stock sells for $.95.

551. Persons who own or hold shares of stock are called *Stockholders.*

552. A *Dividend* is a sum of money distributed among the stockholders out of the gains of the business. An *Assessment* is a sum required of stockholders to meet losses or expenses.

553. A *Stock Broker* is a person who deals in stocks; that is, one who buys and sells stocks, usually for others.

554. *Brokerage* is a commission charged by brokers for transacting business for others. The rate is usually $\frac{1}{8}$%, but sometimes as high as $\frac{1}{4}$%, and is reckoned on the par value of stock.

555. *Bonds* are interest-bearing obligations to pay certain sums of money at or before some future time.

NOTE. — United States bonds, and bonds issued by states, cities, railroad companies, etc., are bought and sold like stocks.

PROBLEMS

556. 1. Find the cost of forty $100 shares of railroad stock at 101, brokerage $\frac{1}{4}\%$.

OPERATION

$$\$101 + \$\tfrac{1}{4} = \$101\tfrac{1}{4}, \text{ entire cost of 1 share}$$
$$\$101\tfrac{1}{4} \times 40 = \$4050, \text{ entire cost of 40 shares}$$

STATEMENT

$$(\$101 + \$\tfrac{1}{4}) \times 40 = \text{answer}$$

NOTE. — Statements only are required, unless the teacher directs otherwise.

2. Find the cost of 25 shares of railroad stock, at 98, brokerage $\frac{1}{8}\%$.

3. Find the cost of 45 shares of bank stock, $100, at $103\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

NOTE. — In the following examples no brokerage is allowed unless otherwise stated.

4. A man bought 50 shares of Reading R.R. stock, $100, at $4\frac{1}{2}\%$ discount. After receiving a 5% dividend, he sold at 93. How much did he gain?

5. Find the cost of five $1000 United States bonds at $105\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

6. What income will be realized from $10,000 worth of $4\frac{1}{2}\%$ bonds, purchased at par?

7. How many shares, $100, of railroad stock may be bought for $9050, at $113\frac{1}{8}$?

STATEMENT

$$\$9050 \div \$113\tfrac{1}{8} = \text{answer}$$

8. What amount of 4% bonds, at $3\frac{1}{2}$% premium, may be bought for \$4140?

9. How many shares of stock, \$50, at $\frac{1}{4}$% discount, can I buy for \$3591?

10. Mr. Carle sent his broker in Philadelphia \$6390 to invest in Pennsylvania R.R. stock at $53\frac{1}{8}$. How many shares, \$50, did he get, brokerage $\frac{1}{4}$%?

11. My broker sold for me 150 shares of R.R. stock, \$50, at $55\frac{1}{2}$, and invested the proceeds in mining stock, \$100, at $104\frac{1}{2}$, brokerage $\frac{1}{4}$% in each case. How many shares did I get, and how much surplus?

12. How much must be invested in United States 4% bonds to yield an annual income of \$1600?

OPERATION

$$\$1600 \div .04 = \$40{,}000 \quad \textit{Ans.}$$

Since \$1 of stock yields \$.04 income, \$1600 must be .04 of the sum to be invested.

13. I received \$301 as my annual dividend from a $3\frac{1}{2}$% stock. How much stock do I hold?

14. Mr. Harvey received \$225 as his quarterly dividend from a 10% stock. Find the par value of the stock.

15. A railroad company divided \$44,800 among its stockholders as an 8% dividend. Find the face of the stock. How many shares, \$100, do I own if my share of the dividend was \$4000?

16. What sum must I invest in 5% stock, at 103, to yield an income of \$480?

OPERATION

$$\$480 \div .05 = \$9600, \text{ par value of stock}$$
$$\$1.03 \times 9600 = \$9888, \text{ sum invested}$$

Since \$1 of stock yields \$.05 income, \$480 must be .05 of the par value of the stock. Hence the par value must equal as many dollars as .05 is contained times in \$480, or \$9600. Since the cost of \$1 of stock

is \$1.03, the cost of \$9600 of stock is 9600 times \$1.03, which equals \$9888 *Ans.*

STATEMENT

$$\$1.03 \times (480 \div .05) = \text{answer}$$

17. What sum of money must be invested in United States $4\frac{1}{2}\%$ bonds, at 105, to yield an annual income of \$360?

18. What sum must I invest in 5% city school bonds, selling at $101\frac{3}{4}$, to yield an annual income of \$1800?

19. What will be my annual income if I invest \$3120 in a 5% stock, selling at 104?

OPERATION

$$\$3120 \div 1.04 = \$3000, \text{ par value of stock}$$
$$\$3000 \times .05 = \$150, \text{ income}$$

Since \$1.04 is paid for \$1 of stock, for \$3120 there can be bought as many dollars of stock as 1.04 is contained times in \$3120, or \$3000. 5% of \$3000 = \$150 *Ans.*

STATEMENT

$$(\$3120 \div 1.04) \times .05 = \text{answer}$$

20. What income will be derived from \$9785 invested in a $5\frac{1}{2}\%$ stock selling at 103?

21. What income will be derived from \$7225 invested in a $4\frac{1}{2}\%$ stock selling at 85?

22. If I buy 5% stock at 90, what rate of interest do I receive on my investment?

OPERATION

$$\$5 \div \$90 = .05\tfrac{5}{9} = 5\tfrac{5}{9}\% \quad \textit{Ans.}$$

Since each share costs \$90 and yields an income of \$5, the rate of income on the investment equals \$5 ÷ \$90, or $.05\frac{5}{9} = 5\frac{5}{9}\%$.

23. When United States 6% bonds are bought at 112, what rate of income is realized?

24. When stock paying 6% is bought at 120, what rate per cent of income is realized?

25. When stock yielding 9% is bought at a premium of 30%, what rate of income is realized?

26. A man bought bank stock which pays 5% semiannually, at a premium of 40%. What rate of income did he realize on the investment?

27. Stock paying 5% dividend is purchased at 10% discount. What rate per cent is realized on the investment?

28. What must be paid for a 5% stock in order to realize 6% on the investment?

One share, \$100, of 5% stock yields an income of \$5; hence \$5 must equal .06 of the price to be paid. \$5 ÷ .06 = \$83⅓ *Ans.*

29. What must be paid for a 4% stock to realize 5% on the investment?

30. What price must be paid for 8% stock to realize an income of 6%?

31. What must be paid for stock yielding 10% to realize 8% on the investment?

32. A man bought railroad stock at a discount of 3¼%, and sold it at premium of 4¾%, thereby gaining \$640. Find the par value of the stock.

\$1 of stock was bought for 100% − 3¼%, or 96¾% of its par value, and sold for 100% + 4¾%, or 104¾% of its par value. The gain on \$1 of stock = 104¾% − 96¾%, or 8% of its par value. Hence the par value of the stock = \$640 ÷ .08, or \$8000 *Ans.*

STATEMENT

$$\$640 \div (.03\tfrac{1}{4} + .04\tfrac{3}{4}) = \text{answer}$$

33. I paid my broker \$25 for selling Jersey Central stock, \$100, brokerage ⅛%. How many shares did he sell?

34. I paid my broker \$75 for selling Wyoming Valley Traction Company stock, \$100, brokerage ¼%. Find the par value of the stock, and the number of shares sold.

35. My broker bought for me 50 shares of bank stock, \$100, at 125, and sold them at $130\frac{1}{2}$. What was my profit, brokerage $\frac{1}{4}\%$?

36. A man bought 60 shares of D. L. and W. stock, \$100, at 107, and after receiving an 8% dividend sold them at $102\frac{1}{2}$, brokerage $\frac{1}{4}\%$. Did he gain or lose, interest on the money invested not considered?

37. If a man received \$1325 as a 5% dividend on stock bought at 107, how much had he invested?

38. What per cent dividend is declared if I receive \$600 on \$11,420 invested in bank stock at $142\frac{3}{4}$?

STATEMENT

$$\$600 \div (\$11{,}420 \div 1.42\tfrac{3}{4}) = \text{answer}$$

39. What per cent dividend is declared if I receive \$382.50 on \$8372.50 invested in turnpike stock at $98\frac{1}{2}$?

40. I own 300 shares of stock, \$100. July 1, I received a stock dividend of 15 shares; and Jan. 1, I received another of 18 shares and \$90 in money. Find the rates of dividend declared.

41. Johnson rents his farm for \$720 a year, which is 6% of its value, and pays a tax of 2 mills on its valuation, and an annual insurance of \$36. If he sells his farm and invests in a 5% stock at 90, will his annual income be increased or diminished, and how much?

42. Which is the better investment, $4\frac{1}{2}\%$ stock bought at 90, or $5\frac{1}{2}\%$ stock at 110?

43. Which is the better investment, stock yielding 6% dividend, bought at 20% discount, or stock yielding 8% bought at 90?

44. What price must be paid for 5% stock so that it will yield the same per cent of income as 10% stock bought at 160?

45. A man owns 75 shares of bank stock, $100, which pays a semiannual dividend of $4\frac{1}{2}\%$. Will his annual income be increased or diminished, and how much, if he sells his bank stock at par, and invests in 6% bonds at 75?

46. At what premium must 6% bonds be sold so that only 5% is realized on the investment?

47. A man wishing to raise $3000 cash directs his broker to sell sufficient Baltimore and Ohio railroad stock. If the market value of the stock is $108\frac{1}{2}$, how many shares, $100, must he sell, and what will be the surplus, brokerage $\frac{1}{4}\%$?

48. Mr. Voorhis bought Manhattan Elevated R. R. stock, $100, at $147\frac{1}{2}$, and sold it at $150\frac{1}{4}$. After paying brokerage $\frac{1}{4}\%$, he gained $168.75. How many shares did he buy?

STATEMENT

$$\$168.75 \div \overline{(\$150\tfrac{1}{4} - \$\tfrac{1}{4}) - (\$147\tfrac{1}{2} + \$\tfrac{1}{4})} = 75, \text{ answer}$$

49. A capitalist bought a certain number of shares of N. Y. Central stock, $100, which yielded a dividend of 6%, at $109\frac{1}{2}$, and twice as many shares of D. & H., which also yielded an annual dividend of 6%, at 112, brokerage in each case being $\frac{1}{4}\%$. His annual income from both was $720. How much money had he invested in each?

50. A capitalist has $46,800 which he desires to invest in stocks. He can invest in 7% stock at 90, or in 10% stock at 130. Which will be the more profitable for him, and how much?

51. A man bought $18,080 worth of bank stock at 113, and after receiving *two* $2\frac{3}{4}\%$ dividends, he sold the stock at $111\frac{1}{2}$. What was the par value of the stock, and how much did he make?

52. What rate per cent of income will be realized on 6% bonds, payable in 10 yr., if they are purchased at 5% premium?

OPERATION

$$\$1 \times .06 \times 10 = \$.60, \text{ what } \$1 \text{ yields, at par, in 10 yr.}$$
$$\$.60 - \$.05 = \$.55, \text{ net proceeds of } \$1 \text{ for 10 yr.}$$
$$\$.55 \div 10 = \$.055, \text{ net proceeds of } \$1 \text{ for 1 yr.}$$
$$\$.055 \div \$1.05 = .05\tfrac{5}{21} = 5\tfrac{5}{21}\% \quad \textit{Ans.}$$

At par \$1 yields \$.06 in 1 yr., and in 10 yr. it yields \$.60. Since the bonds were purchased at 5% premium, the net proceeds of \$1 is \$.60 − \$.05, or \$.55 for 10 yr., and for 1 yr. it is \$.55 ÷ 10, or \$.055, which is the annual income from \$1.05. Hence the rate of income = \$.055 ÷ \$1.05, or $.05\frac{5}{21}$, or $5\frac{5}{21}$%.

53. What rate of interest will a purchaser receive on his investment, if he buys 5% 10-year bonds at 102?

54. What rate per cent of income will 6% bonds payable in 20 yr. yield, if purchased at 4% premium?

55. What rate of interest will a man receive on his investment, if he buys $4\frac{1}{2}$% 10-year bonds, at 5% discount?

OPERATION

$$\$1 \times .04\tfrac{1}{2} \times 10 = \$.45, \text{ what } \$1 \text{ yields in 10 yr. at par}$$
$$\$.45 + \$.05 = \$.50, \text{ proceeds of } \$1 \text{ for 10 yr.}$$
$$\$.50 \div 10 = \$.05, \text{ proceeds of } \$1 \text{ for 1 yr.}$$
$$\$.05 \div \$.95 = .05\tfrac{5}{19} = 5\tfrac{5}{19}\% \quad \textit{Ans.}$$

56. A bought *three* 10-year \$500-bonds, bearing 5% interest, at 98. What rate of interest was realized on the investment?

57. A man purchased a 5-year \$5000 mortgage, bearing 6% interest, at a discount of 3%. What rate of interest was realized on the investment?

58. Which is the better investment, 15-year 6% bonds, purchased at 104, or 15-year $5\frac{1}{2}$% bonds at $96\frac{1}{2}$?

59. If \$5000 be invested in 5% bonds at par, and an equal sum in 6% bonds at 90, which will be the better investment, and how much? Bonds in each case to run 10 yr.

60. A capitalist invested in city bonds as follows:

a. $5000 in 6% bonds, payable in 10 yr., at 95
b. $5000 in $5\frac{1}{2}$% bonds, payable in 15 yr., at 104
c. $5000 in $4\frac{1}{2}$% bonds, payable in 20 yr., at 106

What rate of income was realized in each case?

61. I bought 100 shares of bank stock, $100, at 95. After holding them for 2 yr. 6 mo. and receiving regular semiannual dividends of $2\frac{1}{2}$%, I sold at par. Did I gain or lose, and how much, money being worth 6%?

DOMESTIC EXCHANGE

557. *Exchange* is the system of making payments to persons living at a distance by means of written orders called drafts or bills of exchange.

558. A *Draft* or *Bill of Exchange* is a written order from one party to another to pay a certain sum of money to a third party at a specified time.

559. The party who directs payment to be made and signs the draft is called the *Maker* or *Drawer;* the party to whom the draft is addressed is called the *Drawee;* the party to whose order the draft is made payable is called the *Payee.*

560. When exchange is made between different places in the same country it is called *Domestic Exchange* or *Inland Exchange;* when it is between different countries it is called *Foreign Exchange.*

561. When the draft is presented to the *drawee,* if he consents to pay it, he writes the word "Accepted," with date of acceptance, and his name across the face of it. This is called an Acceptance, and is necessary only on time drafts.

562. Drafts are drawn *at sight;* that is, on presentation, or at a certain number of days *after sight* or after date. Time

drafts are allowed three days of grace in some states. In some states grace is allowed on sight drafts unless drawn without grace. (See Art. 523.)

563. The cost of a draft is regulated chiefly by the course of trade. If the trade, say between New York and New Orleans, is equal, exchange is said to be at par, and a draft sells for its face value. If New Orleans is indebted to New York, drafts in New York on New Orleans sell at a discount. (That is, drafts on New Orleans are sold cheap in New York to encourage their purchase, since by drawing money from New Orleans they help to discharge her debt toward New York.) But if New York is indebted to New Orleans, drafts in New York on New Orleans sell at a premium. The cost of a time draft is affected not only by the course of trade, but by the discount for the time also.

SIGHT DRAFT

564. Wyoming National Bank of Wilkesbarre.

$2000. WILKESBARRE, PA., Jan. 9, 1898.

At sight pay to the order of John Wiley Two Thousand Dollars.

WILLIS JOHNSON,

To the Third National Bank, *Cashier.*
New York.

TIME DRAFT

565. Wyoming National Bank of Wilkesbarre.

$2000. WILKESBARRE, PA., Jan. 9, 1898.

Ten days after sight pay to the order of John Eaton Two Thousand Dollars.

WILLIS JOHNSON,

To the Third National Bank, *Cashier.*
New York.

NOTE.—In this book time drafts are allowed *three days of grace* unless otherwise specified.

PROBLEMS

566. 1. Find the cost of a draft for \$600, at $\frac{1}{4}\%$ premium.

OPERATION

$$\$600 \times .00\tfrac{1}{4} = \$1.50, \text{ premium}$$
$$\$600 + \$1.50 = \$601.50, \text{ cost}$$

2. Find the cost of a draft for \$800, at $\frac{3}{8}\%$ premium.

STATEMENT

$$\$800 \times 1.00\tfrac{3}{8} = \text{answer}$$

3. Find the cost of a draft for \$750, at $1\frac{1}{8}\%$ premium.

4. Find the cost of a draft for \$1500, at $1\frac{1}{4}\%$ discount.

OPERATION

$$\$1500 \times .01\tfrac{1}{4} = \$18.75, \text{ discount}$$
$$\$1500 - \$18.75 = \$1481.25, \text{ cost}$$

STATEMENT

$$\$1500 \times .98\tfrac{3}{4} = \text{answer}$$

5. Find the cost in New York of a draft on Denver for \$346.80, at $\frac{1}{2}\%$ discount.

6. Find the cost in New York of a draft on Cleveland for \$960, at $\frac{5}{8}\%$ discount.

7. Find the cost of a 30-day draft for \$500, exchange being at $\frac{1}{4}\%$ premium, and interest 6%.

OPERATION

$$\$500 \times .06 \times \tfrac{33}{360} = \$2.75, \text{ discount for 33 da.}$$
$$\$500 - \$2.75 = \$497.25, \text{ cost of draft if exchange were at par}$$
$$\$500 \times .00\tfrac{1}{4} = \$1.25, \text{ premium}$$
$$\$497.25 + \$1.25 = \$498.50, \text{ cost}$$

Since the buyer of a time draft must wait the required time before he receives the money, the draft should be purchased at a discount. The discount of \$500 for 33 da. at 6% = \$2.75. If exchange were at par, the cost of the draft would be \$500 – \$2.75, or \$497.25. But exchange

is at $\frac{1}{4}$% premium. $\frac{1}{4}$% of \$500 = \$1.25 = premium. Hence the entire cost of the draft equals \$497.25 + \$1.25 = \$498.50.

STATEMENT

$$\overline{\$500 - (\$500 \times .06 \times \tfrac{33}{360})} + (\$500 \times .00\tfrac{1}{4}) = \text{answer}$$

$$\text{Or} \quad \$500 \times \overline{1.00\tfrac{1}{4} - (.06 \times \tfrac{33}{360})} = \text{answer}$$

8. Find the cost of a draft for \$320, at 60 da., premium $1\frac{1}{8}$%, interest 6%.

9. How much will a 60-day draft for \$560 cost, when exchange is $\frac{5}{8}$% premium, and interest 6%?

10. What is the value of a 90-day draft for \$3000, at $\frac{3}{8}$% premium, and interest at 5%? No grace.

11. Find the cost of a 60-day draft for \$5600 at $\frac{1}{8}$% discount. Rate of interest 6%.

OPERATION

$$\$5600 \times .06 \times \tfrac{63}{360} = \$58.80, \text{ discount for 63 da.}$$
$$\$5600 - \$58.80 = \$5541.20, \text{ cost if exchange were at par}$$
$$\$5600 \times .00\tfrac{1}{8} = \$7, \text{ discount}$$
$$\$5541.20 - \$7 = \$5534.20, \text{ cost}$$

STATEMENT

$$\overline{\$5600 - (\$5600 \times .06 \times \tfrac{63}{360})} - (\$5600 \times .00\tfrac{1}{8}) = \text{answer}$$

$$\text{Or,} \quad \$5600 \times \overline{(1.00 - .00\tfrac{1}{8}) - (.06 \times \tfrac{63}{360})} = \text{answer}$$

12. Find the cost of a draft for \$2000, payable in 60 da., when exchange is $\frac{1}{2}$% discount, and interest 7%.

13. Find the cost of a draft for \$1000 for 60 da., premium $\frac{3}{4}$%, and interest 6%. No grace.

14. Find the face of a draft costing \$967.20, premium $\frac{3}{4}$%.

OPERATION

$$\$1.00 + \$.00\tfrac{3}{4} = \$1.00\tfrac{3}{4}, \text{ cost of \$1 of draft}$$
$$\$967.20 \div \$1.00\tfrac{3}{4} = \$960, \text{ face}$$

STATEMENT

$$\$967.20 \div (1.00 + .00\tfrac{3}{4}) = \text{answer}$$

15. Find the face of a draft which cost $\$2485$, discount $\frac{3}{8}\%$.

STATEMENT

$$\$2485 \div (1.00 - .00\tfrac{3}{8}) = \text{answer}$$

16. Find the face of a draft which cost $\$3840.37\frac{1}{2}$, discount $\frac{1}{4}\%$.

17. Find the cost of a 30-day draft for $\$2440$, exchange being at a discount of $\frac{3}{4}\%$, and interest 6%. No grace.

18. Find the face of a 90-day draft which cost $\$1182.90$, premium $\frac{1}{8}\%$, interest 6%.

OPERATION

$$\$1.00 \times .06 \times \tfrac{93}{360} = \$.0155, \text{ discount of } \$1 \text{ for 93 da.}$$
$$\$1.00 - \$.0155 = \$.9845, \text{ cost of } \$1 \text{ of draft if exchange were at par}$$
$$\$.9845 + \$.00\tfrac{1}{8} = \$.98575, \text{ cost of } \$1 \text{ of draft}$$
$$\$1182.90 \div .98575 = \$1200, \text{ face}$$

STATEMENT

$$\$1182.90 \div (1.00 - \overline{1.00 \times .06 \times \tfrac{93}{360}} + .00\tfrac{1}{8}) = \text{answer}$$

19. Find the face of a 60-day draft which cost $\$1984$, premium $\frac{1}{4}\%$, interest 6%.

FOREIGN EXCHANGE

567. *Foreign Exchange* is the exchange between places in different countries.

568. A *Bill of Exchange* is a written order upon one party to pay a certain sum of money to another party, or to his order, at a specified time.

NOTE.—Most foreign bills of exchange are drawn upon the leading banking houses of the chief commercial centers, as London, Paris, Berlin, Bremen, etc., and are expressed in the currency of the corresponding country.

569. Bills of exchange drawn upon London are called *Sterling Bills.* The legal par of exchange between the United States and Great Britain is \$ 4.8665 per *pound sterling.*

Bills of exchange drawn upon Paris are reckoned at a certain number of *francs* to the dollar, the value of the franc being about \$ .194.

Bills of exchange drawn upon Berlin, Hamburg, and Bremen are reckoned at a certain number of cents per four reichsmarks (marks), the value of the reichsmark being about $24\frac{1}{8}$ ¢.

NOTE. — The above rates of exchange are not fixed values. Rates of exchange vary with the conditions of trade. They are quoted by the commercial papers.

570. Foreign bills of exchange are usually issued in sets of two or three bills, called the first, second, and third of exchange. In order to provide against loss or delay, these bills are sent, each by a different route, or by a different means of conveyance. Each bill contains a condition that when one has been paid the others become void.

PROBLEMS

571. 1. What is the cost of a bill of exchange on London for £ 130, when exchange is at \$ 4.855?

Since the value of £ 1 is \$4.855, the value of £ 130 is 130 times \$4.855, or \$631.15.

OPERATION

$$\begin{array}{r} \$\,4.855 \\ 130 \\ \hline 145650 \\ 4855 \\ \hline \$\,631.150 \end{array}$$ *Ans.*

2. Find the cost in New York of a bill of exchange on London for £ 375 10 *s.* when exchange is at \$ 4.875.

3. Find the cost of a bill of exchange on Liverpool for £ 475 16 *s.* 9 *d.*, when exchange is at \$ $4.81\frac{1}{2}$.

4. Find the cost of a bill of exchange on Paris for 3090 francs, exchange being 5.15 francs to the dollar.

OPERATION

$$5.15)3090(\$600$$
$$\underline{3090.}$$
$$00$$

Since 5.15 francs cost \$1, the cost of 3090 francs will be as many dollars as 5.15 is contained times in 3090, which equals \$600.

5. What will be the cost in Baltimore of a bill of exchange on Paris for 12,315 francs, when exchange is quoted at 5.3 francs to the dollar?

6. What will be the cost in Boston of a bill of exchange on Berlin for 5520 marks, exchange being quoted at $96\frac{1}{2}$?

Since the cost of 4 marks is \$$.96\frac{1}{2}$, the cost of 1 mark is $\frac{1}{4}$ of \$$.96\frac{1}{2}$, or \$$.24\frac{1}{8}$, and the cost of 5520 marks is 5520 times \$$.24\frac{1}{8}$, or \$1331.70.

OPERATION

$$\$.96\tfrac{1}{2} \div 4 = \$.24\tfrac{1}{8} = \text{cost of 1 mark}$$
$$\$.24\tfrac{1}{8} \times 5520 = \$1331.70 \textit{ Ans.}$$

7. Find the cost in Philadelphia of a bill of exchange on Bremen for 3680 marks, exchange being $94\frac{3}{4}$.

8. Find the cost of a bill of exchange on Dublin for £1550 15 *s.*, sterling exchange being \$4.845.

9. Find the cost of a bill of exchange on Frankfort for 11,040 marks, when exchange is quoted at $96\frac{1}{2}$.

NOTE. — The principles involved in the preceding examples are applicable to exchange with any other nation. By consulting the table of foreign money and applying the methods illustrated in the preceding examples, any problem in foreign exchange may be easily solved.

MISCELLANEOUS PROBLEMS

(Solve by analysis)

572. 1. A farmer raised 1035 bushels of potatoes in the years 1897 and 1898. How many bushels did he raise each year if he had 30% more bushels in 1897 than in 1898?

2. A man gave his son 128 A. 80 P. of land, which was $66\frac{2}{3}\%$ of what he had left. How many acres had he before he made this gift to his son?

3. A merchant marked goods 25% above cost. If he sells for 25% less than his marked price, will he gain or lose, and how many per cent?

4. A merchant wishes to average 20% profit on his sales after deducting 3% for bad debts and allowing 5% to an agent for collecting. At what per cent above cost must he mark his goods?

5. If goods are bought 25% below their value, and an allowance of 5% is made for cash payment, what per cent profit will be made if the goods are sold 10% above their value?

6. A man invested \$6315 in bonds at 105, and after receiving a 4% dividend, he sold them at 115. How much did he gain, brokerage in each case $\frac{1}{4}\%$?

7. Find the compound interest of \$360 at 6% from Jan. 1, 1897, to June 12, 1898, interest semiannually.

8. A note for \$3600, dated April 1, 1895, bearing interest at 6%, had the following payments indorsed on it: June 5, 1895, \$54; April 1, 1896, \$75; Aug. 20, 1897, \$400; Dec. 31, 1897, \$700. How much was due April 1, 1898?

9. In what time will a sum of money double itself at 8%, simple interest?

10. In what time will a sum of money gain twice itself at 8%, simple interest?

11. Find the interest of \$836.25 for $2\frac{1}{2}$ years at 5%, compounded semiannually.

12. Find the face of a 60-day note which, being discounted at bank at 6%, will yield \$200.

13. An agent sold flour at 5% commission, and invested the net proceeds in sugar, charging $2\frac{1}{2}$% for buying. His entire commission was \$210. Find value of flour and sugar.

14. What sum must be invested in 6% bonds at $94\frac{5}{8}$, brokerage $\frac{1}{8}$%, in order to secure an annual income of \$1200?

15. The amount of \$2550, at 6%, compounded semiannually for 2 yr. 8 mo. 15 da., will equal the amount of what sum at simple interest for the same time and rate?

16. I bought a bill of goods amounting to \$2500, receiving 5% off for cash. I immediately sold the goods on a credit of 6 months, at an advance of 10% on the list price. What per cent did I gain on the investment, money being worth 6%?

17. The amount of a certain principal, at simple interest, for 3 yr. 6 mo. is \$968, and for 4 yr. 8 mo., at the same rate, the amount is \$1024. Find the principal and rate.

18. For how much must flour which cost \$7.50 a barrel be sold to gain $16\frac{2}{3}$%, after deducting a commission of 5%?

19. Find the amount due on a note for \$1325, dated Jan. 18, 1895, and paid Jan. 8, 1898, at 6% annual interest, if the interest for the *first* year only was paid when due.

20. A merchant purchased a bill of goods amounting to $1800 on 60 days' credit. He is offered 3% for cash. Will he gain or lose, if he accepts the offer, money being worth 6% ?

21. Mr. King bought a building lot for $3000 cash. To raise part of the money he directed his agent to sell for him 20 shares D. L. & W. stock at 122½. The balance he raised by having his note for 60 da. discounted at a bank at 6%. What was the face of the note, allowing the agent ⅛% brokerage for selling the stock ?

22. Find the cost of a 60-day draft for $5161, without grace, when the rate of exchange is ¼% premium, and interest 6%.

23. What premium can I afford to pay for 6½% bonds in order to realize 5% on my investment?

24. The face of a 6% interest-bearing note, at 90 days, was $750, and the proceeds $759.952. If the date of the note was Aug. 1, 1898, and the rate of discount 8%, what was the date of discount?

25. A certain sum of money, at 6% simple interest, for a certain time amounts to $960, and at 8% for the same time to $1030. Find the principal and time.

26. What single discount is equal to 33⅓%, 20%, and 1% off ?

27. Stewartstown Lumber & Manufacturing Co., in 1898, declared a cash dividend of 6%, and a stock dividend of 9%. How much stock does that person own who received $360 as his cash dividend, and 9 shares, $100, as his stock dividend?

28. A school district wishes to erect a high school building to cost $18,000. It is found that a tax of 8 mills on the assessed valuation of the property will be sufficient to produce the required sum, and the collector's commission of 5%. Find the assessed value of the district.

RATIO AND PROPORTION

RATIO

573. *Ratio* has its origin in the comparison of similar numbers.

574. Numbers are compared in two ways:

1st. By considering them with respect to their difference.

2d. By considering them with respect to their quotient.

575. The expression of the relative magnitude of two similar quantities by means of their difference is called *Arithmetical Ratio.*

576. The expression of the relative magnitude of two similar quantities by means of their quotient is called *Geometrical Ratio.*

577. *Ratio,* therefore, may be defined, in general terms, as the expression of the relative magnitude of two similar quantities.

NOTE. — **Only Geometrical Ratio will be considered at this time.**

578. *Geometrical Ratio* is usually indicated by placing the colon between the two numbers compared. Thus, $6:2$ means the ratio of 6 to 2. Ratio is sometimes expressed by writing the numbers in the form of a common fraction; thus, $\frac{6}{2}$.

579. The first of the two numbers compared is called the *Antecedent;* the second, the *Consequent.* In the ratio $6:2$, the antecedent is 6 and the consequent 2.

580. The ratio of two numbers is found by dividing the antecedent by the consequent. For example, the ratio of 8 to 4 is 2; it is obtained by dividing 8 by 4.

Note.—A ratio is always an abstract number. There is no ratio between unlike quantities.

581. A *Simple Ratio* is the ratio of one antecedent and one consequent; as $9:3$.

582. A *Compound Ratio* expresses the product of two or more simple ratios; thus, $4:12 \times 5:6 = \frac{4}{12} \times \frac{5}{6}$.

583. There are several different kinds of ratios, but only two kinds are usually considered in arithmetic, viz. Direct and Inverse.

584. *Direct Ratio* is the quotient obtained from dividing the antecedent by the consequent. *Inverse Ratio* is obtained from dividing the consequent by the antecedent.

PROBLEMS

585. 1. What is the ratio of 126 to 882? Of \$425 to \$25?

2. What is the ratio of $72\frac{3}{4}$ to $363\frac{3}{4}$? Of $\frac{3}{15}$ to $\frac{16}{45}$?

3. What is the ratio of \$8 to 80¢? Of 3 ft. 5 in. to 7 ft.?

4. What is the ratio of $87\frac{1}{2}$¢ to \$7? Of $5\frac{1}{2}$ to $16\frac{1}{2}$?

5. What is the ratio of 43.2 to 7.2? Of .3 to $2.6\frac{1}{3}$?

6. Find the value of the following ratios:

$(4:8) \times (7:6)$; $(9:3) \times (6:5)$; $(24:3) \times (76:28)$.

7. The antecedent is 54, and the ratio 18. Find the consequent.

8. The consequent is 26.5, and the ratio 8. Find the antecedent.

9. The ratio is $4\frac{1}{2}$, and the antecedent $157\frac{1}{2}$. Find the consequent.

10. The consequent is $24\frac{1}{6}$, and the ratio $\frac{1}{6}$. Find the antecedent.

PROPORTION

586. When four quantities are arranged so that the ratio of the *first* to the *second* equals the ratio of the *third* to the *fourth*, the four terms form a proportion. Thus, $12:4=9:3$ is a proportion, and is read, the ratio of 12 to 4 equals the ratio of 9 to 3; or, 12 is to 4 as 9 is to 3.

587. A proportion is often indicated by writing a double colon between the ratios compared. Thus, $12:4::9:3$.

588. The first and fourth terms of a proportion are called the *Extremes*, and the second and third the *Means*.

Principles

1. *The product of the extremes equals the product of the means.*

2. *Either extreme equals the product of the means divided by the other extreme.*

3. *Either mean equals the product of the extremes divided by the other mean.*

PROBLEMS

589. Find the omitted term in each of the following:

1. $12:6=14:(\;)$
2. $16:4=(\;):8$
3. $9:(\;)=27:9$
4. $(\;):24=36:72$
5. $32:96=30:(\;)$
6. $\$12:\$30=(\;):190$ bu.
7. $\$34:\$136=70$ yd. $:(\;)$
8. $(\;):1\frac{5}{7}=1\frac{21}{35}:1\frac{3}{5}$
9. $8\frac{3}{4}:1\frac{1}{4}=487\frac{1}{2}:(\;)$
10. $1:(\;)=1.36:35.36$

Simple Proportion

590. A *Simple Proportion* consists of two simple and equal ratios. Simple proportion was formerly called the *Single Rule of Three*, because three terms are given to find the fourth.

591. The first and second terms of a proportion are called the first couplet, and the third and fourth the second couplet.

592. In stating a problem in proportion it is most convenient to arrange the quantities so that the required answer will be the fourth term. Therefore, the third term must be of the same kind of quantity as the answer.

NOTE. — Twenty years' experience in the schoolroom has led the author to believe that the most simple and natural method of arranging a proportion is to make the required answer the fourth term.

PROBLEMS

593. 1. If 8 barrels of flour cost $40, how much will 12 barrels cost?

STATEMENT

$$8 \text{ bbl.} : 12 \text{ bbl.} = \$40 : \text{answer}$$
$$(12 \times 40) \div 8 = 60$$
$$\$60 \quad \textit{Ans.}$$

Since the *cost* of 12 bbl. is required, we make the given cost, $40, the third term of the proportion. It is evident that the cost of 12 bbl. will be more than the cost of 8 bbl. Therefore, the greater of the two remaining quantities must be the second term. The reason for this is plain. Since the consequent of the second couplet will be greater than the antecedent, the consequent of the first couplet must also be greater than the antecedent; for the two couplets must represent equal ratios. Dividing the product of the means by the given extreme, we find the other extreme to be 60.

2. If 25 yards of cloth cost $75, how much will 15 yards cost?

STATEMENT

$$25 \text{ yd.} : 15 \text{ yd.} = \$75 : \text{answer}$$
$$(15 \times 75) \div 25 = 45$$
$$\$45 \quad \textit{Ans.}$$

Since the answer, or fourth term, will be the cost of 15 yd., we make the given cost, $75, the third term. From the nature of the problem, it is evident that the cost of 15 yd. will be less than the cost of 25 yd. Therefore, the smaller of the two remaining quantities must be made the second term. Again, the reason for this is obvious. Since the consequent of the second ratio will be less than the antecedent, the consequent of the first ratio must also be less than the antecedent.

After making the third term of the proportion the same kind of quantity as the required answer, the pupil should notice, from the nature of the example, whether the answer (fourth term) is to be greater or less than the third term. If greater, then the greater of the two remaining quantities must be the second term; but if the answer is to be less than the third term, then the smaller of the two remaining quantities must be made the second term.

3. If 14 yards of silk cost $42, how much will 56 yards cost?

4. If a train running 35 miles an hour completes a certain trip in 8 hours, in what time will a limited express running 45 miles an hour complete the same trip?

5. If 120 sheep were sold for $480, how much would 89 sheep bring at the same rate?

6. If 80 tons of coal cost $220, how much will 124 tons cost?

7. If it requires 200 pounds of coal to run an engine for 6 hours, how many pounds will be consumed by the engine in making 15 trips of 8 hours each?

8. A man walked 171 mi. 140 rd. in 9 da. At the same rate, how far would he walk in 21 da.?

9. If 154 bu. of wheat are required to make 44 bbl. of flour, how many bushels will be required to make 26 bbl.?

10. If 36 carpenters earn $416 in one week, in how many days will they earn $624?

11. If $840 gains $42 in one year, how much will $945 gain in the same time?

12. If 9 masons build a wall in 24 da., working 8 hr. daily, in what time will they build it working 10 hr. daily?

13. A staff 27 ft. high casts a shadow $18\frac{1}{2}$ ft. What must be the height of a staff to cast a shadow 30 ft. 9 in. at the same time of day?

14. If a ton of iron ore yields .65 of a ton of pure iron, how much iron ore will be required to yield 376.22 tons?

15. How many barrels, each containing 31.5 gal., will be required to hold 1299.375 gal.?

16. If $13\frac{5}{7}$ tons of iron cost \$ $582\frac{6}{7}$, how many tons will \$ $2833\frac{1}{3}$ buy at the same price?

17. When $53\frac{5}{11}$ bu. of corn are given for $13\frac{1}{15}$ tons of coal, how many bushels must be given for $92\frac{3}{8}$ tons?

18. The cost of reaping 10 A. 140 sq. rd. of grain is \$ 6.80. At the same rate, what will be the cost of reaping 18 A.?

19. If a force of 1320 laborers clears half the streets of a city of snow in 5 hours, how many extra men must be employed to complete the work in 3 hours more?

20. If 31 yd. of cloth cost \$ 76, how much will 18 pieces, each containing $39\frac{1}{2}$ yd., cost?

21. John's money is to Henry's money as 3 is to 7. If Henry's money is \$ 6000, how much money has John?

22. If the freight on 20 bbl. of flour from New York to Scranton is \$ 9.60, what will be the freight on 78 bbl.?

23. If an 8-cent loaf weighs 20 oz. when flour is selling at \$ 5 a barrel, how much should it weigh when flour is selling at \$ 8 a barrel?

24. A piece of land 40 rd. long, and 80 rd. wide, contains just 20 A. How long must a lot be to contain 20 A. if its width is 16 rd.?

25. If 80 loaves of bread supply 150 men 9 da., how many loaves will be required to supply 180 men the same time?

26. If 1260 bu. of oats last 46 horses 7 weeks, how many horses will 3780 bu. last the same time?

27. A bicycle wheel 72 inches in circumference revolves 880 times in going a mile. How many times will it revolve in going 47,520 feet?

28. If 12 overcoats can be made from 141 yards of cloth, how many yards will be required to make 35 overcoats?

29. If it requires 64 yd. of matting $\frac{7}{8}$ of a yard wide to cover a floor, how many yards $\frac{4}{5}$ yd. wide will be sufficient to cover the same floor?

30. If 1440 men have provisions enough for 30 da., at the rate of 20 oz. a day to each man, how many additional men could be received for the time if the daily allowance of each man is reduced to 18 oz.?

31. If \$800, at 5% for a certain time, yields \$133 interest, at what rate will the same principal yield \$186.20 interest in the same time?

Compound Proportion

594. *Compound Proportion* expresses an equality of two ratios, one or both of which always are compound.

$$\text{I.}\ \left\{\begin{matrix} 10:6 \\ 12:10 \\ 16:24 \end{matrix}\right\} = 4:3 \qquad \text{II.}\ \left\{\begin{matrix} 8:24 \\ 14:28 \\ 6:24 \end{matrix}\right\} = \left\{\begin{matrix} 10:20 \\ 6:18 \\ 3:12 \end{matrix}\right\}$$

In the first of the preceding proportions, only one ratio is compound; in the second, both are compound.

PROBLEMS

595. 1. If 16 men build 12 rd. of wall in 20 days, how many rods will 40 men build in 24 days?

Since the required term, or the answer, is to be rods, we make 12 rd. the third term.

It is evident that 40 men will build more wall than 16 men will. Therefore we make 40 men the second term, and 16 men the first. More wall will be built in 24 da.

STATEMENT

$$\begin{matrix} 16:40 \\ 20:24 \end{matrix} = 12\text{ rd.} : \text{answer}$$

OPERATION

$$\frac{40 \times 24 \times 12\text{ rd.}}{16 \times 20} = 36\text{ rd.}\quad \textit{Ans.}$$

than in 20 da. Therefore we make the larger of the two numbers compared (24 da.) the second term, and 20 da. the first.

Dividing the product of the second and third terms by the product of the first terms, we get 36 rd. as the answer.

2. If 4 men dig 250 rd. of ditch in 150 days, in how many days will 36 men dig 486 rd.?

STATEMENT

$$36 : 4 = 150 \text{ da.} : \text{answer}$$
$$250 : 486$$

OPERATION

$$\frac{4 \times 486 \times 150}{36 \times 250} = 32\tfrac{2}{5} \text{ da. } \textit{Ans.}$$

Since the answer is to be days, we make 150 da. the third term. If 4 men do the work in 150 da., 36 men will do it in less time; hence we make the smaller of the two numbers compared (4 men) the second term, and 36 men the first. Again, it will take more days to dig 486 rd. than 250 rd., hence we make 486 the second term and 250 the first.

Dividing the product of the second and third terms by the product of the first terms, we get $32\frac{2}{5}$ da. as the answer.

3. If 36 men cut 360 cords of wood in 5 da. of 10 hr. each, how many cords will 40 men cut in 3 da. of 8 hr. each?

4. If a piece of land 80 rd. long and 40 rd. wide costs $326, what will be the cost, at the same rate, of a piece 140 rd. long and 80 rd. wide?

5. If 32 yd. of sheeting $2\frac{1}{4}$ yd. wide cost $7.36, how much will 45 yd. $2\frac{1}{2}$ yd. wide cost?

6. If a block of ice 16 in. long, 12 in. wide, and 10 in. thick weighs 25 lb., how much will a block 30 in. long, 10 in. wide, and 8 in. thick weigh?

7. If 10 joists 30 ft. long, 10 in. wide, and 4 in. thick cost $14, how much will 60 joists 20 ft. long, 9 in. wide, and $3\frac{1}{2}$ in. thick cost?

8. If $400 yield $114 interest in 4 yr. 9 mo., how much interest will $750 yield in 3 yr. 8 mo. at the same rate?

9. If \$1200, in 9 yr., at 3% gains \$324 interest, how much will \$2400 gain in 6 yr. 4 mo. at 8%?

10. If \$500 gains \$100 interest in 4 yr. at 5%, what principal will gain \$108 in 3 yr. at 6%?

11. If it requires 21,600 bricks for a wall 80 ft. long, 6 ft. high, and 20 in. thick, each brick being 8 in. long, 4 in. wide, and 2 in. thick, how many bricks 8 in. long, $4\frac{1}{2}$ in. wide, and 2 in. thick will it take to build a wall 100 ft. long, 8 ft. high, and 24 in. thick?

12. If 28 men, working 14 da. of 10 hr. each, dig a cellar 160 ft. long, 40 ft. wide, and 8 ft. deep, in how many days of 8 hr. each will 21 men dig a cellar 120 ft. long, 30 ft. wide, and 10 ft. deep?

Partitive Proportion

596. *Partitive Proportion* treats of the division of a number into parts proportional to certain other numbers.

PROBLEMS

597. 1. Divide 152 into three parts proportional to 3, 7, and 9.

If 152 is divided into 3 + 7 + 9, or 19 equal parts, $\frac{3}{19}$ of 152, or 24, will equal the first number, $\frac{7}{19}$ of 152, or 56, will equal the second number, and $\frac{9}{19}$ of 152, or 72, will equal the third number.

OPERATION

$$3 + 7 + 9 = 19$$

$\frac{3}{19}$ of 152 = 24, first number

$\frac{7}{19}$ of 152 = 56, second number

$\frac{9}{19}$ of 152 = 72, third number

2. Divide 198 into three parts proportional to 5, 6, and 7.

3. Divide 374 into four parts proportional to 2, 3, 8, and 9.

4. Divide 207 into three parts proportional to $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

NOTE. — Reduce the fractions to their least common denominator and divide 207 in proportion to the numerators of the fractions.

5. Divide \$819 into three parts proportional to $\frac{1}{4}$, $\frac{3}{5}$, and $\frac{2}{3}$.

6. Three men gained \$4800 on a joint investment. Find each man's share of the gain if their respective investments were proportional to 7, 8, and 9.

7. A man divided 434 acres of land between his two sons so that their respective shares were proportional to $\frac{3}{7}$ and $\frac{5}{9}$. How many acres did each receive?

8. A miller invested \$5776 in wheat, corn, and rye in parts proportional to 3, 5, and 8. Find how much was invested in each kind of grain.

9. A farmer shipped to market a car load of corn and rye containing 468 bu. in parts proportional to $\frac{1}{5}$ and $\frac{1}{8}$. Find the number of bushels of each.

10. Divide \$3203.20 among three men so that their shares shall be proportional to $\frac{7}{12}$, $\frac{5}{8}$, and $\frac{11}{16}$.

11. Three brothers raised 31,620 bushels of potatoes. How many bushels did each raise if their amounts are to each other as 9, 10, and 12?

12. Three partners divided \$37,640 profits among themselves in the proportion of 4, 7, and 9. What was the share of each?

PARTNERSHIP

598. *Partnership* is the association of individuals for the purpose of carrying on a certain branch of business. The persons constituting the association are called *partners,* and the association thus formed a *house, firm,* or *company.*

599. The money or property put into the business by the partners is called the *Capital* of the company.

600. The *Resources* or *Assets* of a firm embrace its property of all kinds, its cash on hand, and the amounts due it. Its *Liabilities* are its debts.

601. For convenience of treatment, partnership is sometimes separated into *simple* and *compound.*

602. In *Simple Partnership* each partner has his capital invested for the same time.

PROBLEMS

603. 1. A and B engage in the hardware business. A puts in \$6000 and B \$4000. They gain \$1250. What is each one's share of the gain?

OPERATION

$\$6000 + \$4000 = \$10{,}000 =$ capital invested by both

$\frac{6000}{10000}$, or $\frac{3}{5} =$ A's share of the capital

$\frac{4000}{10000}$, or $\frac{2}{5} =$ B's share of the capital

Hence, $\frac{3}{5} \times \$1250$, or $\$750 =$ A's share of the gain

and $\frac{2}{5} \times \$1250$, or $\$500 =$ B's share of the gain

2. A and B enter into partnership to carry on a retail grocery business. A invests \$2500 and B \$2000. They gain \$675. What is each one's share of the gain?

3. Three men, A, B, and C, form a partnership to deal in oysters. A puts in \$450, B \$575, and C \$740. They gain \$353. Find each one's share of the gain.

4. A, B, and C engage in the coal business. A contributes \$8500, B \$9000, and C \$12,000. They gain \$13,275. Find each one's share of the gain.

5. Three persons, A, B, and C, form a partnership to deal in lumber, with a joint capital of \$22,400. At the end of a year they divide the profits, of which A's share is \$2250, B's \$1950, and C's \$2520. How much capital had each invested?

604. In *Compound Partnership* each partner's capital is not always employed for the same period of time.

PROBLEMS

605. 1. A and B enter into partnership to deal in potatoes. A puts in \$8000 for 3 yr., and B puts in \$6500 for 2 yr. They gain \$2960. What is the share of each?

A's \$8000 invested for 3 yr. is equivalent to 3 times \$8000, or \$24,000, for 1 yr. B's \$6500 invested for 2 yr. is equivalent to 2 times \$6500, or \$13,000 for 1 yr. They both have invested equivalent to the sum of \$24,000 and \$13,000, or \$37,000 for 1 yr.

OPERATION

$$\begin{aligned}
\$8000 \text{ for } 3 \text{ yr.} &= \$24000 \text{ for } 1 \text{ yr.} = \text{A's} \\
\$6500 \text{ for } 2 \text{ yr.} &= \underline{\$13000} \text{ for } 1 \text{ yr.} = \text{B's} \\
&\ \$37000 = \text{both for } 1 \text{ yr.}
\end{aligned}$$

$$\tfrac{\cancel{24000}}{\cancel{37000}}, \text{ or } \tfrac{24}{37} \times \$2960 = \$1920 = \text{A's gain}$$

$$\tfrac{\cancel{13000}}{\cancel{37000}}, \text{ or } \tfrac{13}{37} \times \$2960 = \$1040 = \text{B's gain}$$

2. A, B, and C form a partnership. A puts in \$3000 for 1 yr., B puts in \$2500 for 8 mo., and C \$1500 for 6 mo. At the end of the year they divide \$2470. Find the share of each.

3. Smith and Jones enter into partnership, Smith contributing \$ 3600, and Jones \$ 1800. At the end of 8 mo. Jones puts in \$ 600 more. At the end of the year they divide \$ 1680. Find each man's share of the gain.

4. Jan. 1, 1897, Mr. Samson commenced dealing in coal with a capital of \$ 5000. May 1, Mr. Straw became a partner with a capital of \$ 3000. Sept. 1, Mr. Riddle put in \$ 4000. Jan. 1, 1898, they found their loss to be \$ 1250. How much of the loss should each man bear?

5. Wilson, Allen, and Thompson entered into partnership April 1, 1896. Wilson put in \$ 2500, Allen \$ 3000, and Thompson \$ 3500. At the end of one year they each withdrew \$ 1000 from the business. April 1, 1898, they found their loss to be \$ 1050. Find what part of the loss each should bear.

6. Anson and Roll entered into partnership to continue for $1\frac{1}{2}$ yr. During the first 9 mo. Anson's capital was $\frac{3}{8}$ as much as Roll's, and during the last 9 mo. $\frac{5}{8}$ as much as Roll's. They gained \$ 13,230. What was the share of each?

7. Payne, Collins, and Arnold formed a partnership for speculative purposes. Payne invested \$ 1600 for 9 mo., Collins \$ 1200 for 1 yr. 1 mo., and Arnold \$ 2400 long enough to entitle him to receive \$ 1456 profit. Find the time Arnold's capital was invested, and the shares of Payne and Collins respectively, the entire gain being \$ 2756.

EQUATION OF PAYMENTS

AVERAGE TERM OF CREDIT

606. The *Average Term of Credit* is the time to elapse before several sums of money, without interest, due at different times, may be paid at once, without loss to the parties concerned.

PROBLEMS

607 1. A owes B $500 due in 4 mo.; $800 due in 5 mo.; $1400 due in 6 mo. Find the average term of credit.

A credit of $500 for 4 mo. = a credit of $1 for 500 times 4 mo., or 2000 mo. A credit of $800 for 5 mo. = a credit of $1 for 800 times 5 mo., or 4000 mo. A credit of $1400 for 6 mo. = a credit of $1 for 8400 mo.

By adding we find $1 has a credit of 14,400 mo. If $1 has a credit of 14,400 mo., $2700 will have a credit of $\frac{1}{2700}$ of 14,400 mo., or $5\frac{1}{3}$ mo., the average term of credit.

OPERATION

$$\begin{array}{rcl} 500 \times 4 &=& 2000 \\ 800 \times 5 &=& 4000 \\ \underline{1400} \times 6 &=& \underline{8400} \\ 2700 & &)\underline{14400} \\ & & 5\frac{1}{3} \end{array}$$

2. I owe $500 due in 6 mo., $600 in 8 mo., and $900 in 4 mo. What is the average term of credit?

3. A man owes $600 due in 4 mo., $800 due in 5 mo., and $1400 due in 6 mo. What is the average term of credit?

4. A merchant bought a bill of goods amounting to $2250, of which $1250 was to be paid in 30 da., $750 in 60 da., and the remainder in 90 da. Find the average term of credit.

5. C owes D \$8000, which he agrees to pay as follows: \$3000 cash, \$2000 in 4 mo., and the balance in 8 mo. Find the average term of credit.

OPERATION

$$
\begin{array}{l}
3000 \text{ cash} \ldots \text{ no credit} \\
2000 \times 4 = 8000 \\
\underline{3000} \times 8 = \underline{24000} \\
8000 \qquad \underline{)32000} \\
\qquad\qquad\quad 4 \ldots 4 \text{ mo. } \textit{Ans.}
\end{array}
$$

Since the first payment is cash, it has no credit.

6. Johnson owes Smith \$1800, of which he agrees to pay \$800 down, \$600 in 4 mo., and \$400 in 9 mo. Find the average term of credit.

7. If Morgan lends his brother \$1350 for 6 mo., how long ought his brother to lend him \$900 to balance the favor?

OPERATION

The use of \$1350 for 6 mo. is equivalent to the use of \$1 for 1350 times 6 mo., or 8100 mo., or to the use of \$900 for $\frac{1}{900}$ of 8100 mo., or 9 mo.

$$
\begin{array}{r}
1350 \times 6 = 8100 \\
8100 \div 900 = 9 \\
9 \text{ mo. } \textit{Ans.}
\end{array}
$$

8. If I borrow \$150 from a friend for 3 mo., for how many months should I lend him \$300 to repay the favor?

9. I owe \$1100 to be paid in 8 mo. without interest. If I pay \$300 at the end of 4 mo., and \$400 at the end of 6 mo., how long after maturity may I retain the remainder as an equivalent for the sum prepaid?

OPERATION

Since the first payment (\$300) is made 4 mo. before it is due, I am entitled to a credit of \$300 for 4 mo., or of \$1 for 1200 mo. Likewise on the second payment I am entitled to a credit of \$1 for 800 mo. In both I am entitled to a credit of \$1 for 2000 mo. Hence to balance this credit, I may retain the remainder (\$400) after maturity for $\frac{1}{400}$ of 2000 mo., or 5 mo.

$$
\begin{array}{r}
300 \times 4 = 1200 \\
\underline{400} \times 2 = \underline{800} \\
700 \qquad 2000 \\
1100 - 700 = 400 \\
2000 \div 400 = 5 \\
5 \text{ mo. } \textit{Ans.}
\end{array}
$$

10. C owes D \$2400 payable in 8 mo. without interest. At the end of 2 mo. he pays \$600; at the end of 3 mo., \$800; and at the end of 4 mo., \$200. How long in equity may C retain the remainder after maturity?

THE EQUATED TIME

608. The *Equated Time* is the date at which several sums due at different times may be paid at once, without loss to the parties concerned.

PROBLEMS

609. 1. A man bought goods at a vendue April 1, 1898, as follows: \$700 on a credit of 2 mo.; \$1000 on a credit of 3 mo.; \$1400 on a credit of 6 mo. What is the equated time of payment?

OPERATION

$$\begin{array}{r} 700 \times 2 = 1400 \\ 1000 \times 3 = 3000 \\ \underline{1400} \times 6 = \underline{8400} \\ 3100 \quad \underline{)12800} \\ 4\frac{4}{31} \end{array}$$

We find the average term of credit to be $4\frac{4}{31}$ mo. Hence the equated time is $4\frac{4}{31}$ mo. from April 1, or Aug. 5.

April 1 + $4\frac{4}{31}$ mo. = Aug. 5.

2. Mr. Day bought merchandise Jan. 1, 1898, as follows: \$1050 on 2 mo.; \$700 on 4 mo.; and \$350 on 6 mo. What is the equated time of payment?

3. Find the equated time for the payment of \$3000 from June 11, 1898, due as follows: $\frac{1}{5}$ in 30 da.; $\frac{1}{4}$ in 60 da.; $\frac{3}{10}$ in 90 da.; and the remainder in 120 da.

4. I bought goods Aug. 8, 1898, to the amount of \$4200 to be paid as follows: \$2100 cash; \$1400 in 4 mo.; and the remainder in 6 mo. What is the equated time of payment?

5. I borrowed \$720 for 10 mo., and \$1080 for 8 mo. At the end of 7 mo. I paid \$1200. How long after the equated time for the payment of the whole may I retain the remainder?

MISCELLANEOUS PROBLEMS

610. 1. How much wheat at \$.98 a bushel can be bought for \$823.20, after deducting 5% commission?

2. If $\frac{3}{8}$ of a lot of goods was sold for $\frac{5}{8}$ of its value, what was the loss per cent?

3. In what time will a principal double itself at 6% simple interest?

4. In what time will a principal double itself at 6% compound interest?

5. A merchant sold sugar at 4¢ a pound, and thereby lost 12%. Find the cost of the sugar.

6. Find the face of a 90-day draft which cost \$4611.07, at $\frac{1}{2}$% premium, interest 6%.

7. For how much must a building which ccst \$10,900 be insured, at $3\frac{1}{2}$%, to cover the value of the building and cost of the premium?

8. How long will it take 10 men to do as much work as 2 men can do in $3\frac{1}{2}$ da. of 8 hr. each?

9. On March 23, a man gave notes as follows: \$400 payable in 3 mo.; \$300 in 2 mo.; and \$800 in 4 mo. Find the average term of credit, and the equated time of payment.

10. Divide 96 into two such parts that the second shall be 6 more than the first.

11. A man raised 390 bu. of potatoes in one field, which was 20% more than he raised in another field. How many bushels did he raise in both fields together?

12. A merchant sold goods to the amount of $1392.25, and received in payment a 90-day note without interest. At the end of 20 da. he had the note discounted at a bank at 6%. How much did he realize from the sale?

13. A man owed $5000 to be paid in 1 yr. 3 mo. At the end of 5 mo. he paid $1600, and 2 mo. later he paid $1800 more. When in equity should he pay the balance?

14. On a bill of goods amounting to $2430, a discount of 10% from the list price was made, and 6% additional was allowed for cash payment. Find the cash value of the goods.

15. A street 200 ft. long and 25 ft. wide is to be paved with bricks $8\frac{1}{2}$ in. long by $4\frac{1}{2}$ in. wide. How many will be required?

16. What per cent of income will be realized on 10% stock bought at $112\frac{1}{2}$?

17. A farmer sold a storekeeper 30 bu. of potatoes at 50¢ a bushel. Allowing 5 bu. for waste in measuring, for how much per peck must he sell the potatoes in order to realize 20% on the transaction?

18. A man has 135 sheep in three fields; in the second field there are twice as many as in the first, and in the third three times as many as in the second. How many sheep are there in each field?

19. A grocer bought a barrel of molasses containing $31\frac{1}{2}$ gal., at 40¢ a gallon. He retailed it at $.12 a quart. Find his gain, and gain per cent.

20. An electric car ran from Wilkesbarre to Pittston and returned in $1\frac{1}{2}$ hr. It was 45 min. going, and returned at the rate of $13\frac{1}{2}$ mi. an hour. What is the distance between the two places?

21. My crop of wheat this year is $16\frac{2}{3}\%$ less than it was last year. Find my last year's crop if I raised 2760 bu. during the two years.

22. What sum must be invested in 6% bonds at $112\frac{1}{2}$, to realize an annual income of \$960?

23. A certain principal, at a certain rate per cent, amounts to \$1131 in 5 yr., and \$1339.80, at the same rate, in 9 yr. Find the principal and rate.

24. Find the value of a car load of wheat weighing 12,015 lb., at 68¢ a bushel.

25. If 8 men can do a piece of work in 10 days, how many men can do $\frac{5}{8}$ of the work in 5 days?

26. At 21¢ a square yard, how much will it cost to plaster a schoolroom 40 ft. long, 22 ft. wide, and 9 ft. high?

27. What will be the cost of slating a roof 42 ft. long, the length of the rafters on each side being 30 ft., at \$8 per square?

28. A brick wall contains 2112 cu. ft. If it is 18 in. thick and 16 ft. high, how long is it?

29. What part of 10 A. 140 sq. rd. is 7 A. 80 sq. rd.?

30. How many horses, at \$80 each, can be bought for \$14,241.60 after deducting a commission of $3\frac{1}{2}\%$ for buying?

31. A, B, and C enter into partnership to deal in clover seed. How many bushels can they purchase at \$2.10 a bushel, if A puts into the business \$5600, B \$1200 more than $\frac{1}{2}$ as much as A, and C \$200 less than $\frac{1}{3}$ as much as A and B together?

32. How many rolls of paper, each 18 yd. long and 18 in. wide, will be required to paper the sides and ceiling of a room 30 ft. long, 24 ft. wide, and 12 ft. high?

33. How many board feet are there in 24 pieces of timber, each 12 ft. long, and 13 in. square at the ends?

34. How much will it cost to line the sides and bottom of a tank 6 ft. long, 4 ft. wide, and 3 ft. deep, with zinc worth 24¢ per pound, allowing $3\frac{1}{2}$ lb. to cover a square yard?

35. I am offered a building lot for $1200 cash, or for $1375, payable in $1\frac{3}{8}$ yr. without interest. Which is the better offer, money being worth 6%?

36. How much must a plow, costing $7.20, be marked so that 10% may be deducted from the marked price, and a profit of 20% be made?

37. Three brothers, A, B, and C, engaged in business. A furnished $\frac{1}{4}$ of the capital, B 35%, and C the remainder. A's capital was in the business 7 mo., B's 6 mo., and C's 5 mo. They gained $561.60. Find each partner's share of the gain.

38. The premium paid for insuring goods for $\frac{2}{3}$ of their value, at $1\frac{3}{4}$%, was $1365. Find the value of the goods.

39. A bin is 6 ft. square. If the bottom is covered with grain to the depth of 15 in., how many bushels does the bin contain?

40. Find the face of a note which, when discounted at bank for 9 mo. 27 da. at 8%, will yield a proceeds of $3582.

41. Mr. Day's asking price is 30% above cost. If he sells for 30% of his asking price less, does he gain or lose and how much?

42. A merchant had a 90-day note discounted at a bank at 6%, the proceeds being $600. At maturity he took up the note by paying $100 cash, and giving a new 30-day note, discounted at 6%, for the balance. For what sum was the second note drawn?

43. How many board feet are there in the studding of the outside walls of a house 20 ft. by 40 ft., if the studs used are two-by-fours, 18 ft. long, placed 16 in. apart, and doubled at the corners?

44. In a school there are 175 male and 350 female pupils. What per cent of the whole number is each?

45. A merchant commenced business with a capital of \$3000. The first year he gained $33\frac{1}{3}\%$, which he invested in his business. The second year he gained 25%, which he also invested in his business. The third year he lost 15%. What was his average annual gain?

46. If 42 qt. of water are mixed with 84 qt. of milk, what per cent of the mixture is each?

47. A train runs 20 miles in 25 minutes, and another runs 12 miles in 15 min. Find the ratio of the speed of the second to that of the first.

48. It requires 12 yd. of material $1\frac{1}{4}$ yd. wide to make a suit of clothes. How many yards must be bought if the material shrinks in sponging 5% in length and 4% in width?

49. A note for \$900, dated July 1, 1895, at 6%, is indorsed as follows: Sept. 15, \$25; Jan. 1, 1896, \$60; Aug. 9, 1897, \$100. Find the amount due April 1, 1898.

50. A note for \$1200, dated Jan. 1, 1897, at 6%, bears the following indorsements: March 1, \$150; April 1, \$160; July 3, \$250; Oct. 9, \$125. Find the amount due Jan 1, 1898.

51.

\$$580\frac{25}{100}$. CHICAGO, Ill., April 24, 1898.

Ninety days after date, for value received, I promise to pay Smith and Weil, or order, at the Wyoming National Bank, Five Hundred Eighty and $\frac{25}{100}$ Dollars, with interest at 6%.

WILLIS SIMONS.

Discounted May 1, 1898, at 6%. No grace. Find the proceeds.

52. A wheel makes 300 revolutions in $4\frac{2}{7}$ seconds; another makes 150 revolutions in $1\frac{2}{3}$ seconds. Find the ratio of the speed of the first wheel to that of the second.

53. What per cent is gained on goods bought for 20% less than their value and sold for 20% more than their value?

54. How many feet, board measure, in 10 pieces of timber 12 in. by 16 in., and 20 ft. long?

55. What premium can I afford to pay for stock yielding $4\frac{1}{2}\%$ semiannually, in order to realize 8% on the investment?

56. If 124 men, in 5 da. of 10 hr. each, excavate a street 200 yd. long, 15 yd. wide, and 2 ft. deep, in how many days of 8 hr. each will 12 men excavate a street 320 yd. long, 20 yd. wide, and 18 in. deep, if the digging is twice as hard?

57. The School Board of Pittston wishes to raise $15,000 for building purposes. Allowing 3% of the amount levied as not collectible, and 5% as the collector's commission, what sum must be levied?

INVOLUTION

611. *Involution* treats of the formation of powers of numbers.

612. A *Power* of a number is the product arising from using the number a certain number of times as a factor. The number itself is the *First Power*, or the *Root* of the other powers.

613. The *Second Power* or *Square* of a number is the product of the number taken twice as a factor. Thus, $4 \times 4 = 16$ = the second power, or square of 4.

614. The *Third Power* or *Cube* is the product of the number taken three times as a factor. Thus, $4 \times 4 \times 4 = 64$ = the third power, or cube of 4.

615. The *Fourth Power* is the product of the number taken four times as a factor. Thus, $4 \times 4 \times 4 \times 4 = 256$ = the fourth power of 4.

616. The required power of a number is indicated by a small figure called an *Exponent.* It is placed to the right of the number and a little above it. Thus, 4^2 = the second power, or the square of 4; 4^3 = the third power, or cube of 4; 4^4 = the fourth power of 4.

PROBLEMS

617. 1. What is the fourth power of 9? The third power of 13?

2. What is the fifth power of 7? The fifth power of 8?

3. What is the value of $4^2 \times 5^2$? Of $4^3 \times 8^2$? Of $10^3 \times 12^2$?

4. What is the value of $(8\frac{2}{3})^2$? Of $(2\frac{1}{3})^4$? Of $(16\frac{2}{3})^2$?

5. What is the value of $(6.2)^2 \div (4.5)^2$? Of $(\frac{1}{12})^2 \times 4^3$?

6. What is the value of $(2\frac{1}{4})^3 + (5\frac{1}{2})^2$? Of the third power of $\frac{4\frac{1}{5}}{9}$?

7. What is the value of $.006^3$? Of $(30.06\frac{1}{2})^2$?

8. If 7^3 be multiplied by 7^4, what power of 7 shall we have?

9. What power of 9 equals $9^2 \times 9^4 \times 9^3 \times 9^5$?

10. What power of $\frac{3}{4}$ is equal to $(\frac{3}{4})^2 \times (\frac{3}{4})^3 \times (\frac{3}{4})^4 \times (\frac{3}{4})^5$?

11. What power of a number is the square of the square of a number?

12. What power of a number is the square of the cube of a number?

13. What power of a number is the cube of the square of a number?

14. What is the square of $\frac{1}{2}$ of 8?

15. What is $\frac{1}{2}$ of the square of 8?

16. The square of $\frac{1}{2}$ of a number is what part of $\frac{1}{2}$ of the square of the same number?

17. The cube of $\frac{1}{3}$ of a number is what part of $\frac{1}{3}$ of the cube of the same number?

18. What number multiplied by 5 equals the square of that number?

EVOLUTION

618. *Evolution* treats of finding any root of a number.

619. A *Root* of a number is the number itself or one of its equal factors.

620. The *Square Root* of a number is one of its two equal factors. Thus the square root of 16 is 4, for $4 \times 4 = 16$.

621. The *Cube Root* of a number is one of its three equal factors. Thus, the cube root of 27 is 3, for $3 \times 3 \times 3 = 27$.

622. The *Fourth Root* of a number is one of its four equal factors; the *Fifth Root* one of its five equal factors, etc.

623. The *Radical Sign* ($\sqrt{}$) is the symbol of evolution. Thus, $\sqrt{25}$ indicates the square root of 25; $\sqrt[3]{27}$ indicates the cube root of 27; $\sqrt[4]{256}$ indicates the fourth root of 256.

624. The small figure placed in the angle of the sign is the index of the root. It indicates what root is to be found. When the sign is used without any figure added, the square root is indicated. Evolution may be indicated also by a common fraction. Thus, $9^{\frac{1}{2}}$ denotes the square root of 9; $27^{\frac{1}{3}}$ denotes the cube root of 27; $4^{\frac{3}{2}}$ denotes the square root of the third power of 4; $6^{\frac{2}{3}}$ denotes the cube root of the second power of 6.

SQUARE ROOT

625. A *Perfect Square* is the product of two equal factors. A perfect square, therefore, is a number whose square root can be found exactly.

626. Following are examples of perfect squares:

Equal Factors	Perfect Squares	Equal Factors	Perfect Squares
1×1 =	1	10×10 =	100
2×2 =	4	99×99 =	9801
3×3 =	9	100×100 =	10000
4×4 =	16	999×999 =	998001
9×9 =	81	1000×1000 =	1000000

627. From the preceding examples, it may be seen that the square of a number consisting of *one* figure is composed of one or two figures; the square of a number consisting of *two* figures is composed of three or four figures; the square of a number consisting of *three* figures is composed of five or six figures, etc. Likewise, it may be shown that the square of any number is composed of twice as many figures as the number, or twice as many *minus* one. Therefore, the square root of a number composed of *one* or *two* figures consists of *one* figure; the square root of a number composed of three or four figures consists of two figures; the square root of a number composed of five or six figures consists of three figures, etc.

It will be seen, therefore, that for every two figures in the square there will be one figure in the root.

Hence to ascertain the number of figures in the square root of a number, we begin at units and separate the number into periods of two figures each, and the number of periods will equal the number of figures in the root. Thus, the square root of 2025 will consist of two figures (20′25); and the square root of 63,846 will consist of three figures (6′38′46).

628. 1. Find the square of 45.

45 equals 40 + 5, or 4 tens and 5 units

$$
\begin{array}{lrr}
 & 40 \;\; + 5 = & 45 \\
 & 40 \;\; + 5 = & 45 \\
\hline
(t \times u) + u^2 = & 5 \times 40 \;\; + 5^2 = & 225 \\
t^2 + (t \times u) \qquad = 40^2 + & 5 \times 40 \qquad = & 1800 \\
\hline
t^2 + 2(t \times u) + u^2 = 40^2 + & 2\,(5 \times 40) + 5^2 = & 2025
\end{array}
$$

Commencing at units, 5 times 5 = 5^2 = the square of the units. 5 times 40 = 5×40 = the product of the tens and units. 40 times 5 = 5×40 = the product of the tens and units. 40 times 40 = 40^2 = the square of the tens. Adding, we have $40^2 + 2\,(5 \times 40) + 5^2$. Hence it will be seen that the square of 45, or of any number consisting of tens and units, equals the square of the tens, plus twice the product of tens and units, plus the square of the units.

629. 1. Find the square root of 2025.

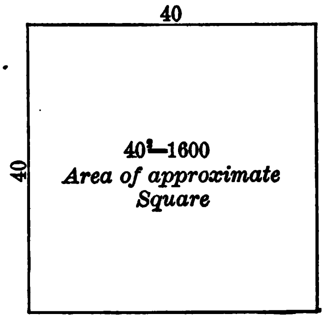
ANALYTIC PROCESS

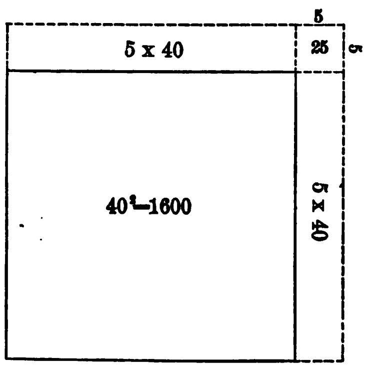
$$
\begin{array}{rr|r|l}
 & & 20'25 & 40 = 4 \text{ tens} \\
\text{Square of the tens} = 40^2 = & & 1600 & \;\;5 = 5 \text{ units} \\
\hline
\text{Trial divisor} = \text{twice the tens} = 2 \times 40 = & 80 & 425 & 45 = \textit{Ans.} \\
\text{Twice the product of the tens and units} & & & \\
= 2\,(5 \times 40) = & 400 & & \\
\text{Square of the units} = 5^2 = & 25 & 425 & \\
\end{array}
$$

We find by Art. 627 that the root must consist of two figures, tens and units. The greatest number of tens whose square is contained in 2025 is 4 tens, or 40, whose square is 1600. This, subtracted from 2025, leaves 425, which, according to Art. 628, must equal twice the product of the tens and units, plus the square of the units. To find the units we must divide 425 by twice the tens, or 80. Dividing 425 by 80, we find the units to be 5. Then two times the product of the tens and units $= 2\,(5 \times 40) = 400$. Adding to this the square of the units, or 25, we have 425, which, subtracted, leaves no remainder.

Illustration by Areas

If 2025 is a perfect square, it can be illustrated by the area of a square the side of which is the square root of 2025.





The greatest number of tens whose square is contained in 2025 is 4 tens, or 40. If we let Fig. 1 represent an approximate square whose sides are 40 units, its area will exhaust 40×40, 1600 square units of the area. Subtracting 1600 from 2025, we have 425 square units of surface to be added to two sides of the approximate square, 1600, to make it the required square, 2025. This requires two rectangular pieces each 40 units long, and a square corner piece with sides equal to the width of the rectangles. The entire length of the two rectangular side pieces equals $40 + 40$, or 80 units. Dividing 425, which is the approximate area of the two side pieces, by 80, we get their width, which is 5. The areas of the two rectangular sides, and of the corner piece, are

$$\begin{aligned} 5 \times 40 &= 200 = \text{one side piece} \\ 5 \times 40 &= 200 = \text{the other side piece} \\ 5 \times \ \ 5 &= \underline{\ \ 25} = \text{the corner piece} \\ & \quad\ 425, \text{ total.} \end{aligned}$$

This exhausts the area, and the side of the required square is $40 + 5$, or 45. Therefore the square root of 2025 is 45.

Note. — In practice we shorten the process as in the following examples.

2. Find the square root of 104,976.

OPERATION

$$\begin{array}{r|l} & 10'49'76(324 \\ 3^2 = & 9 \\ \hline 62) & 149 \\ & 124 \\ \hline 644) & 2576 \\ & 2576 \\ \hline \end{array}$$

Pointing off into periods of two figures each, we find there are three periods, and therefore three figures in the root. (Art. 627.)

The greatest number whose square is contained in the left-hand period is 3. We place the 3 in the root, and subtract its square from 10, and bring down the next period, 49, making the dividend 149. We double the first figure of the root, 3, making 6 for a trial divisor, which we write at the left of 149. Six is contained in 14 *twice*. We write the 2 in the root and in the divisor also, making the divisor 62. We multiply 62 by the second figure of the root and obtain the product 124. Subtracting this from 149 and bringing down the next period, we have 2576 for a dividend. We next double the root already found, 32, thus making 64, which we write at the left of 2576, for a trial divisor. It will be seen that 64 is contained in 257 *four* times. We write the 4 in the root, and also in the divisor, thus increasing the divisor to 644. We next multiply 644 by the last figure of the root, 4, and obtain the product 2576. As nothing remains, the square root of 104,976 equals 324.

3. Find the square root of 249,001.
4. Find the square root of 316,969.
5. Find the square root of 349,281.
6. Find the square root of 484,416.
7. Find the square root of 574,564.
8. Find the square root of 788,544.

630. The left-hand period may consist of but one figure. When the trial divisor is larger than the dividend, exclusive of the right-hand figure, the corresponding figure of the root will be a cipher. When this is the case we annex a cipher to the trial divisor and bring down the next period.

1. Find the square root of 42,025.

It will be seen that the left-hand period contains but one figure. The greatest number whose square is contained in 4 is 2. We write the 2 in the root and its square under the 4 and bring down the next period, 20. We next double the first figure of the root, 2, making 4 for a trial divisor, which is written at the left of 20. Since the trial divisor is larger than the dividend, exclusive of the right-hand figure, it is evident that the next figure of the root will be a cipher. We annex a cipher to both the root and the trial divisor, bring down the next period, thus making a new dividend, 2025, and then proceed as in Art. 629.

OPERATION

```
     4'20'25(205 Ans.
2²=4
   405)2025
       2025
```

631. To extract the square root of a decimal we point off the decimal *toward the right* into periods of two figures each, instead of toward the left, as in whole numbers, and extract the root as in the preceding examples.

Frequently the last period will contain but *one* figure. In this case we annex a cipher to complete the period, since adding a cipher to the right of a decimal does not alter its value.

1. Find the square root of .119025.

OPERATION

```
     .11'90'25(.345 Ans
3² = 9
     64)290
        256
    685)3425
        3425
```

632. When there is a remainder after all the periods have been brought down, we annex periods of ciphers and continue the operation to as many decimal places as may be desired.

1. Find the square root of .59049.

OPERATION

.59′04′9(.76843+ *Ans.*
$7^2 = 49$
146)1004
876
1528)12890
12224
15364)66600
61456
153683)514400
461049
53351 Rem.

633. To extract the square root of a whole number and a decimal, we point off the whole number toward the left and the decimal toward the right.

1. Find the square root of 522.064.

OPERATION

5′22.06′4)22.848+ *Ans.*
$2^2 = 4$
42)122
84
448)3806
3584
4564)22240
18256
45688)398400
365504
32896 Rem.

634. To extract the square root of a common fraction, when both terms are perfect squares, we extract the square root of the numerator and denominator separately. Thus, $\sqrt{\frac{9}{16}} = \frac{3}{4}$, for $\sqrt{9} = 3$ and $\sqrt{16} = 4$.

635. When both terms of a common fraction are not perfect squares, we reduce the fraction to a decimal and extract the root of the decimal. Thus, $\sqrt{\frac{7}{8}} = \sqrt{.875} = .935+$. Sometimes the fraction may be reduced so that both terms become perfect squares. Thus, $\sqrt{\frac{12}{27}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$; $\sqrt{\frac{18}{32}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$; $\sqrt{21\frac{7}{9}} = \sqrt{\frac{196}{9}} = \frac{14}{3} = 4\frac{2}{3}$.

636. To extract the square root of a mixed number it is usually best to reduce the fraction to a decimal. Thus,

$$\sqrt{7\tfrac{2}{3}} = \sqrt{7.6666+} = 2.76+.$$

637. When the denominator is a perfect square, we may extract the square root of the numerator and denominator separately. Thus, $\sqrt{\frac{11}{64}} = \frac{\sqrt{11}}{\sqrt{64}} = \frac{3.316+}{8} = .414+$.

638. Find the square root of:

1. 196	12. .1225	23. $\frac{147}{192} = \frac{49}{64}$
2. 361	13. .0064	24. $\frac{3}{7}$
3. 576	14. .00261	25. $7\frac{1}{2}$
4. 2116	15. .170569	26. $9\frac{4}{7}$
5. 6241	16. 53.29	27. $4\frac{11}{12}$
6. 46,656	17. 1162.25	28. $76\frac{9}{16} = \frac{1225}{16}$
7. 94,249	18. 7397.7201	29. $2\frac{1}{4} \times 6\frac{1}{2}$
8. 189,225	19. $\frac{64}{81}$	30. $\frac{576}{676} = \frac{144}{169}$
9. 527,076	20. $\frac{196}{225}$	31. $12.6\frac{7}{8}$
10. 649,636	21. $\frac{324}{361}$	32. $25.0\frac{3}{4}$
11. .0576	22. $\frac{441}{625}$	33. $.000\frac{4}{9}$

PROBLEMS

639. 1. If it requires 144 yd. of matting 1 yd. wide to cover the floor of a square room, what is the length of each side of the room in feet?

NOTE. — Since the area of a square is the product of two of its sides, the length of one side is obtained by extracting the square root of its area.

2. I own a square piece of land which contains just 40 A. What is the length of each side in rods?

3. What is the side of a square whose area is $272\frac{1}{4}$ sq. ft.?

4. One half the square of a certain number is 275,282. What is the number?

5. How much will it cost, at \$.80 a rod, to fence a square lot containing 245,025 sq. ft.?

6. How many yards long is each side of a square reservoir which contains 10 acres?

7. How large a square table can be made from two boards, each 12 ft. long, and 18 in. wide?

8. How much less will it cost to inclose a square piece of land containing 40 A., than one containing the same area in the form of a rectangle 90 rd. long, the price per rod being \$.81 in each case?

9. A rectangular piece of land is 4 times as long as it is wide, and contains just 10 acres. Find its dimensions.

10. A gentleman has a garden which contains 3600 sq. ft. Its length is to its width as 9 to 4. Find its dimensions.

CUBE ROOT

640. A *Perfect Cube* is the product of three equal factors. It is, therefore, a number whose cube root can be found exactly. The following shows how perfect cubes are produced.

641.

Equal Factors	Perf. Cubes	Equal Factors	Perf. Cubes
$1 \times 1 \times 1 =$	1	$10 \times 10 \times 10 =$	1000
$2 \times 2 \times 2 =$	8	$46 \times 46 \times 46 =$	97,336
$3 \times 3 \times 3 =$	27	$99 \times 99 \times 99 =$	970,299
$4 \times 4 \times 4 =$	64	$100 \times 100 \times 100 =$	1,000,000
$5 \times 5 \times 5 =$	125	$444 \times 444 \times 444 =$	87,528,384
$9 \times 9 \times 9 =$	729	$999 \times 999 \times 999 =$	997,002,999

$$1000 \times 1000 \times 1000 = 1{,}000{,}000{,}000$$

642. From the preceding examples of perfect cubes it may be seen that the cube of a number consisting of *one* figure is composed of *one, two,* or *three* figures; the cube of a number consisting of *two* figures is composed of *four, five,* or *six* figures; the cube of a number consisting of *three* figures is composed of *seven, eight,* or *nine* figures; the cube of a number consisting of *four* figures is composed of *ten, eleven,* or *twelve* figures, etc. Likewise it may be shown that the cube of any number is composed of three times as many figures as the number itself, or three times as many, minus *one* or minus *two.*

Therefore the cube root of a number composed of *one, two,* or *three* figures consists of but *one* figure; the cube root of a number composed of *four, five,* or *six* figures consists of *two* figures; the cube root of a number composed of *seven, eight,* or *nine* figures consists of *three* figures; the cube root of a number composed of *ten, eleven,* or *twelve* figures consists of *four* figures, etc.

Therefore for every *three* figures in the cube there will be *one* figure in the root.

Hence, to ascertain the number of figures in the cube root of a number, we separate the number into periods of three figures each, beginning at units and pointing off toward the left.

The number of periods will denote the number of figures in the root.

643. 1. Find the cube of 45.

OPERATION I

$$45 = 40 + 5, \text{ or } 4 \text{ tens and } 5 \text{ units}$$

$$\begin{array}{rl}
45^2 = (40+5)^2 = 40^2 + 2\,(40 \times 5) + 5^2 = & 2025 \text{ (Art. 628)} \\
40 + 5 = & 45 \\
\hline
(40^2 \times 5) + 2\,(40 \times 5^2) + 5^3 = & 10125 \\
40^3 + 2\,(40^2 \times 5) + \ (40 \times 5^2) \qquad = & 81000 \\
\hline
40^3 + 3\,(40^2 \times 5) + 3(40 \times 5^2) + 5^3 = & 91125
\end{array}$$

OPERATION II

$$\begin{array}{rr}
40 + 5 = & 45 \\
40 + 5 = & 45 \\
\hline
200 + 25 = & 225 \\
1600 + 200 \qquad = & 1800 \\
\hline
1600 + 400 + 25 = & 2025 \\
40 + 5 = & 45 \\
\hline
8000 + 2000 + 125 = & 10125 \\
64000 + 16000 + 1000 \qquad = & 81000 \\
\hline
64000 + 24000 + 3000 + 125 = & 91125
\end{array}$$

A careful inspection of the two preceding operations will show us that the cube of a number consisting of tens and units embraces the following four parts:

1. The cube of the tens, $40^3 = 64,000$.

2. Three times the product of the square of the tens by the units, $3(40^2 \times 5) = 24,000$.

3. Three times the product of the tens by the square of the units, $3(40 \times 5^2) = 3000$.

4. The cube of the units, $5^3 = 125$.

644. 1. Find the cube root of 91,125.

ANALYTIC PROCESS

			91'125	40 = 4 tens
Cube of the tens	$= 40^3 =$		64000	5 = 5 units
Trial divisor = 3 times the square of the tens	$= 3 \times 40^2 =$	4800	27125	45 *Ans.*
3 times the square of the tens times the units	$= 3(40^2 \times 5) =$	24000		
3 times the tens times the square of the units	$= 3(40 \times 5^2) =$	3000		
Cube of the units	$= 5^3 =$	125	27125	

According to Art. 642, the number, 91,125, has a cube root of two figures, viz. *tens and units.* This number, 91,125, according to Art. 643, must equal the cube of the tens plus 3 times the square of the tens multiplied by the units, plus 3 times the tens multiplied by the square of the units, plus the cube of the units.

The greatest number of tens whose cube is contained in 91,125 is 4 tens, or 40, whose cube is 64,000. This subtracted from 91,125 leaves 27,125, which, according to Art. 643, must equal 3 times the square of the tens multiplied by the units, plus 3 times the tens multiplied by the square of the units, plus the cube of the units.

We now have the *tens* of the root. We must next find the units. As the remainder, 27,125, must contain the products just mentioned, of each of which the units form one factor, we may find the other factor by dividing by the known factor, viz. *three times the square of the tens*, or 4800. This is called the trial divisor. By way of trial, we divide 27,125 by 4800, and obtain the quotient, 5, which we write as the trial units' figure of the root.

Completing the required products we find they amount to 27,125. Hence 5 is the units' figure of the root. If the products, when completed, amount to more than the dividend, a smaller quotient figure must be taken. The same method of explanation will apply when there are more than two figures in the root.

In practice, we usually omit the ciphers, and delay multiplying by the units until we have found the sum of the other factors. We then multiply them all at once, as in the following operation.

OPERATION

$$
\begin{array}{r|l}
 & 91'125(45 \\
4^3 = & 64 \\
\hline
\text{Trial divisor} = 3 \times 40^2 = 4800 & 27125 \\
3 \times 40 \times 5 = 600 & \\
5^2 = 25 & \\
\hline
\text{Complete divisor} = 5425 & 27125 \\
\hline
\end{array}
$$

ILLUSTRATION BY SOLIDS

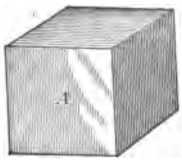


FIG. 1

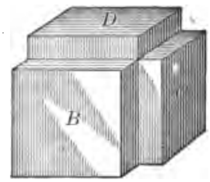


FIG. 2

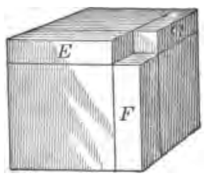


FIG. 3

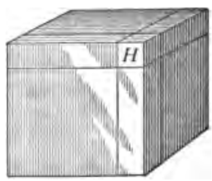


FIG. 4

If 91,125 is a perfect cube, it can be illustrated by a cubical block the length of whose edge is the cube root of 91,125.

The greatest number of tens whose cube is contained in 91,125 is 4 tens, or 40. Now, if we let A, Fig. 1, represent an approximate cube, the length of whose edge is 40 units, it will exhaust $40 \times 40 \times 40$, or 64,000 cubic units of the volume. Subtracting 64,000 from 91,125, we have left 27,125 cubic units to be added to the sides of the approximate cube, 64,000, to make it a cube equal to 91,125 cubic units.

To do this we require:

1. Three side-slabs, each 40 units square (B, C, D, Fig. 2).

2. Three corner pieces (E, F, G, Fig. 3), each 40 units long, with a width and thickness equal to the thickness of the side-slabs.

3. A corner cube (H, Fig. 4) to fill the vacancy left by the corner pieces (E, F, G, Fig. 3).

Since the remainder, 27,125, consists chiefly of the three square side-slabs, we may find, nearly, their thickness by dividing 27,125 by the area of one side of the three slab-additions, which equals $40 \times 40 \times 3$, or 4800. Dividing 27,125 by 4800, we get a quotient of 5, which we find to be the exact thickness. Therefore the volume of the several additions to the approximate cube is as follows:

Three side-slabs, $B + C + D$, Fig. 2	$= 40 \times 40 \times 5 \times 3 =$	24,000
Three corner pieces, $E + F + G$, Fig. 3	$= 40 \times 5 \times 5 \times 3 =$	3000
The corner cube H, Fig. 4	$= 5 \times 5 \times 5 =$	125
	Total $=$	27,125

This exhausts the volume, and the edge of the required cube is $40 + 5$, or 45 units. Therefore, the cube root of 91,125 is 45.

2. $\sqrt[3]{150,568,768} = ?$

OPERATION

			150'568'768	532
	$5^3 =$		125	
Partial, or trial divisor . .	$50^2 \times 3 =$	7500	25568	
	$50 \times 3 \times 3 =$	450		
	$3^2 =$	9		
First complete divisor		7959	23877	
Partial, or trial divisor . .	$530^2 \times 3 =$	842700	1691768	
	$530 \times 2 \times 3 =$	3180		
	$2^2 =$	4		
Second complete divisor		845884	1691768	

We first separate the number into periods of three figures each, beginning at units and counting to the left. The greatest number whose cube is contained in 150 is 5. We write the 5 in the root, and subtract its cube, 125, from the first period 150, which leaves a difference of 25, to which we annex the next period, thus forming the dividend 25,568.

Annexing a cipher to the first figure of the root, squaring it, and multiplying by 3, we obtain 7500 as a partial or trial divisor ($50^2 \times 3 = 7500$). Placing 7500 to the left of 25,568 and dividing, we get the quotient 3,

which forms the second figure of the root. In order to complete the divisor, we annex a cipher to the first figure of the root, multiply it by the second figure of the root, and the product by 3, and obtain 450($50 \times 3 \times 3 = 450$) as the second part of the first complete divisor. We next square 3, the second figure of the root, write it under 450, and add these partial divisors together, which gives 7959 for the first complete divisor. Multiplying the complete divisor by the second figure of the root, 3, and subtracting the product, 23,877, from 25,568, we have a difference of 1691, to which we annex the next period, and obtain 1,691,768 for a dividend.

We now annex a cipher to the root already found, 53, square it, multiply by 3, and obtain 842,700 as a partial, or trial divisor ($530^2 \times 3 = 842{,}700$). Placing the trial divisor to the left of 1,691,768 and dividing, we get the quotient 2, which forms the third figure of the root. Annexing a cipher to the first two figures of the root, multiplying it by 2, the third figure of the root, and this product by 3, we obtain 3180($530 \times 2 \times 3 = 3180$), which we write under 842,700, as the second part of the second complete divisor. We next square 2, the third figure of the root, write it under 3180, and add these partial divisors together, and get for the second complete divisor 845,884, which we multiply by 2, the last figure of the root; we subtract the product from 1,691,768. As there is no remainder, the cube root of 150,568,768 is 532.

645. The left-hand period may contain only one or two figures.

1.

$$
\begin{array}{rr|l}
 & 5'832 & \underline{18} \\
1^3 = & 1 & \\
10^2 \times 3 = & 300 & 4832 \\
10 \times 8 \times 3 = & 240 & \\
8^2 = & \underline{64} & \\
 & 604 & \underline{4832}
\end{array}
$$

2.

$$
\begin{array}{rr|l}
 & 97'336 & \underline{46} \\
4^3 = & 64 & \\
40^2 \times 3 = & 4800 & 33336 \\
40 \times 6 \times 3 = & 720 & \\
6^2 = & \underline{36} & \\
 & 5556 & \underline{33336}
\end{array}
$$

646. When the dividend will not contain the trial divisor, we write a cipher in the root, annex two ciphers to the trial divisor, bring down the next period, and proceed to complete the divisor as before.

647. If the number is not a perfect cube, there will be a remainder after all the periods have been brought down. When this is the case we annex periods of ciphers, and continue the operation as far as desirable. For every period of three ciphers annexed there will be a decimal figure in the root.

648. To find the cube root of a common fraction, we extract the cube root of both numerator and denominator separately, when they are perfect cubes, but when they are not, it is usually best to reduce the fraction to a decimal and extract its root.

649. To find the cube root of a decimal, we separate the decimals into periods of three figures each, beginning at the decimal point and counting to the right.

650. Find the cube root of the following:

1. 2744	6. 223,648,543	11. $26\frac{1}{2}$
2. 15,625	7. 350,402,625	12. .216
3. 175,616	8. 259.694072	13. .000343
4. 1,860,867	9. 12.812904	14. $\frac{27}{343}$
5. 37,933,056	10. 16503.467336	15. $\frac{512}{2197}$

PROBLEMS

651. 1. Find the cube root of .1 to four decimal places.

2. Find the cube root of 6 to five decimal places.

3. Find the value of $\sqrt[3]{\frac{5}{8}}$; $\sqrt[3]{18\frac{26}{27}}$; $\sqrt[3]{4699\frac{27}{64}}$; $\sqrt[3]{\frac{8}{9}}$; $\sqrt[3]{\frac{4}{27}}$.

4. Find the length of the edge of a cube whose volume is 13,724 cu. in.

5. Find the number of square feet in one face of a cube containing $91\frac{1}{8}$ cu. ft.

6. Find the depth of a cubical bin that contains just 80 bu.

7. Find the number of square feet of zinc that will be required to line a cubical tank whose contents are $421.87\frac{1}{2}$ cu. ft.

MENSURATION

LINES AND ANGLES

652. A *Line* is that which has length only.

653. A *Straight Line* is one that does not change its direction.

654. A *Curved Line* is one that continually changes its direction. 

655. *Parallel Lines* are those that have the same direction. 

656. When two straight lines are drawn from the same point, the opening between these two lines is called an *Angle*. 

657. When one straight line meets another straight line so as to form two equal angles, the lines are said to be *Perpendicular* to each other, and the angles thus formed are called *Right Angles*. 

658. When the angle is less than a right angle it is called an *Acute Angle*. 

659. When the angle is greater than a right angle it is called an *Obtuse Angle*. 

SURFACES

660. A *Surface* is that which has the dimensions length and breadth.

661. A *Plane Surface* is a level surface.

662. A *Plane Figure* is a plane surface bounded by straight or curved lines.

663. A *Polygon* is a plane surface bounded by three or more straight lines.

664. A polygon of three sides is called a *Triangle;* of four sides, a *Quadrilateral;* of five sides, a *Pentagon;* of six sides, a *Hexagon;* of seven sides, a *Heptagon;* of eight sides, an *Octagon*, etc.

TRIANGLES

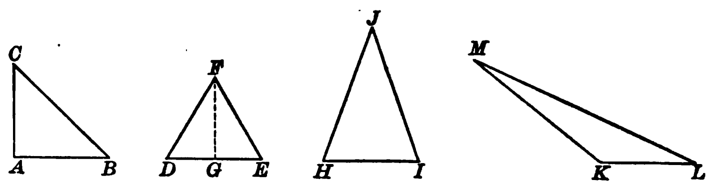


FIG. 1 FIG. 2 FIG. 3 FIG. 4

665. A *Triangle* is a plane surface bounded by three straight lines.

666. A *Right-angled Triangle*, or Right Triangle, has one right angle (Fig. 1).

667. An *Equilateral Triangle* has three equal sides (Fig. 2).

668. An *Isosceles Triangle* has two equal sides (Fig. 3).

669. A *Scalene Triangle* has three unequal sides (Fig. 4).

670. The *Base* of a triangle is the side upon which it seems to stand; as AB (Fig. 1).

671. The *Vertex* is the point opposite the base; as F (Fig. 2).

672. The *Altitude* is the perpendicular distance from the vertex to the base; as FG (Fig. 2).

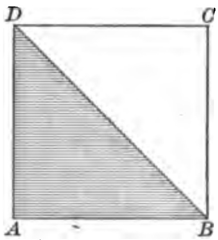


FIG. 5

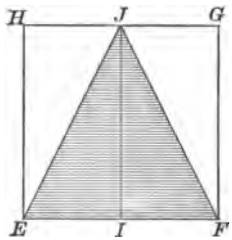


FIG. 6

673. A little inspection of Figs. 5 and 6 will show us that the triangle ABD is just one half of the rectangle $ABCD$, and the triangle EFJ is just one half of the rectangle $EFGH$.

The area of a triangle equals one half of a rectangle having the same base and altitude. Hence,

To find the area of a triangle, take one half of the product of its base and altitude. (See Art. 341.)

1. What is the area of a triangle whose base is 24 ft. and altitude 14 ft.?

$$(24 \times 14) \div 2 = 168 \text{ sq. ft. } Ans.$$

674. Either the base or altitude of a triangle is found by dividing the area by the given dimension, and multiplying the quotient by 2. (See Art. 342.)

1. The area of a triangle is 96 sq. ft., and the base is 16 ft. What is the altitude?

$$(96 \div 16) \times 2 = 12 \text{ ft., altitude}$$

2. The area of a triangle is 96 sq. ft., and its altitude 12 ft. Find the base.

675. *To find the area of a triangle when its three sides are given, from half the sum of the three sides subtract each side separately. Find the product of the half sum and the three remainders. The square root of the product will be the area of the triangle.*

1. Find the area of a triangle whose sides are 6, 8, and 10 ft.

OPERATION

$(6 + 8 + 10) \div 2 = 12$, half the sum of the three sides

$$\left.\begin{array}{r} 12 - 6 = 6 \\ 12 - 8 = 4 \\ 12 - 10 = 2 \end{array}\right\} = \text{each side subtracted separately}$$

$\sqrt{12 \times 6 \times 4 \times 2} = 24$ sq. ft., area

PROBLEMS

676. 1. Find the area of a triangle whose base is 36 rd. and altitude 28 rd.

2. Find the area of a triangle whose base is 80 yd. and altitude 63 ft.

3. Find the area of a triangle whose base is 24.5 chains and altitude 16.8 chains.

4. The area of a triangular piece of land is 15 acres. The altitude is 60 rd. Find the base.

5. A triangular piece of land contains 1980 sq. yd. The base is 72 yd. Find the altitude.

6. Find the area of a triangle whose sides are respectively 15, 20, and 25 ft.

7. Find the value, at $85 per acre, of a triangular piece of land, the sides being respectively 30, 40, and 50 chains.

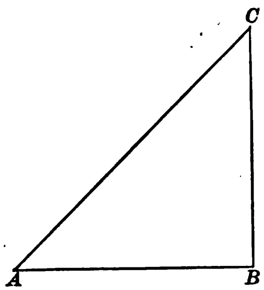
8. Find the area of an isosceles triangle whose base is 50 rd., and each of the equal sides 40 rd.

9. How much will it cost, at 5¢ a square foot, to paint the two gables of a house, if the width of the house is 32 ft., and the length of the rafters on each side is 20 ft.?

10. Which is greater and how much, a triangle whose sides are respectively 30, 40, and 50 ft., or one whose base is 60 ft. and altitude 20 ft.?

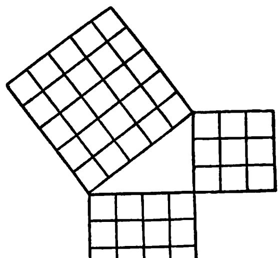
The Right-angled Triangle

677. In the right-angled triangle *ABC*, the side *AC*, or the side opposite the right angle, is called the *Hypotenuse*, the side *AB* the *Base*, and the side *BC* the *Perpendicular*.



678. It is an established principle of Geometry *that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the base and the perpendicular.*

679. The diagram at the right illustrates the preceding principle. It will readily be seen that the number of small squares in the square of the hypotenuse equals the sum of the small squares in the base and the perpendicular. Hence,



The hypotenuse equals the square root of the sum of the squares of the base and perpendicular; the base equals the square root of the difference of the squares of the hypotenuse and the perpendicular; and the perpendicular equals the square root of the difference of the squares of the hypotenuse and the base.

PROBLEMS

680. 1. The base of a right-angled triangle is 32, and the perpendicular 60. Find the hypotenuse.

STATEMENT

$$\sqrt{B^2+P^2} = \text{answer}$$

2. The perpendicular of a right-angled triangle is 60, and the hypotenuse 65. Find the base.

STATEMENT

$$\sqrt{H^2-P^2} = \text{answer}$$

3. Find perpendicular if hypotenuse is 85, base 51.

STATEMENT

$$\sqrt{H^2-B^2} = \text{answer}$$

4. Two men travel from the same place. One travels directly east at the rate of 16 mi. a day, and the other goes due north at the rate of 12 mi. a day. How far apart will they be at the end of 6 days?

5. The rafters of a house are 17 ft. long, and the height of the gable is 8 ft. What is the width of the house?

6. A rope 190 ft. long will reach from the top of a pole to a point on the opposite bank of a river whose width is 152 ft. What is the height of the pole?

7. How far from the bottom of a building 44 ft. high must the foot of a ladder 55 ft. long be placed so that the top of the ladder will just reach the top of the building?

8. I own a lot 200 ft. by 184 ft. What is the distance through it from opposite corners?

9. A flag pole 80 ft. high stands at the edge of a street. A rope 96 ft. long fastened at the top of the pole reaches to the middle of the street. Find the width of the street.

10. Find the distance from a lower corner to the opposite upper corner of a room 32 ft. long, 24 ft. wide, and 18 ft. high.

Quadrilaterals

681. A *Quadrilateral* is a polygon of *four* sides.

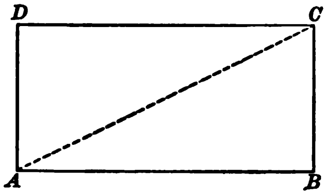


Fig. 1

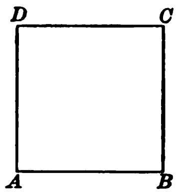


Fig. 2

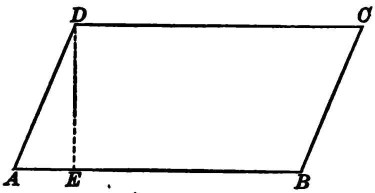


Fig. 3

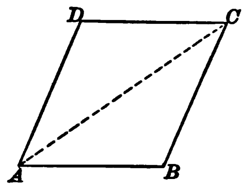


Fig. 4

682. A *Parallelogram* is a quadrilateral having its opposite sides parallel. (Figs. 1, 2, 3, 4.)

683. A *Rectangle* is a plane surface having four square corners. (Fig. 1.)

684. A *Square* is a rectangle whose four sides are of equal length. (Fig. 2.)

685. A *Rhomboid* is an oblique-angled parallelogram. (Fig. 3.)

686. A *Rhombus* is a rhomboid whose four sides are of equal length. (Fig. 4.)

687. The *Base* of a parallelogram is the side upon which it seems to stand; as *AB*, Fig. 1.

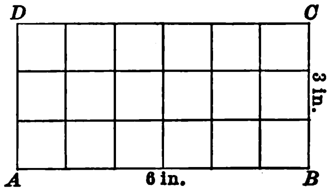
688. The *Altitude* of a parallelogram is the perpendicular distance between the base and the side opposite; as *DE*, Fig. 3.

689. The *Diagonal* of a quadrilateral is a straight line joining its opposite angles; as *AC*, Figs. 1 and 4.

690. The *Area* of a figure is the number of square units of surface of a certain value it contains.

The Rectangle

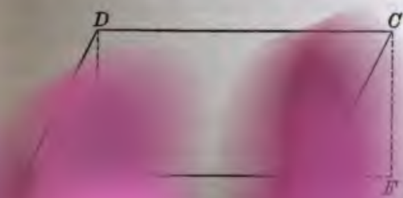
691. If we suppose the rectangle *ABCD* to be 6 in. long and 3 in. wide, the unit of measure will be the square inch. If the rectangle be divided by lines as represented in the figure, it will be found to contain 3 times 6, or 18, square inches. Hence,



The area of a rectangle equals the product of the two numbers representing its base and altitude. (Art. 330.)

The Rhomboid

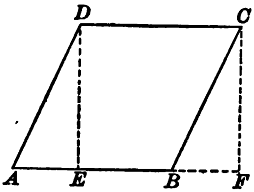
692. If the triangle *ADE* be taken from the rhomboid *ABCD*, and placed at the right of the figure, we shall have the rectangle *EFCD*. It is evident that the base and altitude of the rectangle are the same as the base and altitude of the rhomboid. Therefore their areas must be equal. Hence,



[illegible]rea of a [illegible] multiply the base by the altitude.

THE RHOMBUS

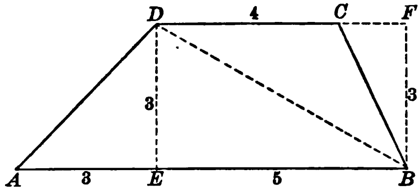
693. If the triangle ADE be taken from the rhombus $ABCD$, and placed at the right of the figure, we shall have the square $EFCD$. It is evident that the base and altitude of the square are the same as the base and altitude of the rhombus. Therefore their areas must be equal. Hence,



To find the area of a rhombus, multiply the base by the altitude.

THE TRAPEZOID

694. A *Trapezoid* is a quadrilateral having only two of its sides parallel. The parallel sides are AB and DC. The altitude is the perpendicular distance between the parallel sides; as DE. If the trapezoid is divided into two parts by the line BD, it will consist of the two triangles ABD and BCD. The base of the first triangle is AB, and the altitude DE. The base of the second triangle is CD, and the altitude BF. Since $DE = BF$, the altitude of both triangles must be the same. The area of the trapezoid must equal the sum of the areas of the two triangles.



According to Art. 673,

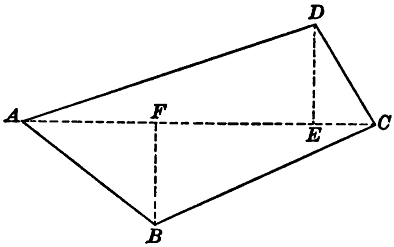
$$\begin{array}{lll} \text{The area of } ABD & = (AB \times DE) \div 2 = (8 \times 3) & \div 2 = 12 \\ \text{The area of } BCD & = (CD \times BF) \div 2 = (4 \times 3) & \div 2 = 6 \\ \hline \left.\begin{array}{l}\text{Adding, the area}\\ \text{of the trapezoid}\end{array}\right\} & = \overline{(AB + CD) \times DE} \div 2 = \overline{(8 + 4) \times 3} \div 2 = 18 & \textit{Ans.} \end{array}$$

From the preceding explanation we derive the following:

To find the area of a trapezoid, multiply the sum of the parallel sides by half the altitude.

The Trapezium

695. A *Trapezium* is a quadrilateral having none of its sides parallel.



If the trapezium is divided into two parts by the diagonal AC, it will consist of the two triangles ABC and ACD, the diagonal forming the base of both triangles. The altitude of the first triangle is BF and of the second DE. Now, it is evident that the area of the trapezium must equal the sum of the areas of the two triangles. Hence,

To find the area of a trapezium, divide the trapezium by a diagonal into two triangles, and find the sum of the areas of the two triangles.

PROBLEMS

696. 1. What is the area of a piece of land in the form of a parallelogram, whose length is 30 rd. and altitude 25 rd.?

2. The base of a rhombus is 30 yd. and altitude 70 ft. Find its area.

3. The base of a rhomboid is 30 ch. and the altitude 25 rd. What is its area?

4. Find the area of a trapezoid, one side of which is 12 ft., the other 8 ft., and the altitude 4 ft.

5. The diagonal of a trapezium is 60 ft., and the altitudes of the triangles into which the trapezium is divided are 40 ft. and 30 ft. respectively. Find the area of the trapezium.

6. How many acres are there in an irregular-shaped piece of land whose sides are 15, 20, 30, and 35 rd. respectively, and the diagonal 25 rd.? (See Art. 675.)

7. The diagonal of a square is 72 ft. Find its area.

NOTE. — The area of a square equals one half of the square of its diagonal.

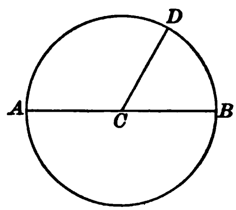
8. The diagonals of a rhombus are 60 ft. and 32 ft. Find the area.

9. The diagonal of a rectangle is 45 yd. What is its area, its length being to its width as 4 to 3?

THE CIRCLE

697. A *Circle* is a plane figure bounded by a curved line, called its *Circumference*, every part of which is equally distant from a point within called the center.

698. The *Diameter* of a circle is a straight line drawn from any point in the circumference, through the center, and terminating in the circumference opposite; as *AB*.



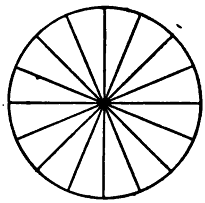
699. The *Radius* of a circle is a straight line drawn from the center to the circumference; as *CD*.

700. An *Arc* of a circle is any part of the circumference; as *AD* and *BD*.

701. If we measure the diameter and the circumference of a given circle, we shall find that the circumference equals about 3.1416 times the diameter. Hence,

To find the circumference of a circle when the diameter is given, multiply the diameter by 3.1416. To find the diameter, when the circumference is given, divide the circumference by 3.1416.

702. If we regard the circle as composed of a very large number of triangles, as shown in the figure, then the sum of the bases of these triangles will form the circumference of the circle, and their altitude the radius of the circle. Therefore, the rule for finding the area of a triangle may be applied in finding the area of a circle.



To find the area of a circle, multiply the circumference by one half of the radius.

Since $\text{Area} = \text{Circumference} \times \frac{1}{2}\text{ Radius}$

and $\text{Circumference} = 2 \times \text{Radius} \times 3.1416,$ (Art. 701)

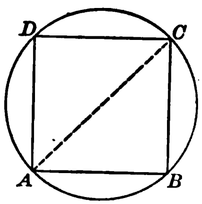
$\text{Area} = 2 \times \text{Radius} \times 3.1416 \times \frac{1}{2}\text{ Radius}$, or,

$\text{Area} = \text{Radius}^2 \times 3.1416.$ Hence,

To find the area when the circumference is not given, multiply the square of the radius by 3.1416.

703. To find the side of an inscribed square when the circumference is given.

From the accompanying figure the following facts are apparent:



1. The diameter of the circle AC forms the diagonal of the square $ABCD$.
2. The diagonal forms the hypotenuse of the right-angled triangle ABC.
3. The square of the diagonal equals the sum of the squares of the sides AB and BC.
4. The side AB equals the side BC.
5. The square root of one half of the square of the diagonal of a square equals one side of the square. Hence,

$$AC^2 = AB^2 + BC^2 \qquad \frac{AC^2}{2} = AB^2$$

$$AC^2 = 2\,AB^2 \qquad \sqrt{\frac{AC^2}{2}} = AB$$

To find the side of an inscribed square when the diameter is given, extract the square root of one half of the square of the diameter.

704. *To find the diameter of a circle when the area is given, divide the area by .7854, and the square root of the quotient will equal the diameter.*

To find the circumference, divide this area by .07958, and the square root of the quotient will equal the circumference.

PROBLEMS

705. 1. Find the circumference of a circle whose diameter is 30 rd.

2. Find the diameter of a circle whose circumference is 34 rd.

3. Find the area of a circle whose diameter is 42 ft.

4. Find the area of a circle whose circumference is 21.5 yd.

5. The area of a circle is 300 sq. ft. What is the diameter? The circumference?

6. The area of a circle is 1 acre. What is the diameter? The circumference?

7. Find the side of the largest square that can be drawn within a circle whose diameter is 20 ft.

8. Find the side of the largest square that can be drawn within a circle whose diameter is 32 yd.

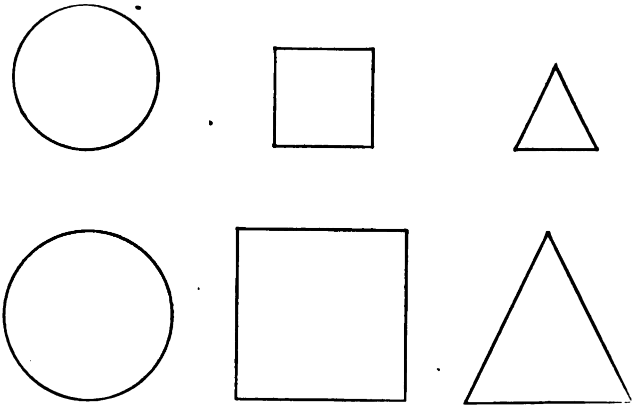
9. Find the area of the largest square that can be drawn within a circle whose radius is 4 ft.

10. The area of an inscribed square is 25 sq. ft. Find the diameter of the circle.

SIMILAR FIGURES

706. *Similar Figures* are those that have the same form, but differ in size.

Note.—To have the same form, two figures must have corresponding equal angles, the same number of sides, and the sides containing the equal angles must be proportional.

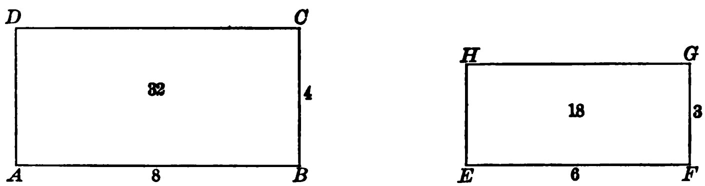


707. The following principles of similar figures are derived from geometry:

1. *The areas of similar figures are to each other as the squares of their like dimensions.*

2. *The like dimensions of similar figures are to each other as the square roots of their areas.*

1. The area of a rectangle whose longer side is 8 ft. is 32 sq. ft. What is the area of a similar rectangle whose longer side is 6 ft.?



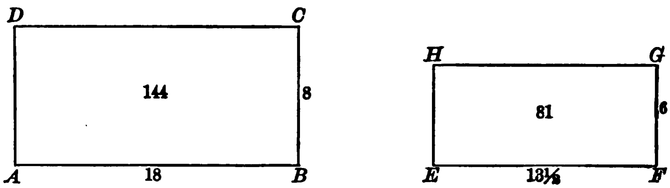
Let *ABCD* represent the larger rectangle, and *EFGH* the smaller.

From the conditions of the problem it is evident that the area *EFGH* is less than the area *ABCD*. According to Prin. 1 the area *ABCD* must sustain the same relation to the area *EFGH* that the square of the length *AB* does to the square of the length *EF*.

Hence $$8^2 : 6^2 :: 32 : x$$

$$x = \frac{6^2 \times 32}{8^2} = 18 \quad \textit{Ans.}$$

2. The area of a rectangle whose longer side is 18 ft. is 144 sq. ft. Find the longer side of a similar rectangle whose area is 81 sq. ft.



Let *ABCD* represent the larger rectangle, and *EFGH* the smaller.

From the conditions of the problem it is evident that the length *EF* is not so great as the length *AB*.

According to Prin. 2 the length *AB* sustains the same relation to the length *EF* that $\sqrt{\text{area } ABCD}$ does to $\sqrt{\text{area } EFGH}$.

Hence $$\sqrt{144} : \sqrt{81} :: 18 : x$$

Or $$12 : 9 :: 18 : x$$

$$x = \frac{9 \times 18}{12} = 13\tfrac{1}{2} \quad \textit{Ans.}$$

PROBLEMS

708. 1. A triangle whose altitude is 8 ft. has an area of 36 sq. ft. What is the area of a similar triangle whose altitude is 12 ft.? (Prin. 1.)

STATEMENT

$$8^2 : 12^2 :: 36 : x$$

2. The diameter of a circle whose area is 78.54 sq. rd. is 10 rd. What is the diameter of a circle whose area is 490.875 sq. rd.? (Prin. 2.)

STATEMENT

$$\sqrt{78.54} : \sqrt{490.875} :: 10 : x$$

3. There are two circles, one 6 ft. in diameter, and the other 30 ft. The second is how many times as large as the first?

STATEMENT

$$6^2 : 30^2 :: 1 : x$$

4. The side of a square field that contains 10 A. is 40 rd. Find the side of a similar field that will contain 30 A.

5. If a pipe 2 in. in diameter fills a cistern in 45 min., in what time will a pipe $1\frac{1}{2}$ in. in diameter fill it?

6. The area of an equilateral triangle is 20.5 sq. ft. What is the area of a similar triangle whose sides are one half as long?

7. The area of a circle whose diameter is 12 ft. is 113.0976 sq. ft. What is the diameter of a circle whose area is 12.5664 sq. ft.?

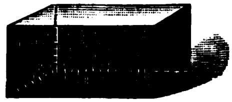
8. The area of a right-angled triangle whose perpendicular is 18 ft. is 216 sq. ft. Find the perpendicular of a similar triangle whose area is 384 sq. ft.

9. Mr. Singly has a rectangular piece of land 120 rd. long and 80 rd. wide. Find the dimensions of a similar field that will contain $93\frac{3}{4}$ A.

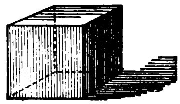
SOLIDS

709. A *Solid* or *Volume* is a figure that has the dimensions length, breadth, and thickness.

710. A *Rectangular Solid* is one bounded by six rectangles.



711. A *Cube* is a solid bounded by six equal squares.



PRISM AND CYLINDER

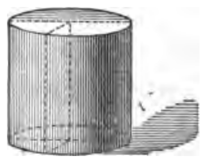
712. A *Prism* is a solid whose sides are parallelograms, and whose ends or bases are equal polygons parallel to each other.

NOTE.—A prism is named from the form of its base as triangular, quadrangular, pentagonal, etc.

TRIANGULAR PRISM

QUADRANGULAR PRISM

PENTAGONAL PRISM



CYLINDER

713. The *Convex Surface* of a prism is the area of its parallelograms taken together.

714. A *Cylinder* is a solid bounded by a uniformly curved surface, having for its two ends, or bases, equal circles parallel to each other.

715. The *Convex Surface* of a cylinder is the area of its curved surface.

716. The *Altitude* of a prism or cylinder is the perpendicular distance between its ends, or bases.

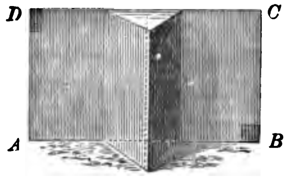


FIG. 1

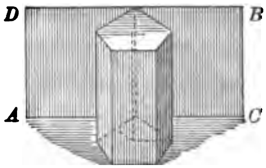


FIG. 2

717. If a piece of paper is fitted to a prism or cylinder so as to cover its convex surface, and then unrolled, its form will be that of a rectangle, as *ABCD* in Figs. 1, 2, 3. Hence,

FIG. 3

To find the convex surface of a prism or cylinder, multiply the perimeter or circumference of the base by the altitude. To find the entire surface add the area of the two ends to the convex surface.

PROBLEMS

718. 1. Find the convex surface of a triangular prism whose sides are each 3 ft., and altitude 5 ft.

STATEMENT

$$3 \times 3 \times 5 = \text{answer}$$

OPERATION

$$3 + 3 + 3 = 9 = \text{perimeter of the base}$$
$$9 \times 5 = 45, \text{ sq. ft., convex surface}$$

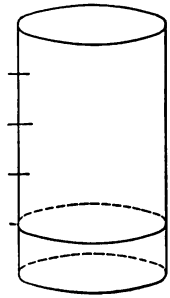
2. Find the convex surface of a prism whose bases are pentagons, each side of them being 4 ft., and altitude 8 ft.

3. Find the convex surface of a cylinder whose altitude is 12 ft. 6 in., and circumference 9 ft.

4. Find the convex surface of a quadrangular prism, the altitude being 10 ft., and each side of the base 3 ft. 6 in.

5. Find the entire surface of a rectangular solid whose ends are 3 ft. by $2\frac{1}{2}$ ft., and length 8 ft.

719. It is evident from the accompanying figure, that a cylinder, for example, 5 in. high, will contain 5 times as many cubic inches as a cylinder with an equal base 1 in. high. The same is true of the prism. Hence,



To find the solid contents of a cylinder or of a prism, multiply the area of the base by the altitude.

PROBLEMS

720. 1. Find the solid contents of a cylinder whose diameter is 4 ft., and altitude 16 ft.

2. Find the solid contents of a triangular prism, the altitude being 30 ft., and each side of the base 4 ft.

3. How many cubic feet of earth must be taken out in digging a well 30 ft. deep, and 6 ft. in diameter?

4. How many cubic feet are there in a log 25 ft. long, and 16 in. in diameter?

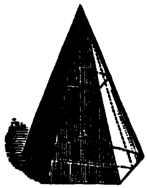
5. The solid contents of a square prism is 6727 cu. in. The altitude is 7 ft. Find the length of each side of the base.

Pyramid and Cone

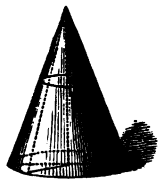
721. A *Pyramid* is a solid that has for its base a polygon, and for its sides three or more triangles meeting at a common point called its vertex.

722. A *Cone* is a solid that has for its base a circle, and whose convex surface tapers regularly to a point called its vertex.

723. The *Altitude* of a pyramid or cone is the perpendicular distance from the vertex to the plane of the base.



PYRAMID



CONE

724. The *Slant Height of a Pyramid* is the perpendicular distance from the vertex to any side of the base. The *Slant Height of a Cone* is the distance from the vertex to any point in the circumference of the base.

725. *To find the convex surface of a pyramid or cone, multiply the perimeter or circumference of the base by one half of the slant height. Add the area of the base to the convex surface to find entire surface.*

PROBLEMS

726. 1. What is the convex surface of a triangular pyramid whose slant height is 18 ft. and each side of the base 6 ft.?

STATEMENT

$$(6 \times 3) \times (\tfrac{1}{2} \times 18) = \text{answer}$$

2. What is the convex surface of a cone, the circumference of the base being 36 in. and slant height 26 in.?

STATEMENT

$$36 \times (\tfrac{1}{2} \times 26) = \text{answer}$$

3. What is the entire surface of a pyramid whose base is 40 ft. square and slant height 80 ft.?

4. Find the entire surface of a cone, the diameter of whose base is 4 ft. and slant height 25 ft.

727. *To find the solid contents of a pyramid or cone, multiply the area of the base by one third of the altitude.*

PROBLEMS

728. 1. Find the solid contents of a pyramid whose altitude is 24 ft. and base a rectangle 3 ft. by 4 ft.

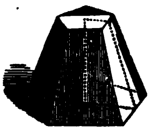
2. Find the solid contents of a cone whose altitude is 25 ft. and circumference of base 12 ft.

3. Find the solid contents of a cone whose altitude is 33 ft. and radius of base $2\frac{1}{2}$ ft.

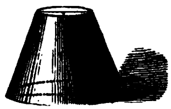
4. Find the entire surface of the cone described in the preceding example.

5. Find the solid contents of the pyramid described in Ex. 3, Art. 726.

6. Find the solid contents of a pyramid whose base is an equilateral triangle, each side being 8 ft. and altitude 32 ft.



FRUSTUM



FRUSTUM

729. The *Frustum* of a pyramid or cone is the part that remains after cutting off the top by a plane parallel to the base.

730. *To find the convex surface of a frustum, multiply the sum of the perimeters, or of the circumferences of the two bases, by one half of the slant height.*

PROBLEMS

731. 1. Find the convex surface of a frustum of a cone whose slant height is 18 ft., the circumference of the lower base being 15 ft. and upper base 8 ft.

2. Find the entire surface of a frustum of a square pyramid, each side of the lower base being 12 ft. and upper base 6 ft. and the slant height 75 ft.

732. *To find the solid contents of a frustum, take the sum of the areas of the two bases; to this add the square root of their product, and multiply this sum by one third of the altitude.*

PROBLEMS

733. 1. Find the solid contents of a frustum of a square pyramid whose altitude is 21 ft., each side of the lower base being 4 ft. and of the upper base 3 ft.

OPERATION

$4 \times 4 = 16 =$ area of lower base
$3 \times 3 = 9 =$ area of upper base
$\sqrt{16 \times 9} = 12 =$ square root of their product
$16+9+12 = 37 =$ sum of the bases and sq. root of their product
$37 \times (\frac{1}{3} \times 21) = 259$ cu. ft. *Ans.*

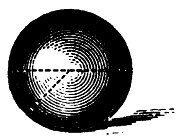
STATEMENT

$$(4^2 + 3^2 + \sqrt{4^2 \times 3^2}) \times (\tfrac{1}{3} \times 21) = \text{answer}$$

2. Find the solid contents of a frustum of a cone whose altitude is 18 ft., diameters of bases being 8 ft. and 6 ft.

3. How many cubic feet in 50 fence posts 8 ft. long, the larger end being 8 in. square and the smaller end 6 in. square?

734. A *Sphere* is a solid bounded by a uniformly curved surface, every part of which is equally distant from a point within, called the center.


SPHERE

735. The *Diameter* of a sphere is a straight line passing from any point in the circumference through the center and terminating in the surface directly opposite.

736. *To find the convex surface of a sphere, multiply the circumference by the diameter, or the square of the diameter by 3.1416.*

PROBLEMS

737. 1. Find the surface of a sphere 3 ft. 4 in. in diameter.

2. Find the surface of a globe 12 in. in diameter.

738. *To find the solid contents of a sphere multiply the cube of the diameter by .5236.*

PROBLEMS

739. 1. Find the solid contents of a sphere whose diameter is 6 ft.

2. Find the solid contents of a globe whose radius is 3 ft. 6 in.

3. The surface of a globe is 50.2656 sq. ft. The diameter is 4 ft. Find the circumference.

SIMILAR SOLIDS

740. The following principles of similar solids are derived from geometry:

1. *Similar solids are to each other as the cubes of their like dimensions.*

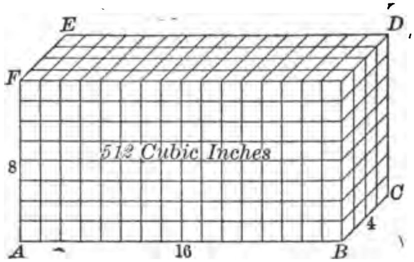
2. *Like dimensions of similar solids are to each other as the cube roots of their contents.*

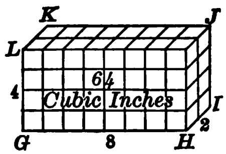
1. A rectangular solid 16 in. long contains 512 cu. in. How many cubic inches are contained in a similar solid 8 in. long?

Let *ABCDEF* represent the first rectangular solid, and *GHIJKL* the second.

From the conditions of the problem it is evident that the volume *GHIJKL* is less than *ABCDEF*. According to Prin. 1 the volume *ABCDEF* must sustain the same relation

to volume *GHIJKL* that the cube of the length *AB* does to the cube of the length *GH*.





Hence $$16^3 : 8^3 :: 512 : x.$$

$$x = \frac{\cancel{8^3} \times 512}{\cancel{16^3}_{\,2^3}} = 64 \text{ Ans.}$$

2. A rectangular solid 16 in. long contains 512 cu. in. Find the length of a similar solid that contains 64 cu. in.

Using the same figures as in the first illustrative example, it is plain that the length *GH* is not so great as the length *AB*.

According to Prin. 2 the length *AB* sustains the same relation to *GH* that the cube root of the volume *ABCDEF* does to the cube root of volume *GHIJKL*.

Hence $$\sqrt[3]{512} : \sqrt[3]{64} :: 16 : x$$

$$x = \frac{\cancel{\sqrt[3]{64}} \times 16}{\cancel{\sqrt[3]{512}}_{\,\sqrt[3]{8}}} = 8 \text{ Ans.}$$

PROBLEMS

741. 1. A ball 6 inches in diameter is how many times as large as one 3 inches in diameter?

STATEMENT

$$3^3 : 6^3 :: 1 : x$$

2. If a 3-inch cube weighs 27 lb., how much will a 7-inch cube of the same material weigh?

STATEMENT

$$3^3 : 7^3 :: 27 : x$$

3. If a 4-inch cube weighs 64 lb., what will be the length of the edge of a cube that weighs 216 lb.?

STATEMENT

$$\sqrt[3]{64} : \sqrt[3]{216} :: 4 : x$$

4. If a ball of metal 6 in. in diameter weighs 135 lb., what is the diameter of a ball of the same metal whose weight is 5 lb.?

5. If a man 5 ft. high weighs 145 lb., what will be the weight of a man of similar proportions 6 ft. high?

6. A cylinder whose diameter is 4 ft. contains 125 cu. ft. Find the diameter of a similar cylinder containing 512 cu. ft.

7. A cone 8 ft. high contains 100 cu. ft. What is the altitude of a similar cone that contains 50 cu. ft.?

8. A pyramid 6 in. high weighs 32 lb. What is the altitude of a similar pyramid that weighs 256 lb.?

9. How many 2-inch cubes are equal to one 6-inch cube?

10. A stack of hay 20 ft. in diameter contains 5 tons. How many tons are there in a similar stack 12 ft. in diameter?

REVIEW PROBLEMS

742. 1. Find the surface of a 6-inch cube.

2. Find the surface of a rectangular solid 8 ft. long, 4 ft. wide, and 3 ft. thick.

3. How many cubic feet of earth must be removed in digging a well 6 ft. in diameter and 30 ft. deep?

4. The base of a right-angled triangle is 60 ft., and the perpendicular 80 ft. Find the hypotenuse.

5. Find the length of the longest stick that can be placed within the well described in Ex. 3.

6. Find the side of a square the area of which is equal to the area of a rectangle 280 ft. long and 70 ft. wide.

7. The floor of a room 24 ft. long contains $42\frac{2}{3}$ sq. yd. Find the width.

8. A farmer took 2200 steps of 3 ft. each to walk around a rectangular field. How many acres are there in the field, if its length is to its width as 2 to 1?

9. What is the height of a room 13 ft. long and 12 ft. wide, if it contains 2,695,680 cu. in.?

10. Find the side of the largest square that can be drawn within a circle 20 in. in diameter.

11. How many gallons of water will a cylindrical cistern hold, if it is 6 ft. 4 in. deep, and 4 ft. 3 in. in diameter?

12. Find the area of a square that can be drawn within a circle whose radius is 8 ft.

STATEMENT

$$(8 \times 2)^2 \div 2 = \text{answer}$$

NOTE.—The diameter of a circle equals the diagonal of the inscribed square. To find the area of a square when the diagonal is given, divide the *square* of the diagonal by 2.

13. Find the largest square stick of timber that can be cut from a log 16 in. in diameter.

14. Find the cost of fencing a circular driving park 40 rd. in diameter, at $1.25 a rod.

15. The volume of a cone is 384 cu. in. The area of the base is 64 sq. in. Find the altitude.

16. How many gallons of water will a tub hold, 18 in. deep, 26 in. wide at the top, and 20 in. at the bottom?

17. A pipe $1\frac{1}{2}$ in. in diameter will fill a cistern with water in $1\frac{1}{2}$ hr. In what time will a pipe 2 in. in diameter fill it?

18. The length of a rectangular field containing 30 acres is to its breadth as 8 to 6. Find its dimensions.

19. A ball of brass 4 in. in diameter weighs 9 lb. What will be the diameter of a ball of the same material which weighs 576 lb.?

20. Find the volume of a pyramid whose base is a rectangle 6 ft. by 8 ft., and whose edges which meet at the apex are 14 ft.

21. Find the entire surface of the figure described in Ex. 20.

22. Find the cost of digging a ditch at 20¢ a cubic yard, 300 rd. long, 20 in. deep, 18 in. wide at the bottom, and 2 ft. wide at the top.

23. Find the value of a piece of timber 20 ft. long, 12 in. square at one end, and 8 in. square at the other, at 22¢ a cubic foot.

24. Find the distance from one of the lower corners of a room to the opposite upper corner, if the room is 30 ft. long, 20 ft. wide, and 12 ft. high.

25. How many spheres 3 in. in diameter can be made from 2 spheres 6 in. in diameter?

26. A building 80 ft. long and 60 ft. wide has a pyramidal roof 15 ft. high. Find the length of the rafters reaching from the corners to the highest point of the roof.

27. How much, at 4¢ a square foot, will it cost to cover, with tin, the roof described in Ex. 26?

28. How many cones 3 ft. in diameter and 8 ft. high can be made from a cylinder of the same diameter and height?

29. Find the slant height of a pyramid whose base is 4 ft. square, and altitude 10 ft.

30. Find the entire surface of a cube containing 5832 cu. ft.

ALGEBRAIC EQUATIONS

743. *Algebra* is a branch of mathematics in which the quantities to be considered are represented by letters.

744. An *Equation* is the expression of equality between two quantities, or sets of quantities. Thus, $9 + 8 = 17$, and $x + 2x = 21$ are equations. a, b, x, etc., are used for $1a, 1b, 1x$.

PROBLEMS

745. 1. James and John have 21 marbles. How many has each if John has twice as many as James?

OPERATION

Let $x =$ James's number
then $2x =$ John's number
and $x + 2x = 21$
$3x = 21$
$x = 7$, James's number
$2x = 14$, John's number

We let x represent the number of marbles which James has; then, since John has twice as many as James, $2x$ will represent John's number, and $x + 2x$, or $3x$, the number both have, or 21. If $3x$ is equal to 21, x is equal to 7, or James's number, and $2x$, or 14, equals John's number.

2. Willis and John picked 36 bushels of apples. How many did each pick if Willis picked twice as much as John?

3. The sum of two numbers is 60. The larger is 3 times the smaller. What are the numbers?

OPERATION

Let $x =$ the smaller number
then $3x =$ the larger number
and $x + 3x = 60$, the sum
$4x = 60$, four times the smaller number
$x = 15$, the smaller number
$3x = 45$, the larger number

4. Divide 80 cents between Robert and Elizabeth so that Robert shall have 3 times as many cents as Elizabeth.

5. A man bought a horse and carriage for $250. He paid 4 times as much for the horse as for the carriage. Find the cost of each.

6. Divide 108 into two parts so that the larger part shall equal 5 times the smaller.

7. The sum of three numbers is 96. The second is twice the first, and the third 3 times the first. What are the numbers?

OPERATION

$$
\begin{aligned}
\text{Let } x &= \text{the first number} \\
\text{then } 2x &= \text{the second number} \\
\text{and } 3x &= \text{the third number} \\
x + 2x + 3x &= 96, \text{ the sum} \\
6x &= 96, \text{ six times the first number} \\
x &= 16, \text{ the first number} \\
2x &= 32, \text{ the second number} \\
3x &= 48, \text{ the third number}
\end{aligned}
$$

8. A man bought a hat, a pair of shoes, and an overcoat for $33. He paid 2 times as much for the shoes as for the hat, and 8 times as much for the overcoat as for the hat. How much did he pay for each?

9. $38,000 is to be divided among three persons. The second is to receive 3 times as much as the first, and the third 5 times as much as the first. Find the share of each.

10. What number added to 3 and 4 times itself will give 72?

11. What number added to 4 and 7 times itself will give 108?

12. The difference of two numbers is 42, and the larger is 3 times the smaller. Find the numbers.

OPERATION

Let $x =$ the smaller number
then $3x =$ the larger number
and $3x - x = 42$, the difference
$2x = 42$, twice the smaller number
$x = 21$, the smaller number
$3x = 63$, the larger number

13. The difference of two numbers is 72, and the larger number is 4 times the smaller. Find the numbers.

14. A farmer raised 384 bushels more of wheat than of oats. How many bushels of each did he raise if 4 times the number of bushels of oats equals the number of bushels of wheat?

15. Seven times a certain number minus two times the number equals 65. What is the number?

746. Find the value of x, y, z, etc., in the following equations:

1. $x + 2x = 72$	7. $x + 2x + 3x = 144$
2. $2x + 3x = 180$	8. $2x + 4x + 5x = 110$
3. $5y + 2y = 196$	9. $3y + 5y + 6y = 294$
4. $3z + 4z = 161$	10. $5z + 3z - 4z = 48$
5. $8y - 3y = 35$	11. $12w - 3w + 6w = 120$
6. $7w - 4w = 81$	12. $23z - 14z - 3z = 192$

PROBLEMS

747. 1. A man bought an equal number of sheep and calves for $270, paying $4 apiece for the sheep, and $5 apiece for the calves. How many of each did he buy?

2. The difference between 2 times a certain number and 7 times the same number is 70. Find the number.

3. I bought an equal number of pounds of sugar and rice for 96 cents, paying 5 cents a pound for the sugar, and 7 cents for the rice. How many pounds of each did I buy?

4. A man bought an equal number of pigs, calves, and cows for \$260, paying \$2 each for the pigs, \$4 each for the calves, and \$20 each for the cows. How many of each did he buy?

5. Divide the number 120 into three such parts that the second shall equal 3 times the first, and the third 2 times the second.

6. The sum of three numbers is 96. The second is 3 times the first, and the third is 2 times the sum of the first and second. Find the numbers.

7. Three boys, A, B, and C, picked 72 quarts of berries. A picked twice as many quarts as B, and C picked 3 times as many quarts as A. How many quarts did each pick?

8. The sum of two numbers is twice their difference, which is 34. The larger number is 3 times the smaller. What are the numbers?

9. The distance around a rectangular piece of land is 192 rods. Find the dimensions if the length is twice the width.

10. In a mixture of 63 bushels of grain, there are 4 times as many bushels of oats as rye, and one half as many bushels of corn as oats. How many bushels of each are there?

748. $\frac{1}{2}$ of x is written $\frac{1}{2}x$, or $\frac{x}{2}$; $\frac{1}{3}$ of x is written $\frac{1}{3}x$, or $\frac{x}{3}$; $\frac{2}{3}$ of $x = \frac{2x}{3}$.

Just as $1 = \frac{2}{2}, \frac{3}{3}, \frac{4}{4}$, etc., so $x = \frac{2x}{2}, \frac{3x}{3}, \frac{4x}{4}$, etc.

PROBLEMS

749. 1. A man and his son earned \$66. How much did each earn if the son earned one half as much as his father?

OPERATION

Let $x =$ what the father earned

then $\frac{x}{2} =$ what the son earned

$$\text{and} \quad x + \frac{x}{2} = \$\,66, \text{ what both earned}$$

$$\frac{2x}{2} + \frac{x}{2} = \frac{3x}{2} = \$\,66$$

$$\frac{x}{2} = \$\,22, \text{ what the son earned}$$

$$x = \$\,44, \text{ what the father earned}$$

2. A dealer sold a piano for \$400 and thereby gained $\frac{1}{3}$ of what he paid for it. How much did he pay for it?

3. The sum of two numbers is 450, and the smaller is $\frac{2}{3}$ of the larger. What are the numbers?

4. Mary's age increased by $\frac{3}{4}$ of her age equals 21 years. How old is Mary?

5. A man lost $\frac{2}{3}$ of his money, and then had \$2000 remaining. How much had he at first?

6. My age diminished by its $\frac{3}{5}$ equals 18 years. What is my age?

7. After losing $\frac{3}{5}$ of his money, Martin had \$960. How much money had he at first?

8. The sum of $\frac{1}{2}$ and $\frac{1}{3}$ of what a teacher receives in a month equals \$75. How much does he receive per month?

OPERATION

$$\text{Let} \quad x = \text{what he receives}$$

$$\text{then} \quad \frac{x}{2} + \frac{x}{3} = \$\,75$$

$$\text{or} \quad \frac{3x}{6} + \frac{2x}{6} = \$\,75$$

$$\frac{5x}{6} = \$\,75$$

$$\frac{x}{6} = \$\,15$$

$$x = \$\,90 \quad \textit{Ans.}$$

9. The sum of $\frac{1}{3}$ and $\frac{1}{4}$ of what number equals 280?

750. Find the values of x, y, and z in the following equations:

1.

$$x+\frac{x}{3}=24$$
$$\frac{4x}{3}=24$$
$$\frac{x}{3}=6$$
$$x=18 \quad \textit{Ans.}$$

2.

$$x+\frac{2x}{3}=30$$
$$\frac{5x}{3}=30$$
$$\frac{x}{3}=6$$
$$x=18 \quad \textit{Ans.}$$

3.

$$2x-\frac{x}{3}=15$$
$$\frac{5x}{3}=15$$
$$\frac{x}{3}=3$$
$$x=9 \quad \textit{Ans.}$$

4. $x+\frac{x}{4}=25$

5. $x+\frac{3x}{4}=42$

6. $x+\frac{x}{5}=36$

7. $x-\frac{2x}{3}=12$

8. $x-\frac{3x}{4}=11$

9. $y-\frac{2y}{5}=27$

10. $2y-\frac{2y}{3}=28$

11. $2x+\frac{2x}{5}=36$

12. $2z-\frac{3z}{5}=42$

751. Find the value of x.

1.

$$\frac{x}{2}+\frac{x}{3}=25$$
$$\frac{3x}{6}+\frac{2x}{6}=25$$
$$\frac{5x}{6}=25$$
$$\frac{x}{6}=5$$
$$x=30 \quad \textit{Ans.}$$

2.

$$\frac{2x}{3}-\frac{x}{2}=12$$
$$\frac{4x}{6}-\frac{3x}{6}=12$$
$$\frac{x}{6}=12$$
$$x=72 \quad \textit{Ans.}$$

3.

$$\frac{4x}{5}-\frac{x}{2}=21$$
$$\frac{8x}{10}-\frac{5x}{10}=21$$
$$\frac{3x}{10}=21$$
$$\frac{x}{10}=7$$
$$x=70 \quad \textit{Ans.}$$

4. $\frac{2x}{3}+\frac{x}{2}=35$

5. $\frac{x}{4}+\frac{x}{3}=21$

6. $\frac{x}{5}+\frac{x}{2}=42$

7. $\frac{x}{8}+\frac{x}{4}=36$

8. $\frac{5x}{8}-\frac{x}{2}=9$

9. $\frac{4x}{5}-\frac{3x}{4}=20$

10. $\frac{x}{10}+\frac{x}{5}=30$

11. $\frac{3x}{4}-\frac{3x}{10}=27$

12. $\frac{2x}{3}+\frac{3x}{4}=51$

PROBLEMS

752. **1.** A certain number increased by 7 equals 29. What is the number?

We let $x =$ the number; then $x + 7$ will equal the number increased by 7. Now, since x *increased* by 7 equals 29, it is evident that x equals 29 *minus* 7, or 22.

OPERATION

Let $x =$ the number
then $x + 7 = 29$
$x = 29 - 7$
$x = 22$

2. John and William together have \$45. John has \$5 more than William. How much money has each?

We let $x =$ William's money; then, since John's money is \$5 more than William's, $x +$ \$5 will equal John's money. Adding, we get $2x +$ \$5 = \$45.

If $2x$ *increased* by \$5 = \$45, it is evident $2x$ will equal \$45 *minus* \$5, or \$40.

If $2x =$ \$40, $x =$ \$20, William's money; and $x +$ \$5, or \$25 = John's money.

OPERATION

Let $x =$ William's money
then $x +$ \$5 = John's money
and $2x +$ \$5 = \$45
$2x =$ \$45 − \$5
$2x =$ \$40
$x =$ \$20, William's money
$x +$ \$5 = \$25, John's money

3. A man bought a pair of gloves and a necktie for \$2.50. He paid 50 cents less for the necktie than for the gloves. How much did he pay for each?

We let $x =$ cost of gloves; then, since the necktie cost \$.50 less than the gloves, $x -$ \$.50 will equal the cost of the necktie. Adding, we get $2x -$ \$.50 = \$2.50. If $2x$ *diminished* by \$.50 equals \$2.50, it is evident that $2x$ equals \$2.50 increased by \$.50, or \$3.00.

OPERATION

Let $x =$ cost of gloves
then $x -$ \$.50 = cost of necktie
and $2x -$ \$.50 = \$2.50
$2x =$ \$2.50 + \$.50
$2x =$ \$3.00
$x =$ \$1.50, cost of gloves
$x -$ \$.50 = \$1.00, cost of necktie

4. If Theodore's age were increased by 9 years, the sum would equal 25 years. How old is Theodore?

5. A lady bought a coat and hat for \$26. She paid \$6 more for the coat than for the hat. How much did she pay for each?

6. If 2 times the distance between two cities were increased by 39 miles, the sum would equal 109 miles. What is the distance?

7. Mary solved 4 problems more than her sister. They together solved 16. How many did each solve?

8. $\frac{3}{4}$ of a certain number minus 9 equals 36. What is the number?

9. Two brothers, Elmer and Martin, earned \$52 delivering papers. Martin earned \$12 less than Elmer. How much did each earn?

10. A man bought a horse and sold it at a gain equal to $\frac{1}{4}$ of the cost. How much did the horse cost if he received \$175 for it?

753. Find the value of x, y, etc., in the following equations:

1. $x + 16 = 45$
2. $x + 24 = 76$
3. $y + 32 = 79$
4. $x - 12 = 27$
5. $y - 32 = 74$
6. $z - 36 = 49$
7. $y + 32 = 76$
8. $\frac{3x}{4} + 30 = 90$
9. $\frac{2x}{3} - 21 = 49$
10. $\frac{4y}{5} - 54 = 42$
11. $2y + 16 = 64$
12. $3x - 24 = 42$
13. $2z - 41 = 29$
14. $2\frac{1}{2}x - 12 = 193$
15. $3\frac{1}{8}y + 16 = 156$
16. $4\frac{3}{8}x - 18 = 94$

754. Multiplication is usually indicated without the use of the sign of multiplication.

Thus, $a \times b = ab$; $x \times y = xy$; $3 \times a \times b \times c = 3abc$.

755. When $a = 2$, $b = 3$, $c = 4$, $d = 5$, find the numerical value of the following expressions. Thus, $4a + 3b = 8 + 9 = 17$; $3d - 2c = 15 - 8 = 7$.

1. $2a + 3c =$	6. $abc + 3d =$	11. $8a + 2b - 4c =$
2. $3c - 2b =$	7. $5d - 4c =$	12. $3c + 2d - 8 =$
3. $3abc + d =$	8. $8abc + ad =$	13. $abc + abd - 5d =$
4. $5ab - 3c =$	9. $5cd - 75 =$	14. $2ad + d =$
5. $4bc + ac =$	10. $3bc \div 12 =$	15. $3abc + 2c =$

756. Since the two members of an equation are equal, we may *add to*, or *subtract from*, both members equal quantities without destroying the equality. In the equation $7 + 6 = 13$, if we add 5 to each member, we have $7 + 6 + 5 = 13 + 5$.

Again, in the equation $21 = 12 + 9$, if we subtract 8 from each member, we have $21 - 8 = 12 + 9 - 8$. It will be seen that in both cases the members remain equal.

757. 1. Find the value of x in the equation $4x - 9 = 2x + 7$. Since the known and unknown quantities appear in both members of the equation, to find the value of x we must transpose the unknown terms to the first member and the known terms to the second member. We can change any term from one member to the other by simply changing its sign.

Subtracting $2x$ from both sides of the equation, we have

$$4x - 2x - 9 = 7$$

Adding 9 to both members, we have

$$4x - 2x = 7 + 9$$

Reducing, we have $\quad 2x = 16$

$$x = 8$$

Find the value of x in the following equations:

2. $8x = 4x + 20$	6. $12x - 9 = 5x + 12$
3. $5x = 2x + 21$	7. $9x + 12 = 60 - 3x$
4. $7x = 4x + 18$	8. $12x - 4x + 8 = 32 - 16$
5. $12x - 9 = 9x$	9. $6x - 40 + 2x = 88 - 8x$

10. $9x + 18x = 54 - 3x + 6$

11. $8x + 8 - 6x = 32 - 4x$

12. $14x + 6 + 4x = 10x + 40 - 2$

13. $6x - 30 - 20 = 40 - 4x$

PROBLEMS

758. 1. A man bought a horse and carriage for $180. Find the cost of each, if twice what he paid for the carriage plus $15 equals what he paid for the horse.

OPERATION

Let $x =$ cost of carriage

then $2x + \$15 =$ cost of horse

Adding, we have $2x + x + \$15 = \$180 =$ cost of both

Transposing and collecting, we have

$$3x = \$165$$

$$x = \$55, \text{ cost of carriage}$$

$$2x + \$15 = \$125, \text{ cost of horse}$$

2. Walter had twice as many marbles as Alfred. After Walter had lost 10 of his and Alfred had won 5, they together had 55. How many had each at first?

OPERATION

Let $x =$ Alfred's number

then $2x =$ Walter's number

$x + 5 =$ Alfred's after winning 5

$2x - 10 =$ Walter's after losing 10

$$2x - 10 + x + 5 = 55$$

$$3x - 5 = 55$$

$$3x = 60$$

$$x = 20, \text{ Alfred's number}$$

$$2x = 40, \text{ Walter's number}$$

3. I had 200 lb. of sugar put up in 10-pound and 5-pound packages. There were 22 more 5-pound packages than 10-pound packages. Find the number of packages of each kind.

OPERATION

$$
\begin{aligned}
\text{Let}\qquad x &= \text{number 10-pound packages}\\
\text{then}\quad x + 22 &= \text{number 5-pound packages}\\
10\,x &= \text{lb. in the 10-pound packages}\\
5\,x + 110 &= \text{lb. in the 5-pound packages}\\
10\,x + 5\,x + 110 &= 200\\
15\,x &= 90\\
x &= 6,\ \text{number 10-pound packages}\\
x + 22 &= 28,\ \text{number 5-pound packages}
\end{aligned}
$$

4. A man paid 25% of a month's wages for an overcoat. If he had \$60 remaining, how much were his monthly earnings?

5. Six times a certain number minus 10 equals 4 times the same number plus 8. What is the number?

OPERATION

$$
\begin{aligned}
&\text{Let} & x &= \text{the number}\\
&\text{then} & 6\,x - 10 &= 4\,x + 8\\
&\text{Transposing,} & 6\,x - 4\,x &= 8 + 10\\
&\text{Reducing,} & 2\,x &= 18\\
& & x &= 9
\end{aligned}
$$

6. Five times a certain number equals 4 times the sum obtained by increasing the same number by 2. What is the number?

7. Three brothers have together \$1690. Twice what the first has, diminished by \$220, equals what the second has, and the third has \$250 more than the second. How much money has each?

8. A and B commenced business with an equal sum of money. The first year A gained 25% of his money, and B lost 20%. They then had together \$400 more than they had at first. How much money had each at first?

GENERAL REVIEW PROBLEMS

759. 1. The premium for insuring my house at $2\frac{1}{2}\%$ was \$90. For what sum was it insured?

2. I bought 50 shares of stock at 105, and sold them at $1\frac{1}{2}\%$ discount. Find the loss.

3. The amount of \$1600 at a certain rate for 2 yr. 6 mo. is \$1880. Find the rate.

4. An agent sold 1600 barrels of flour at \$6.52$\frac{1}{2}$. After deducting his commission he remitted \$10,179.00. Find his rate of commission.

5. What must be paid for 6% stock in order to realize 10% on the investment?

6. What part of a mile is 120 rd. 3 yd. 2 ft.?

7. If I buy 300 pears at the rate of 2 for 1¢, and 270 at the rate of 3 for a cent, and sell them all at the rate of 5 for 2¢, shall I gain or lose, and how much?

8. When it is 10 A.M. at the Hawaiian Islands, 155° west longitude, what is the time at Washington, 77° 1′ west longitude?

9. The difference in time between the Hawaiian Islands and Cincinnati is 4 hr. 42 min. 24 sec. Find the longitude of Cincinnati.

10. How many square yards of plastering are there in a room 30 ft. long, $18\frac{1}{2}$ ft. wide, and 10 ft. high, allowing for 3 doors, each $7\frac{1}{2}$ ft. by $2\frac{1}{2}$ ft., and 4 windows, each $6\frac{1}{2}$ ft. by $2\frac{1}{2}$ ft.?

11. Find the cost of 7380 ft. of pine boards at \$ 36 per M.

12. Find the cost of 36,842 lb. of wheat at \$ .87½ per bushel.

13. What sum must I invest in 4½% stock at 94½, brokerage ⅛%, to realize an annual income of \$ 3600?

14. Find the true discount and present worth of \$ 944 due in 2 yr. 6 mo., at 6%.

15. Which is worth most, \$ 1800 cash down, \$ 1872 in 6 mo., or \$ 1904 in 9 mo., money being worth 6%?

16.

\$ 5480. SCRANTON, PA., Oct. 14, 1896.

Two years after date, for value received, I promise to pay Asa Tubbs, or order, Five Thousand Four Hundred and Eighty Dollars, with interest at 6%.

T. B. TUCK.

On this note were the following indorsements: June 20, 1897, \$ 325; Oct. 10, 1897, \$ 750; Jan. 30, 1898, \$ 3200; May 9, 1898, \$ 300. Find the amount due Oct. 14, 1899.

17. Find the cost of a 30-day draft on Philadelphia, exchange at ¼% discount, that will pay a bill of \$ 1900, money worth 6%.

18. What sum payable in 8 mo. 15 da., if discounted at bank at 6%, will yield \$ 4116.20 proceeds? No grace.

19. Find the cost of a draft for \$ 3040, exchange being ¼% discount.

20. Find the face of a note which, when discounted at bank for 6½ mo. at 8%, will yield \$ 697.88.

21. The interest on a note for \$ 434.50 at 8% was \$ 115.86⅞. What was the time?

22. Find the cost of a 30-day draft for \$ 1600, when exchange is ½% premium, and interest 5%.

23. Find the face of a 60-day draft costing \$6276.80, when exchange is at a discount of $\frac{3}{4}\%$ and interest 6%.

24. Find the face of a sight draft for \$3945, exchange being $1\frac{3}{8}\%$ discount.

25. In how many days can 72 men, by working 9 hr. a day, do as much work as 56 men in 20 da. of 10 hr. each?

26. A, B, and C engaged in business. A put in \$5000, B \$6000, and C $\frac{3}{5}$ of the entire capital. They lost \$9240. Find the loss of each.

27. Find the cash value of a bill of goods amounting to \$12,861.80, at 20% discount, and 5% off for cash.

28. A, B, and C form a partnership with a capital of \$37,500. A's capital is in the business 6 mo., B's 9 mo., and C's 1 yr. 4 mo. A's share of the gain is \$1800, B's \$2250, and C's \$3200. Find the capital of each.

29. How many boards 15 feet long will be required to build a fence 5 boards high around a rectangular field 120 rods long and 30 rods wide?

30. A, B, and C took a contract to build a bridge. A furnished 30 men for 50 da., B 25 men for 96 da., C 75 men for 28 da. What was the share of each, if they received \$10,125 for the work?

31. A man sold two farms for \$2400 each. On one he gained 20%, and on the other he lost 20%. Did he gain or lose on the sale, and how much?

32. Find the cost, at \$35 per M, board measure, of 10 pieces of sawed timber, each 20 ft. long, 16 in. wide, and 12 in. thick.

33. How many acres are in a field 80 chains long, and 65 chains and 20 links wide?

34. A ladder 35 ft. long is so planted as to reach a window 20 ft. from the ground on the one side of the street, and on the other a window 15 ft. from the ground. Find the width of the street.

35. A merchant asked for a certain lot of goods 20% more than cost; but owing to a depression in business he was obliged to sell for 90% of his asking price. Find his gain per cent.

36. A man bought a car load of potatoes, 25% of which he found to be damaged. What per cent must he gain on the remainder that he may make 10% on the cost of the potatoes?

37. What must I ask for goods which cost 80¢ a yard, that, after falling $33\frac{1}{3}$%, I may gain 10%?

38. How many board feet are there in a board which is 20 ft. long, 18 in. wide at one end, and tapers regularly until it is 12 in. wide at the other?

39. Which is further around, a circular field containing 5 acres, or a square one of the same area?

40. How far from the base of a building must a ladder 50 ft. long be placed to reach a point 30 ft. from the ground?

41. Mr. Jones bought 20 shares of stock, $100, at $2\frac{1}{2}$% discount. He sold 75% of the shares at a premium of $1\frac{3}{4}$%, and the rest at $\frac{1}{2}$% discount. How much did he gain?

42. If I borrow $5000 from a bank for a year, at 6% discount, and lend the proceeds at $6\frac{1}{2}$% interest, for the same time, shall I gain or lose, and how much? No grace.

43. An agent sold goods at 5% commission, and invested the proceeds in other goods at 3% commission. His entire commission was $360. What was the amount of the sale?

44. If I pay $120 four months before it is due, how long after it is due may I keep $160 to balance it?

45. A merchant sold goods to the amount of \$5936, payment for the same to be made as follows: 50% down, and the remainder in 1 year. To how much cash was the selling price equal, money commanding 6% per annum?

46. What must be paid for 4% stock in order to realize 5% on the investment?

47. What premium can I afford to pay for bonds bearing 6% interest so that I may realize $4\frac{1}{2}$% on the investment?

48. A merchant bought goods at 25 and 5% off list price. If he sold them 10% above list price, what was his gain per cent?

49. Find the solid contents of a frustum of a pyramid whose lower base is 10 ft. square, upper base 5 ft. square, and altitude 10 ft.

50. Find the length of a rope reaching from the top of a pole 40 ft. high to the top of another 10 ft. high, if the poles are 32 ft. apart.

ANSWERS.

Page 22. — **2.** 2951. **3.** 39,341. **4.** 254,508. **Art. 53.** — **2.** \$38,247.462. **3.** \$98,199.335. **4.** \$95,763.412.

Page 23. — **1.** 365 da. **2.** 108°. **3.** 5850 A. **4.** 7,106,199. **5.** 4900 bu.; \$3359. **6.** 19,361 T. **7.** 401,700,000 T. **8.** 150,628.

Page 26. — **1.** 3145. **2.** 3187. **3.** 73,048. **4.** \$884.309. **5.** \$436.79. **6.** 2521 rd. **7.** 61,989 bu. **8.** \$6407.83. **9.** 9,061,980. **Art. 68.** — **1.** 73,630.

Page 27. — **2.** 11,192. **3.** 206 bu.; 858 bu. **4.** 1370 sq. mi. **5.** \$599,117. **6.** 236 mills; 2,388,000 spindles; 76,000 looms. **7.** 7033 T. **8.** 139,806 ft. **9.** 15,350 sq. mi.

Page 30. — **2.** 37,702. **3.** 32,552. **4.** \$672,552. **5.** \$275,436. **6.** \$34,563.42. **7.** \$42,651.588. **Art. 78.** — **2.** 29,040. **3.** 24,600. **4.** 291,000. **5.** 550,200. **6.** 7,128,000. **7.** \$676,800. **8.** \$7,321,000. **9.** \$12,682,000. **10.** \$37,804,000. **11.** \$19,206. **12.** \$216,720. **13.** \$281,700. **14.** \$3,709,450.

Page 31. — **2.** 3,060,144. **3.** 2,938,628. **4.** 3,467,880. **5.** 4,728,528. **6.** 5,316,597. **7.** 7,809,660. **8.** 81,329,636. **9.** 55,190,854. **10.** 791,801,312. **Art. 80.** — **1.** 7,977,782 lb. **2.** 744,611,286 T. **3.** \$7,893,903.78. **4.** \$703,182,420. **5.** 6,140,459,192 bu. **Art. 81.** — **1.** 2,115,827,840.

Page 32. — **2.** \$6240. **3.** 172,800 sec. **4.** 319,088 mi. **5.** 6,213,704. **6.** \$5,983,256. **7.** 384 mi. **8.** 3600. **9.** \$10,112.72. **10.** 2,711,040 lb.

Page 35. — **3.** 80,064 — 3 Rem. **4.** 90,630 — 2 Rem. **5.** 960,365 — 2 Rem. **6.** 193,076 — 4 Rem. **7.** 171,884 — 2 Rem. **8.** 192,960 — 2 Rem. **9.** 42,210 — 1 Rem. **Art. 91.** — **3.** 96,036 — 3¢ Rem. **4.** 103,577 — 3 qt. Rem.

Page 36 — **2.** 1559 sec. **3.** 1168 min. **4.** $1154\frac{5}{8}$ hr. **5.** $631\frac{8}{9}$ da. **6.** $605\frac{4}{7}$ wk. **7.** $1673\frac{4}{5}$ mo. **Art. 93.** — **1.** 104 wk. **2.** 80 pk. **3.** 136 sq. yd. **4.** 108 qt. **5.** \$4.25.

Page 37. — **2.** 465 bbl. **3.** 806 wk. **4.** 5720. **5.** 8800 yd. **6.** \$31.45. **7.** 1793. **8.** 356. **9.** 891.

Page 38. — **2.** 1566 mi. **3.** 6 lb. **4.** 1760. **5.** \$124,508. **6.** 126.

Page 39. — **7.** $386\frac{9}{10}$; $193\frac{9}{20}$; $128\frac{29}{30}$; $96\frac{29}{40}$; $48\frac{29}{80}$. **8.** $796\frac{88}{100}$; $132\frac{488}{600}$; $113\frac{588}{700}$; $79\frac{638}{1000}$; $11\frac{2638}{7000}$. **9.** \$$5\frac{150}{200}$. **10.** 3 lb.

Page 40. — **2.** $102\frac{11}{23}$. **3.** 694. **4.** $1325\frac{74}{127}$. **5.** $104\frac{361}{436}$. **6.** $1419\frac{25}{37}$ yd. **7.** $5166\frac{18}{43}$ mi. **8.** 2409. **9.** 1621. **Art. 100.** — **3.** 735,800. **4.** 843,600. **5.** 5283. **6.** 6475. **7.** 5280 ft. **8.** $279\frac{287}{313}$. **9.** 324 cd. **10.** 2753.

Page 43. — **2.** 2, 2, 2, 5; 2, 3, 3, 3; 2, 2, 2, 3, 3; 2, 2, 3, 7; ~~2, 2, 23~~; 5, 19; 2, 5, 11; 2, 3, 19; 2, 2, 3, 11. **3.** 2, 2, 2, 2, 2, 5; 2, 2, 2, 2, 11; 2, 3, 5, 7; 2, 2, 2, 2, 3, 5; 2, 2, 2, 3, 3, 5; 2, 5, 41; 2, 2, 7, 17; 5, 97. **4.** 2, 3, 5, 17; 2, 2, 2, 2, 2, 2, 3, 3; 2, 2, 2, 3, 29; 2, 2, 3, 73; 3, 3, 3, 5, 7; 2, 2, 3, 5, 19; 2, 2, 2, 2, 2, 2, 2, 2, 5; 2, 3, 3, 5, 7, 13.

Page 44. — **2.** 2. **3.** 2, 3. **4.** 2. **5.** 2. **6.** 2. **7.** 3, 5, 7. **8.** 2, 3, 5, 7. **9.** 2, 5.

Page 45. — **2.** 5. **3.** 5. **4.** 12. **5.** 18. **6.** 42. **7.** 5. **8.** 26. **9.** 6. **10** 120. **11.** 105. **12.** 66. **13.** 5. **14.** 54. **15.** 108. **16.** 144.

Page 46. — **2.** 192. **3.** 144. **4.** 112. **5.** 105. **6.** 148. **7.** 54. **8.** 112. **9.** 72. **10.** 51. **11.** 198.

Page 47. — 1. 12 T. 2. \$7; A, 161; B, 183; C, 206. 3. 14 ft.; 996 panels; 5976 boards.

Page 49. — 2. 60. 3. 144. 4. 280. 5. 288. 6. 600. 7. 88. 8. 720. 9. 756. 10. 1080. 11. 2310.

Page 50. — 3. 576. 4. 504. 5. 576. 6. 2880. 7. 2240. 8. 240. 9. 900. 10. 3580. 11. 5040. 12. 13,200.

Page 51. — 4. 8. 5. 18. 6. 70. 7. $139\frac{1}{2}$. 8. 162. 9. 12. 10. $6\frac{3}{4}$. 11. 1350. 12. 456. 13. 12. 14. 2. 15. 15. 16. 18. 17. 28. 18. 18.

Page 52. — 3. $4\frac{2}{3}$ wk. 4. \$21. 5. 62 lb. 6. 60 bbl. 7. 75 doz. 8. 30 mi.

Page 55. — 2. $1\frac{21}{65}$. 3. $1\frac{15}{72}$. 4. $3\frac{18}{25}$. 5. 3. 6. $2\frac{4}{84}$. 7. $2\frac{36}{89}$. 8. $29\frac{2}{9}$ rd. 9. $4\frac{169}{176}$ ft. 10. $1\frac{429}{462}$ mi. 11. $14\frac{185}{263}$ T. 12. 2 da. 13. $1\frac{2922}{3472}$ hr.

Page 56. — 2. $\frac{91}{4}$. 3. $\frac{806}{11}$. 4. $\frac{473}{15}$. 5. $\frac{707}{18}$. 6. $\frac{1057}{25}$. 7. $\frac{1641}{32}$. 8. $\frac{1202}{19}$. 9. $\frac{2152}{29}$. 10. $\frac{3055}{37}$. 11. $\frac{3418}{39}$. 12. $\frac{3307}{41}$. 13. $\frac{13341}{67}$.
Art. 152. — 2. $\frac{10}{15}$. 3. $\frac{6}{16}$. 4. $\frac{20}{24}$. 5. $\frac{15}{40}$. 6. $\frac{42}{56}$. 7. $\frac{70}{120}$.
Art. 153. — 2. $\frac{15}{36}$; $\frac{28}{36}$. 3. $\frac{54}{60}$; $\frac{9}{60}$. 4. $\frac{64}{72}$; $\frac{33}{72}$. 5. $\frac{27}{39}$. 6. $\frac{52}{84}$. 7. $\frac{195}{205}$.

Page 57. — 2. $\frac{4}{5}$. 3. $\frac{9}{10}$. 4. $\frac{5}{7}$. 5. $\frac{3}{4}$. 6. $\frac{3}{7}$. 7. $\frac{7}{8}$. **Art. 157.** — 2. $\frac{1}{3}$. 3. $\frac{2}{3}$. 4. $\frac{3}{4}$. 5. $\frac{6}{7}$. 6. $\frac{8}{9}$. 7. $\frac{5}{7}$. 8. $\frac{2}{3}$. 9. $\frac{1}{7}$. 10. $\frac{7}{10}$. 11. $\frac{1}{2}$. 12. $\frac{249}{448}$. 13. $\frac{1}{5}$.

Page 58. — 2. $\frac{135}{360}$; $\frac{90}{360}$; $\frac{60}{360}$; $\frac{200}{360}$; $\frac{168}{360}$. 3. $\frac{280}{360}$; $\frac{168}{360}$; $\frac{225}{360}$; $\frac{96}{360}$; $\frac{414}{360}$. 4. $\frac{28}{84}$; $\frac{54}{84}$; $\frac{49}{84}$; $\frac{64}{84}$; $\frac{57}{84}$. 5. $\frac{24}{36}$; $\frac{9}{36}$; $\frac{28}{36}$; $\frac{30}{36}$; $\frac{15}{36}$. 6. $\frac{12}{20}$; $\frac{18}{20}$; $\frac{15}{20}$; $\frac{10}{20}$; $\frac{19}{20}$. 7. $\frac{28}{84}$; $\frac{54}{84}$; $\frac{49}{84}$; $\frac{64}{84}$; $\frac{57}{84}$. 8. $\frac{48}{60}$; $\frac{14}{60}$; $\frac{28}{60}$; $\frac{17}{60}$. 9. $\frac{294}{42}$; $\frac{141}{42}$; $\frac{80}{42}$; $\frac{46}{42}$. 10. $\frac{357}{420}$; $\frac{320}{420}$; $\frac{405}{420}$; $\frac{196}{420}$. 11. $\frac{405}{315}$; $\frac{245}{315}$; $\frac{1785}{315}$; $\frac{2142}{315}$. 12. $\frac{357}{504}$; $\frac{294}{504}$; $\frac{414}{504}$; $\frac{266}{504}$. 13. $\frac{330}{540}$; $\frac{1920}{540}$; $\frac{504}{540}$; $\frac{459}{540}$.

Page 59. — 2. $2\frac{3}{8}$. 3. $1\frac{43}{72}$. 4. $1\frac{5}{36}$. 5. $1\frac{203}{240}$. 6. $1\frac{33}{84}$. 7. $2\frac{55}{96}$. 8. $1\frac{221}{546}$. 9. $1\frac{14}{15}$. 10. $2\frac{15}{112}$. 11. $1\frac{41}{132}$.

Page 60. — 2. $32\frac{1}{3}$. 3. $51\frac{5}{6}$. 4. $49\frac{11}{48}$. 5. $75\frac{143}{420}$. 6. $59\frac{61}{360}$. 7. $195\frac{389}{840}$. 8. $251\frac{29}{45}$. 9. $284\frac{61}{140}$. 10. $291\frac{1}{70}$. 11. $270\frac{59}{252}$. **Art. 165.** — 1. $\frac{41}{40}$ bbl. 2. $3\frac{47}{60}$ T. 3. \$$57\frac{37}{40}$. 4. $372\frac{13}{120}$ A. 5. $1124\frac{15}{16}$ lb. 6. \$$471\frac{79}{80}$.

Page 61. — 7. $105\frac{5}{24}$ yd. 8. $70\frac{7}{12}$ mi. **Art. 166.** — 2. $\frac{13}{36}$. 3. $\frac{22}{63}$. 4. $\frac{2}{21}$. 5. $\frac{3}{55}$. 6. $\frac{1}{4}$. 7. $\frac{1}{9}$. 8. $\frac{1}{50}$. 9. $\frac{1}{2}$. 10. $\frac{1}{62}$. 11. $\frac{50}{111}$. 12. $\frac{265}{354}$. 13. $\frac{243}{560}$. 14. $\frac{65}{96}$. 15. $\frac{1}{2}$. 16. $\frac{38}{165}$. **Art. 167.** — 2. $12\frac{1}{6}$. 3. $17\frac{5}{12}$. 4. $21\frac{5}{13}$. 5. $27\frac{1}{13}$. 6. $31\frac{1}{15}$. 7. $35\frac{1}{19}$. 8. $38\frac{2}{21}$. 9. $45\frac{1}{4}$. 10. $53\frac{2}{41}$. 11. $61\frac{13}{54}$. 12. $70\frac{2}{21}$. 13. $81\frac{14}{75}$.

Page 62. — 2. $30\frac{2}{11}$. 3. $1\frac{6}{13}$. 4. $1\frac{7}{15}$. **Art. 169.** — 2. $28\frac{13}{24}$. 3. $45\frac{12}{13}$. 4. $58\frac{1}{2}$. **Art. 170.** — 2. $20\frac{9}{22}$. 3. $28\frac{19}{77}$. 4. $15\frac{1}{18}$. 5. $3\frac{5}{21}$. 6. $55\frac{7}{32}$. 7. $34\frac{31}{49}$. 8. $36\frac{89}{126}$. 9. $16\frac{19}{72}$. 10. $50\frac{19}{56}$. 11. $\frac{88}{99}$. **Art. 171.** — 1. $\frac{197}{360}$. 2. $4\frac{19}{24}$. 3. $14\frac{23}{77}$. 4. $55\frac{5}{42}$. 5. $103\frac{49}{60}$. 6. $17\frac{1}{7}$. 7. $84\frac{13}{30}$. 8. $174\frac{17}{27}$. **Art. 172.** — 1. $105\frac{11}{12}$ bu. 2. $31\frac{5}{72}$ yd.

Page 63. — 3. $1220\frac{11}{12}$ bu. 4. $130\frac{103}{240}$ A. 5. \$$6395\frac{7}{10}$. 6. $\frac{7}{40}$. 7. $72\frac{7}{25}$ cwt. 8. $7\frac{1}{16}$ gal. cider; $24\frac{7}{16}$ gal. water.

Page 64. – 3. $8\frac{2}{5}$. 4. $19\frac{1}{2}$. 5. $30\frac{6}{7}$. 6. $2\frac{1}{2}$. 7. $16\frac{1}{3}$. 8. $5\frac{1}{3}$. 9. $23\frac{3}{9}$. 10. $3\frac{5}{7}$. 11. $26\frac{1}{4}$. 12. $10\frac{7}{8}$. 13. $50\frac{2}{3}$. 14. $66\frac{13}{15}$. 15. $62\frac{1}{2}$. 16. $75\frac{1}{3}$. 17. $26\frac{7}{18}$.

Page 65. — 2. $345\frac{1}{3}$. 3. 516. 4. $2695\frac{1}{2}$. 5. $2326\frac{1}{2}$. 6. 3815. 7. $2763\frac{1}{4}$. 8. $3164\frac{1}{2}$. 9. 4960. 10. $11,489\frac{1}{3}$. 11. $35,601\frac{1}{2}$.

Art. 175. — 1. $46\frac{2}{3}$ bbl. 2. 2079 cu. ft. 3. $52\frac{1}{2}$ P. 4. $\$370\frac{1}{2}$. 5. $269\frac{1}{4}$ bu. 6. 16 T. 7. 400 yd. 8. \$64.40. 9. 513 mi. 10. $29\frac{4}{5}$¢.
Page 66. — 2. 27. 3. 40. 4. $22\frac{1}{2}$. 5. $67\frac{5}{6}$. 6. $76\frac{1}{2}$. 7. $58\frac{1}{2}$. 8. $155\frac{5}{9}$. 9. $399\frac{1}{2}$. 10. $390\frac{1}{2}$. **Art. 177.** — 1. \$128. 2. 48 mi. 3. 710. 4. 26 bu. 5. 20 A.
Page 67. — 2. 306. 3. 924. 4. $10{,}234\frac{2}{3}$. 5. 21,879. 6. 27,202. 7. 16,488. 8. $43{,}357\frac{1}{11}$. 9. 68,325. 10. $77{,}971\frac{1}{4}$. **Art. 179.** — 1. 58 A. 2. 616 lb. 3. $109\frac{3}{8}$ mi. 4. \$11.97. 5. \$242.40. 6. 2938 mi. 7. \$5025. 8. 1323 bu. 9. $\$199.05\frac{7}{12}$. 10. $\$273.50\frac{2}{5}$.
Page 68. — 2. $\frac{6}{11}$. 3. $\frac{11}{14}$. 4. $\frac{2}{7}$. 5. $\frac{4}{9}$. 6. $\frac{2}{9}$. 7. $\frac{1}{2}$. 8. $\frac{96}{425}$. 9. $\frac{1003}{5785}$. 10. $\frac{1}{6}$. 11. $\frac{217}{1902}$. 12. $\frac{21}{25}$. 13. $\frac{1}{2}$. 14. $\frac{2}{5}$. 15. $\frac{25}{66}$. 16. $\frac{9}{32}$.
Page 69. — 2. $6\frac{62}{75}$. 3. $17\frac{114}{147}$. 4. $13\frac{109}{160}$. 5. $11\frac{1}{10}$. 6. $36\frac{18}{35}$. 7. $51\frac{31}{51}$. 8. $2\frac{62}{63}$. 9. $36\frac{1}{3}$. 10. $48\frac{13}{48}$. **Art. 182.** — 2. $82\frac{24}{35}$. 3. $118\frac{8}{21}$. 4. $235\frac{17}{33}$. 5. $325\frac{31}{40}$. 6. $502\frac{1}{12}$. 7. $641\frac{1}{3}$. 8. $104\frac{23}{45}$. 9. 90. 10. $239\frac{11}{56}$. **Art. 183.** — 1. $\$196\frac{7}{8}$. 2. $\$12.12\frac{1}{2}$. 3. \$273.80. 4. $\$136\frac{4}{5}$. 5. $\$18.83\frac{2}{3}$. 6. \$18.13. 7. \$318.75. 8. \$21.56. 9. $\$9.16\frac{7}{8}$. 10. $\$7.21\frac{3}{5}$. 11. $\$3.81\frac{1}{4}$. 12. $\$11.68\frac{3}{4}$. 13. \$24. 14. $\$11.58\frac{3}{4}$.
Page 70. — 2. $\frac{3}{16}$. 3. $\frac{2}{17}$. 4. $\frac{2}{19}$. 5. $\frac{4}{23}$. 6. $\frac{3}{25}$. 7. $\frac{5}{32}$. 8. $\frac{4}{51}$. 9. $\frac{1}{72}$. 10. $\frac{1}{172}$. 11. $\frac{7}{4212}$. 12. $\frac{5}{259}$. 13. $\frac{1}{287}$. **Art. 185.** — 2. $\frac{7}{32}$. 3. $1\frac{3}{98}$. 4. $1\frac{2}{7}$. 5. $2\frac{5}{189}$. 6. $3\frac{3}{301}$. 7. $4\frac{383}{459}$. 8. $5\frac{123}{176}$. 9. $5\frac{157}{162}$.
Page 71. — 1. $87\frac{21}{32}$ mi. 2. $20\frac{187}{280}$ ft. 3. $589\frac{17}{32}$ gal. 4. $35\frac{3}{4}$ bu. 5. 104 yd. 6. $39\frac{3}{8}$ qt. 7. $27\frac{5}{8}$ gal. 8. $\$\frac{11}{36}$. **Art. 187.** — 2. 24. 3. 24. 4. 28. 5. $29\frac{1}{4}$. 6. $33\frac{1}{3}$. 7. $69\frac{3}{5}$. 8. $64\frac{5}{13}$. 9. $109\frac{2}{3}$. 10. $166\frac{10}{11}$. 11. $117\frac{19}{41}$. 12. 365. 13. 285.
Page 72. — 2. $2\frac{6}{7}$. 3. $2\frac{32}{33}$. 4. $3\frac{1}{85}$. 5. $2\frac{25}{28}$. 6. $3\frac{1}{16}$. 7. $3\frac{1}{13}$. 8. $2\frac{46}{49}$. 9. $2\frac{22}{25}$. 10. $2\frac{53}{56}$. 11. $2\frac{28}{41}$. 12. $8\frac{67}{101}$. 13. $34\frac{11}{28}$. **Art. 189.** — 2. $1\frac{7}{15}$. 3. $1\frac{19}{20}$. 4. $\frac{5}{9}$. 5. $4\frac{4}{7}$. 6. $\frac{20}{27}$. 7. $1\frac{5}{9}$. 8. $1\frac{7}{8}$. 9. $1\frac{29}{55}$. 10. $2\frac{5}{57}$. 11. $1\frac{18}{25}$. 12. 8. 13. $\frac{55}{56}$.
Page 73. — 2. $7\frac{1}{3}$. 3. $8\frac{2}{5}$. 4. $9\frac{1}{3}$. 5. 12. 6. 55. 7. 62. 8. $48\frac{20}{21}$. 9. $109\frac{11}{16}$. 10. $109\frac{1}{5}$. **Art. 191.** — 2. $4\frac{5}{28}$. 3. $1\frac{199}{200}$. 4. $1\frac{61}{129}$. 5. $\frac{87}{94}$. 6. $6\frac{27}{44}$. 7. $\frac{211}{352}$. 8. $1\frac{171}{265}$. 9. $7\frac{3}{25}$. 10. $2\frac{23}{44}$. 11. $1\frac{304}{585}$. 12. $2\frac{7}{88}$. 13. $2\frac{77}{86}$. 14. $4\frac{23}{32}$. 15. $2\frac{47}{183}$. 16. $1\frac{157}{225}$.
Page 74. — 3. $4\frac{2}{7}$. 4. 1. 5. 16. 6. $18\frac{2}{3}$. 7. 2. **Art. 193.** — 1. $1401\frac{1}{7}$. 2. $20\frac{19}{36}$ mi. 3. 80 A. 4. $\$779.16\frac{2}{3}$. 5. $\$50\frac{1}{2}$. 6. $\$2675\frac{11}{15}$. 7. Incr. $\frac{1}{56}$. 8. Dim. $\frac{1}{42}$. 0. Unchanged. 10. No effect.
Page 75. — 11. $26\frac{58}{87}$. 12. $\frac{478}{881}$. 13. 54 mi. 14. $\$199.37\frac{11}{48}$.
Page 76. — 2. $\frac{3}{8}$. 3. $\frac{7}{18}$. 4. $\frac{12}{41}$. 5. $\frac{28}{79}$. 6. $\frac{1}{3}$. 7. $\frac{7}{24}$. 8. $\frac{7}{20}$. 9. $\frac{8}{21}$. 10. $\frac{8}{21}$. 11. 5 times; 4 times. 12. 8 times; 2 times. 13. $10\frac{1}{2}$ times; $3\frac{1}{2}$ times. 14. $1\frac{1}{2}$ times; $1\frac{1}{3}$ times. 15. $1\frac{3}{4}$ times; $1\frac{2}{3}$ times. 16. \$4200. 17. \$22.10. 1C. $\frac{5}{9}$. 19. \$6300. **Art. 196.** — 2. $\frac{2}{171}$. 3. $\frac{1}{180}$. 4. $\frac{1}{130}$. 5. $\frac{10}{1887}$. 6. $\frac{11}{448}$. 7. $\frac{2}{15}$. 8. $\frac{1}{225}$; $\frac{2}{315}$. 9. 6 times; $4\frac{4}{5}$ times. 10. 8¢. 11. $\$3\frac{9}{22}$.
Page 77. — 2. $\frac{1}{3}$. 3. $\frac{27}{20}$. 4. $\frac{3}{2}$. 5. $\frac{5}{21}$. 6. $\frac{4}{243}$. 7. $\frac{5}{158}$. 8. $\frac{14}{51}$. 9. $\frac{189}{670}$. 10. $\frac{1428}{6655}$. 11. $2\frac{4}{5}$ times; $2\frac{4}{7}$ times. 12. $\frac{5}{8}$. **Art. 198.** — 1. $\frac{1}{3}$. 2. $\frac{3}{4}$. 3. $\$187\frac{1}{2}$. 4. 8 da. 5. 11. 6. 6 da. 7. 105 bu. 8. $\frac{153}{4096}$.
Page 78. — 9. \$1924. 10. $106\frac{14}{33}$ rd. 11. \$55.20. 12. 85 T. 13. 96 bu. 14. $54\frac{4}{9}$. 15. 59 ft. 16. $\$68\frac{3}{4}$. 17. $\$134\frac{3}{5}$. 18. $\$3413\frac{1}{3}$. 19. $41\frac{1}{7}$ A. 20. 200 mi. 21. $64\frac{4}{5}$ mi.

Page 79. — **22.** \$ $2\frac{3}{25}$. **23.** \$ 55. **24.** 18 yd. **25.** \$ 1200. **26.** $17\frac{1}{3}$ mi. **27.** $98\frac{4}{5}$ yd. **28.** $77\frac{1}{3}$ T. **29.** $531\frac{20}{27}$ bu. **30.** $15\frac{5}{8}$ ft.
Art. 199. — **1.** $\frac{1}{3}$. **2.** \$ $\frac{1}{3}$. **3.** 15.
Page 80. — **4.** $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$. **5.** 60 da. **6.** $\frac{1}{5}$; $\frac{3}{5}$. **7.** 5 hr. **8.** 10. **9.** \$ 10. **10.** $\frac{5}{13}$. **11.** \$ $\frac{3}{20}$. **12.** \$ $5\frac{1}{4}$. **13.** \$ 30. **14.** $\frac{7}{40}$. **15.** 640. **16.** \$ 250; \$ 100; \$ 90.
Page 81. — **17.** A $\frac{1}{4}$ A.; D $\frac{1}{5}$ A.; $2\frac{2}{3}$ da.; $4\frac{2}{3}$ da. **18.** $1\frac{1}{3}$ da. **19.** 150 lb. **20.** $\frac{3}{5}$. **21.** \$ $8\frac{1}{2}$. **22.** 30 da. **23.** \$ 6400. **24.** Sallie, 45; Hattie, 30. **25.** $\frac{1}{3}$ da.; $1\frac{1}{3}$ da.; $1\frac{2}{3}$ da. **26.** 36 ¢.
Page 82. — **27.** 45 larger; 9 smaller. **28.** 48 larger; 36 smaller. **29.** \$ $28\frac{4}{5}$. **30.** 60. **31.** $117\frac{3}{20}$ A. **32.** \$ $1122\frac{5}{9}$. **33.** $15\frac{1}{2}$ rd. **34.** $31\frac{1}{2}$ rd. **35.** \$ $4\frac{33}{100}$. **36.** 108 mi. **37.** $12\frac{1}{10}$ A.
Page 83. — **38.** \$ $2587\frac{1}{2}$; \$ $1138\frac{1}{2}$ Fissel; \$ 1449 Hartman. **39.** \$ 540 A; \$ $472\frac{1}{2}$ B. **40.** $1962\frac{54}{113}$ times. **41.** $254\frac{5}{92}$. **42.** $8\frac{11}{80}$ mi. **43.** $\frac{1}{16}$. **44.** \$ $12.01\frac{1}{5}$. **45.** 210. **46.** 138. **47.** 340 bu. **48.** \$ $524\frac{2}{3}$.
Page 84. — **49.** $603\frac{1}{3}$. **50.** $647\frac{1}{7}$. **51.** 490 bu.; 1323 bu. **52.** \$ 11,968; \$ 17,952. **53.** $75\frac{3}{4}$ bu. **54.** $184\frac{4}{5}$ mi. **55.** 540. **56.** $15\frac{9}{19}$ da. **57.** \$ 3.08. **58.** $8\frac{38}{39}$ hr. **59.** $3\frac{1}{3}$ da.
Page 85. — **60.** $\frac{5}{11}$. **61.** $\frac{12}{31}$ da. **62.** Corn, $56\frac{4}{11}$ ¢; rye, $72\frac{4}{11}$ ¢. **63.** $15\frac{15}{31}$ da. **64.** 210 bu. **65.** $\frac{11}{20}$. **66.** $3\frac{9}{37}$ da. **67.** 20 mi. **68.** 400 ft. **69.** James, $14\frac{13}{16}$ mi.; John, $17\frac{9}{16}$ mi. **70.** \$ $142\frac{1}{2}$.
Page 87. — **1.** 24. **2.** 26. **3.** 57. **4.** 72. **5.** 126. **6.** 50. **7.** 99. **8.** 61. **9.** 40. **10.** 28. **11.** 54. **12.** $2\frac{14}{25}$. **13.** $298\frac{1}{2}$. **14.** 44. **15.** $47\frac{77}{125}$. **16.** 3. **17.** 16. **18.** 28. **19.** 440. **20.** $45\frac{11}{12}$. **21.** $\frac{9409}{142296}$. **22.** $\frac{21}{125}$. **23.** 1.
Page 90. — **1.** .5; .6; .7; .8; .9. **2.** .02; .08; .25; .32; .89. **3.** .008; .019; .086; .126; .496. **4.** .0006; .0098; .0267; .0938; .1639. **5.** .00005; .00072; .00396; .05689; .72896. **6.** .000007; .000065; .000389; .005968; .098645. **Art. 211.** — **1.** $\frac{3}{10}$; $\frac{4}{10}$; $\frac{7}{100}$; $\frac{22}{100}$; $\frac{98}{100}$. **2.** $\frac{5}{1000}$. $\frac{63}{1000}$; $\frac{198}{1000}$; $\frac{105}{1000}$; $\frac{896}{1000}$. **3.** $\frac{7}{10000}$; $\frac{67}{10000}$; $\frac{159}{10000}$; $\frac{1061}{10000}$; $\frac{9006}{10000}$; **4.** $\frac{5}{100000}$; $\frac{56}{100000}$; $\frac{906}{100000}$; $\frac{1601}{100000}$; $\frac{96019}{100000}$. **5.** $\frac{9}{100000}$; $\frac{86}{1000000}$; $\frac{106}{1000000}$; $\frac{1709}{1000000}$; $\frac{15657}{1000000}$.
Page 91. — **1.** 5.3. **2.** 8.7 **3.** 12.03. **4.** 27.47. **5.** 146.007. **6.** 329.017. **7.** 416.0128. **8.** 169.0329. **9.** 874.05384. **10.** 729.072064.
Art. 216. — **1.** $8\frac{4}{10}$. **2.** $9\frac{8}{10}$. **3.** $7\frac{7}{100}$. **4.** $29\frac{36}{100}$. **5.** $84\frac{2}{1000}$. **6.** $92\frac{81}{1000}$. **7.** $124\frac{107}{1000}$. **8.** $167\frac{101}{10000}$. **9.** $246\frac{1809}{10000}$. **10.** $396\frac{6041}{100000}$.
Page 92. — 21.21; 100.024; .124; .0384; 300.0084; 1000.00008; .01008; .000092; .009009; 9000.000009; 29.03346; 8.070054; 17.0090009; 83.0156.
Page 93. — **2.** $\frac{9}{20}$. **3.** $\frac{12}{25}$. **4.** $\frac{11}{20}$. **5.** $\frac{3}{4}$. **6.** $\frac{1}{8}$. **7.** $\frac{9}{200}$. **8.** $\frac{1}{125}$. **9.** $\frac{21}{40}$. **10.** $\frac{13}{2000}$. **11.** $\frac{169}{20000}$. **Art. 222.** — **2.** $7\frac{1}{4}$. **3.** $8\frac{7}{20}$. **4.** $9\frac{17}{20}$. **5.** $16\frac{1}{8}$. **6.** $18\frac{7}{250}$. **7.** $25\frac{1}{125}$. **8.** $36\frac{13}{40}$. **9.** $41\frac{9}{2000}$. **10.** $84\frac{3}{5000}$. **11.** $93\frac{203}{2000}$.
Art. 223. — **2.** $\frac{5}{8}$. **3.** $\frac{1}{16}$. **4.** $\frac{1}{12}$. **5.** $\frac{1}{8}$. **6.** $\frac{1}{6}$. **7.** $\frac{1}{3}$. **8.** $\frac{5}{8}$. **9.** $\frac{2}{5}$. **10.** $\frac{7}{8}$. **11.** $\frac{17}{80}$. **12.** $\frac{13}{30}$. **13.** $\frac{7}{80}$.
Page 94. — **2.** $9\frac{11}{30}$. **3.** $7\frac{5}{12}$. **4.** $8\frac{7}{8}$. **5.** $19\frac{2}{7}$. **6.** $27\frac{33}{400}$. **7.** $13\frac{1}{18}$
Art. 225. — **2.** .5. **3.** .2. **4.** .25. **5.** .75. **6.** $.12\frac{1}{2}$. **7.** $.62\frac{1}{2}$. **8.** $.87\frac{1}{2}$. **9.** .35. **10.** $.37\frac{1}{2}$. **11.** .025. **12.** .525. **13.** .36. **14.** .648. **15.** .0275. **16.** .9375. **17.** .404. **18.** .3525. **19.** .342. **20.** .54. **21.** $.4861\frac{1}{9}$. **22.** $.1923\frac{1}{13}$.
Page 95. — **2.** 331.65. **3.** 538.8324. **4.** 276.7743. **5.** 17.37525. **6.** \$ $121.70\frac{5}{6}$. **7.** 1.029162.

Page 96. — **8.** 250.5317. **9.** $1076.0763\frac{5}{8}$. **Art. 229.** — **2.** 14.03. **3.** 29.0899. **4.** 121.055. **5.** 144.2 **6.** 1.343. **7.** 8.554. **8.** 12.1261. **9.** .0015. **10.** .10001. **11.** 1.08999. **12.** .77499764. **13.** 9.39905. **14.** .08445.

Page 97. — **2.** .3224. **3.** 40.227. **4.** .01208. **5.** .00202. **6.** 849.849. **7.** 64.6813. **8.** 9.6515016. **9.** .18724. **10.** .0027279. **11.** .0011. **12.** .0045. **13.** .048375. **14.** .01002. **15.** 4.6225. **16.** .000125.

Page 99. — **1.** 68.35. **2.** 68.35. **3.** 6.3069+. **4.** .00305+. **5.** 6.7814+. **6.** .067814. **7.** .6545+. **8.** 654.566+. **9.** 44.437. **10.** .004443+. **11.** 750. **12.** 30,000. **13.** 6,250,000. **14.** 1200. **15.** .003199. **16.** .06857. **17.** .0795+. **18.** 38.4. **19.** $33.3\frac{1}{3}$. **20.** $.133\frac{1}{3}$. **21.** 3.5321+. **22.** .0472+. **23.** 68.583+. **24.** 88,750.

Page 100. — **1.** .002325. **2.** .55575. **3.** 12.09375. **4.** 109.375. **5** 1.97174. **6.** .05139375. **7.** .001275. **8.** .1050625. **9.** 112. **10.** .0000405. **Art. 240.** — **1.** 35 bu. **2.** $\$12\frac{1}{2}$. **3.** 42. **4.** \$878.85. **5.** $\$38.156\frac{1}{4}$.

Page 101. — **2.** $\$33.65\frac{1}{4}$. **3.** $\$73.25\frac{1}{3}$. **4.** \$214.045. **5.** \$34.357. **6.** \$3.25. **Art. 242.** — **2.** \$111.926. **3.** \$248.70. **4.** \$17.59. **5.** 7400 ft.

Page 102. — **2.** \$69.225. **3.** \$146. **4.** \$69.46. **Art. 244.** — **1.** \$1472. **2.** 10,000 lb. **3.** \$1147.50. **4.** $.14\frac{2}{7}$. **5.** 10.5 bu. **6.** 92¢. **7.** $\$2.23\frac{1}{3}$. **8.** 24.684 T.

Page 103. — **9.** \$1.48. **10.** \$.109. **11.** 88.8 bu. **12.** 60 bu. **13.** 1500 bu.; \$498.75. **14.** 45 bu. **15.** \$5080.50 A; \$2540.25 B; \$1693.50, C. **16.** 88 da., Johnson; 33 da., White.

Page 106. — **1.** $\$42.86\frac{3}{4}$. **2.** \$148.074+. **3.** \$1821.25.

Page 107. — **4.** \$1266.646.

Page 108. — **2.** $\$4.89\frac{1}{2}$ due Jones. **Art. 255.** — **2.** \$430.765 due Wilson.

Page 109. — **1.** \$50.22. **2.** \$5.15.

Page 111. — **2.** .04. **3.** .08. **4.** .11. **5.** .25. **6.** .36. **7.** .43. **8.** .47. **9.** .54. **10.** .60. **11.** .72. **12.** .83. **13.** .98.

Page 112. — **2.** 8%. **3.** 9%. **4.** 14%. **5.** 21%. **6.** 64%. **7.** 76%. **8.** 85%. **9.** 92%. **Art. 268.** — **2.** .0625. **3.** $.08\frac{1}{3}$. **4.** .125. **5.** $.16\frac{2}{3}$. **6.** .1475. **7.** .22125. **8.** .46375. **9.** .548. **Art. 269.** — **2.** $37\frac{1}{2}$%. **3.** $8\frac{2}{5}$%. **4.** $6\frac{3}{4}$%. **5.** $16\frac{1}{3}$%. **6.** $8\frac{2}{3}$%. **7.** $12\frac{2}{14}$%. **Art. 270.** — **2.** .0075. **3.** .00375. **4.** .0016. **5.** $.008\frac{1}{3}$. **6.** $.007\frac{7}{9}$. **7.** .001875. **8.** .00625. **9.** .004375. **Art. 271.** — **2.** 1.32. **3.** 1.75. **4.** 1.85. **5.** 1.68. **6.** 1.375. **7.** 1.186. **8.** $2.62\frac{3}{7}$. **9.** $1.08\frac{1}{3}$.

Page 113. — **2.** 60. **3.** 32.6. **4.** 51.48. **5.** $40.17\frac{3}{4}$ mi. **6.** 175. **7.** $\$563.73\frac{1}{3}$. **8.** $\$428.06\frac{1}{4}$. **9.** 241.23 ft.

Page 114. — **2.** \$9180. **3.** \$2437.50.

Page 115. — **4.** \$2250. **5.** \$510. **6.** $\$3333\frac{1}{3}$. **7.** \$8.

Page 116. — **8.** \$5. **9.** \$450.

Page 117. — **2.** 1450. **3.** 357. **4.** 291.40625. **5.** $231\frac{6}{49}$. **6.** $103\frac{1}{3}$. **7.** $712\frac{87}{124}$. **8.** \$7280. **9.** 43,200 T; 60,480 T. **10.** \$65,000.

Page 118. — **2.** 490. **3.** 765. **4.** 400. **5.** $\frac{2}{9}$. **Art. 282.** — **2.** 5625. **3.** 6382. **4.** 8200. **5.** 36 mi. 126 rd. $13\frac{1}{2}$ ft. **6.** 93 mi. **7.** \$59.50. **8.** \$22. **Art. 283.** — **2.** $33\frac{1}{3}$%. **3.** 15%. **4.** $12\frac{1}{2}$%. **5.** 40%. **6.** $83\frac{1}{3}$%. **7.** $12\frac{1}{2}$%.

Page 119. — **8.** 42%. **9.** $48\frac{1}{2}$%. **10.** 32%. **11.** 48%. **12.** 20%. **13.** 80%. **14.** 20%. **15.** $60\frac{1}{2}$%. **16.** 10%. **Art. 285.** — **1.** \$1.10; \$6.60.

Page 120. — **2.** 20%. **3.** \$5.50. **4.** \$2370. **5.** $9\frac{1}{5}$%. **6.** \$16.80. **7.** \$72. **8.** \$8750. **9.** \$18. **10.** \$75.

Page 121. — **11.** 86¢. **12.** 25%. **13.** 25%. **14.** 60¢. **15.** \$49.

Page 122. — **2.** \$258. **3.** \$195.80. **4.** \$126.87½. **5.** \$102.03. ART. **292.** — **2.** \$84.82½. **3.** \$79.86. **4.** \$217.35. **5.** \$163.778. **6.** \$156.957. ART. **293.** — **1.** \$180.20. **2.** \$190. **3.** \$80.095. **4.** \$874.994. **5.** \$1097.19.

Page 125. — **1.** 272 ft. **2.** 426½ ft. **3.** 10,766 ft. **4.** 37,728 ft. **5.** 48,913½ ft. ART. **305.** — **1.** 1,346,580 in. **2.** 24,834 in. **3.** 1,778,834 in. **4.** 2,547,184 in. **5.** 3,249,144 in. ART. **308.** — **1.** 1 mi. 66 rd. 3 yd. 2 ft. **2.** 8 mi. 110 rd. **3.** 30 mi. 84 rd. **4.** 16 mi. 312 rd. 5 yd. **5.** 1 mi. 48 rd. 1 in. **6.** 194 rd. 1 yd. 1 ft. 9 in. **7.** 217 rd. 3 yd. 1 ft. 6 in. **8.** 7 mi. 103 rd. 4 yd. 6 in. **9.** 388 mi. 152 rd. 5 yd. **10.** 6 mi. 68 rd. 1 ft. 5 in. **11.** 6 mi. 22 rd. 4 yd. 2 ft. 6 in. **12.** 10 mi. 42 rd. 4 yd. 11 in. **13.** 144 mi. 242 rd. 5 yd. **14.** 160 mi. 219 rd. 5 yd. 6 in. **15.** 184 mi. 81 rd. 2 yd. 1 ft. 6 in.

Page 126. — **2.** 58 rd. 5 yd. 2 ft. 2 in. **3.** 99 rd. 5 yd. 9 in. **4.** 349 rd. 6 ft. 8 in. **5.** 137 mi. 239 rd. 3 yd. **6.** 80 mi. 37 rd. 1 yd. 2 ft. 11 in.

Page 127. — **2.** 15 rd. 11 ft. 8 in. **3.** 62 rd. 3 yd. 4 in. **4.** 30 mi. 210 rd. 4 yd. 1 ft. 4 in. **5.** 51 mi. 10 rd. 1 yd. 3 in. **6.** 87 mi. 2 yd. 2 ft. 1 in.

Page 128. — **2.** 442 mi. 229 rd. 5 ft. 3 in. **3.** 790 mi. 164 rd. 2 yd. **4.** 2535 mi. 38 rd. 2 yd. 1½ ft. **5.** 383 mi. 219 rd. 6 ft. 11 in. **6.** 1312 rd. 1 ft. 6 in. **7.** 506 mi. 5 rd. 2 ft. 9 in. **8.** 8394 mi. 13 rd. 1 ft. 6 in. **9.** 4196 rd. 4 yd. 4 in. **10.** 5057 mi. 21 rd. 1 yd. 1 ft. 6 in. **11.** 26,611 mi. 170 rd. 1 ft. **12.** 21,381 mi. 309 rd. 8 ft. 2 in. **13.** 11,182 rd. 3 yd. 2 ft. 2 in.

Page 129. — **2.** 7 rd. 4 yd. 1 ft. **3.** 27 yd. 1 ft. $8\frac{7}{11}$ in. **4.** 4 rd. 9 ft. $5\frac{3}{4}$ in. **5.** 7 mi. 22 rd. 15 ft. $5\frac{5}{7}$ in. **6.** 15 mi. 266 rd. 3 yd. $3\frac{5}{6}$ ft. **7.** 23 rd. 5 yd. 1 ft. $3\frac{1}{2}$ in. ART. **313.** — **2.** 21 rd. 4 yd. 2 ft. $9\frac{17}{20}$ in. **3.** 12 mi. 168 rd. $11\frac{21}{26}$ ft.

Page 130. — **4.** 51 rd. $13\frac{1}{2}$ ft. **5.** 236 rd. $13\frac{13}{25}$ ft. ART. **314.** — **2.** 8 hr. **3.** 49,500.

Page 131. — **1.** 40 rd. **2.** 213 rd. 5 ft. 6 in. **3.** 288 rd. **4.** 240 rd. **5.** 60 rd. **6.** 4 yd. 1 ft. $2\frac{2}{5}$ in. **7.** 4 yd. 2 ft. $5\frac{1}{4}$ in. **8.** 91 rd. 2 yd. 1 ft. $\frac{6}{7}$ in. ART. **319.** — **2.** 144 rd. 5 yd. 1 ft. $\frac{21}{25}$ in. **3.** 249 rd. 2 yd. $\frac{16}{35}$ in. **4.** 200 rd. 1 yd. 2 ft. 3 in.

Page 132. — **1.** 3 mi. 47 rd. 13 ft. **2.** 13 mi. 282 rd. 6 ft. 3 in. **3.** 8 mi. 127 rd. 1 ft. $2\frac{2}{5}$ in. **4.** 248 rd. 1 ft. 10 in. **5.** 2 mi. 305 rd. 10 ft. 3.75 in. ART. **323.** — **2.** $\frac{1}{2}$ rd. **3.** $\frac{2}{3}$ mi. **4.** $\frac{23}{10560}$ mi. **5.** $\frac{94}{99}$. **6.** $\frac{249}{440}$. **7.** $\frac{58291}{126720}$ mi. **8.** $\frac{2077}{5280}$ mi. **9.** $\frac{797}{3960}$. **10.** $\frac{1}{8}$. **11.** $\frac{859}{2640}$.

Page 133. — **2.** $.43\frac{2}{11}$ rd. **3.** .225 rd. **4.** .145 mi. **5.** .435 mi. **6.** .277 mi. ART. **325.** — **2.** $\frac{3}{4}$ in. **3.** $\frac{2}{3}$ ft. **4.** $\frac{5}{6}$ in. **5.** $\frac{4}{5}$ ft. **6.** $\frac{5}{6}$ in.

Page 134. — **2.** $\frac{1}{2200}$ mi. **3.** $\frac{1}{22}$ rd. **4.** $\frac{1}{25344}$. **5.** $\frac{7}{288}$ yd. **6.** $\frac{35}{198}$ rd. **7.** .11 yd. **8.** $\frac{7}{50688}$ mi.

Page 135. — **1.** 23931 sq. in. **2.** 5600 sq. ft. **3.** 389,560½ sq. ft. **4.** 100 sq. yd. **5.** 4 A. 140 P.; 25 sq. yd. 2 sq. ft. **6.** 93 sq. rd. 10 sq. yd. 108 sq. in. **7.** 36 sq. rd. **8.** 373 A. 53 sq. rd. 10 sq. yd. 108 sq. in. **9.** 119 sq. rd. 29 sq. yd. 5 sq. ft. 72 sq. in. **10.** $2.92\frac{23}{40}$ A.

Page 136. — **11.** 356 rd. **12.** 2100 sq. ft. **13.** 550 sq. yd. **14.** 240 yd. **15.** \$45.36. **16.** 32 rd. **17.** \$548,800.

Page 137. — **1.** 51.2 A. **2.** 2 sq. rd. **3.** 264 sq. rd. **4.** 410.36 rd.

5. 21 A. 140 P. **6.** 207.501+ mi. **7.** 7 mi. **8.** 93 ft. **9.** $5\frac{1}{6}$ ft. **10.** $197.60. **11.** $\frac{1}{3}$.

Page 138.—**12.** 3 rd. 5 ft. 6 in. **13.** $514.50. **14.** 127.06\frac{1}{2}$. **15.** $114.51. **16.** $30.09.

Page 139.—**1.** 80 sq. ft. **2.** $41\frac{1}{4}$ sq. yd. **3.** 128 sq. ft. **4.** 16 rd. **5.** 31 ch.

Page 140.—**6.** 98.9604 in. **7.** $4\frac{1}{2}$ ft. **8.** 12.732 A. **9.** 50.928 A. **10.** 7.07 sq. ft. **11.** 502.656 A. **12.** 9.5+ A. **13.** 64.456 A.

Page 141.—**1.** 862272 cu. in. **2.** 505416 cu. in. **3.** 18 cu. yd. 4 cu. ft. 259 cu. in. **4.** 32 cu. yd. 15 cu. ft. 664 cu. in.

Page 142.—**5.** 256. **6.** 17 cu. yd. 17 cu. ft. 187 cu. in. **7.** .799+ cu. yd. **8.** $\frac{1}{4}$ cu. yd. **9.** $5\frac{1}{2}$ ft.

Page 143.—**2.** $9\frac{1}{2}$ cd. **3.** $585. **4.** $6.31. **5.** $316\frac{2}{3}$ ft.

Page 144.—**2.** 20 bd. ft. **3.** 35 bd. ft. **4.** 900 bd. ft. **5.** 600 bd. ft.

Page 145.—**8.** 810 bd. ft. **9.** $114\frac{1}{3}$ bd. ft. **10.** $426\frac{2}{3}$ bd. ft. **11.** 1152 bd. ft. Art. **366.**—**1.** $56.32.

Page 146.—**2.** 972.22\frac{2}{9}$. **3.** $9.42. **4.** $6.28. **5.** $144. **6.** $106\frac{2}{3}$ P. **7.** 11,340. **8.** 50,337 bricks. **9.** 41,292.

Page 147.—**1.** 1659 pt. **2.** 60 gal. 1 pt. **3.** 7 gal. 2 qt. 1 pt. **4.** 4 bu. 4 qt. $1\frac{3}{5}$ pt. **5.** 8.

Page 148.—**6.** $212.19. **7.** 365 da. **8.** 24 gal. 3 qt. 1 pt. **9.** 5 bu. $3\frac{1}{5}$ qt. Art. **375.**—**1.** $115\frac{1}{5}$ bu. **2.** $92\frac{4}{25}$ bu. **3.** $587\frac{1}{2}$ cu. ft.

Page 149.—**4.** 1468.698 gal. **5.** $52\frac{4}{5}$ bu. oats; $42\frac{6}{25}$ bu. apples. **6.** 42.11 cu. ft. **7.** 13 ft. **8.** $272. **9.** 563.98 gal. **10.** 6 ft.

Page 150.—**1.** 4271 gr. **2.** 84,288 gr. **3.** 14 lb. 8 oz. 13 pwt. 1 gr. **4.** 16 lb. 9 oz. 15 pwt. 3 gr. **5.** 7 lb. 4 oz. 9 pwt. 8 gr. **6.** 64.21\frac{1}{3}$. **7.** 10 oz. 13 pwt. 6 gr. **8.** $\frac{4}{5}$ lb. **9.** 2 oz. 12 pwt. 11 gr. **10.** .23125 lb.

Page 151.—**1.** 29,545 oz. **2.** 79,983 lb. **3.** 98,952 oz. **4.** 5 cwt. 60 lb. 4 oz. **5.** 9 T. 6 cwt. 94 lb. **6.** 16 cwt. 66 lb. $10\frac{2}{3}$ oz. **7.** 4 T. 7 cwt. 82 lb. $3\frac{1}{3}$ oz. **8.** 15 cwt. 23 lb. 5 oz. **9.** 17 cwt. 31 lb. 9.6 oz.

Page 152.—**1.** 43 ℔ 10 ℥ 4 ʒ 13 gr. **2.** 7 ℥ 1 ʒ 1 ℈ 16 gr. **3.** $.44\frac{4}{9}$ ℔. **4.** 610,800 m. **5.** 15 Cong. 2 O. 3 f. ℥; 1 f. ʒ 2 m.

Page 154.—**1.** 67,222 min. **2.** 63,151,080 sec. **3.** 1610 da. 9 hr. **4.** 94 da. 21 hr. 36 min. **5.** .125 yr. **6.** 280 da. 10 hr. 24 min. **7.** 2 yr. 204 da. 3 hr. 28 min. **8.** 5 wk. 3 da. 19 hr. 45 min. 58 sec. **9.** .125. **10.** 138 da. 16 hr. 1 min. 50 sec. **11.** 8 yr.

Page 155.—**13.** 67 yr. 9 mo. 22 da. **14.** 56 yr. 5 mo. 12 da. **15.** March 11, 1899. **16.** 211 da. **17.** 173 da. **18.** 39 da.

Page 156.—**1.** 20,000 sheets. **2.** 320 pages. **3.** 5c. **4.** .125. **5.** 3391 units. **6.** $96.05. **7.** $41.60.

Page 158.—**1.** 31.33\frac{1}{10}$. **2.** 18,201 *d.* **3.** 81,534 far. **4.** £122 8*s.* 5*d.* 2 far. **5.** £$8\frac{25}{48}$. **6.** 7*s.* 10*d.* 1.36 far. **7.** £.47. **8.** £7 12*s.* 7*d.* 1 far. **9.** £11 7*s.* 6*d.*

Page 160.—**1.** 294,328″. **2.** 23° 31′ 12″. **3.** 165° 4′ 46″. **4.** 15° 35′ 22″. **5.** 72° 24′ 16.3″. **6.** $\frac{1}{6750}$. **7.** 16° 31′ 12″. **8.** 62.244 mi.

Page 161.—**1.** 1° 21′ 3″. **2.** 7° 5′. **3.** 30° 10′.

Page 162.—**4.** 63° 36′ 43″. **5.** 5° 56′ 30″. **6.** 21° 59′ 57″. **7.** 103° 57′ 9″. **8.** 98° 27′. **9.** 48° 23′ 51″. **10.** 30° 17′ 56″ east.

Page 163.—**2.** 1 hr. 39 min. $44\frac{2}{15}$ sec. **3.** 1 hr. 16 min. $19\frac{4}{5}$ sec. **4.** 44 min. 4 sec. **5.** 55 min. $39\frac{11}{15}$ sec. past 12 o'clock, noon. **6.** 5 hr. 17 min. $28\frac{1}{5}$ sec.

Page 164. — **7.** 1 hr. 51 min. $51\frac{2}{15}$ sec. slower. **8.** 59 min. $24\frac{2}{5}$ sec. past 8 A.M. **9.** 29 min. $37\frac{1}{3}$ sec. past 4 A.M. July 25. **10.** 20 min. $24\frac{2}{5}$ sec. past 11 P.M. Thursday. **11.** 55 min. till 1 A.M. July 27. ART. **427.** — **2.** 20° 41′ 15″. **3.** 77° 28″ west. **4.** 86° 32′ 30″ west.

Page 165. — **5.** 84° 30′ 3″ west. **6.** 89° 52′ 30″ west. **7.** 12° 28′ 26″ east. **8.** 13° 23′ 44″. **9.** 7 min. $50\frac{4}{5}$ sec. past 10 P.M. **10.** 75° 9′ 5″. **11.** 90° 12′ 28″ west.

Page 166. — **1.** $7\frac{1}{2}$°. **2.** 7 hr.; 2 hr.; 1 hr. **3.** $7\frac{1}{2}$°. **4.** 8 hr.; 3 hr.; 1 hr. **5.** 9 o'clock A.M.; 8 o'clock A.M.; 7 o'clock A.M.; 2 o'clock P.M.; 1 o'clock P.M.; 11 o'clock A.M.

Page 167. — **7.** 15 min. past 2 P.M.; 15 min. past 1 P.M.; 15 min. past 4 P.M. **8.** 4 min. **9.** 9 min. 40 sec. **10.** 8 min. 12 sec. ART. **430.** — **1.** $144.

Page 168. — **2.** 7 A. 55 sq. rd. $106\frac{1}{4}$ sq. ft. **3.** $3363.75. **4.** 4.5. **5.** $47\frac{1}{4}$ cu. in. **6.** 222 mi. **7.** 15,065,741 sq. li. **8.** 42,240. **9.** 6 oz. 17 pwt. $3\frac{3}{7}$ gr. **10.** 5.84+ ft. **11.** $36.864. **12.** $1,152,000. **13.** $4\frac{1}{2}$ yd. **14.** $141.75. **15.** .8125. **16.** $905.625. **17.** 121 ft. **18.** 6 ft.

Page 169. — **19.** $142.50. **20.** 937.5 bd. ft. **21.** 504. **22.** 1432. **23.** 107.16\frac{2}{3}$. **24.** $66.24. **25.** 3 A. 7 sq. ch. 6250 sq. li. **26.** 159.068 mi. **27.** 159.58+ bbl. **28.** .4125 lb. **29.** 296 cu. in. **30.** $57.29. **31.** 50 hr.

Page 170. — **1.** 320 A. **2.** 120 A.

Page 171. — **3.** 10 A. **4.** 160 A. **5.** 160 ch. 640 rd. **6.** 57 ch. **7.** 8. **8.** 41 A. 65 P. **9.** 480 rd. **10.** 269.2 rd. ART. **434.** — **1.** 24.

Page 172. — **2.** 67. **3.** 114. **4.** $11.50. **5.** $23.04. **6.** $92.89. **7.** $159.54. **8.** $54.20.

Page 173. — **2.** $13.15.

Page 174. — **3.** $426.50. **4.** 85. **5.** $13.60. **6.** $30.30. **7.** 75 yd. **8.** 15 yd. **9.** $81.65. **10.** Crosswise; $45\frac{1}{3}$ yd.

Page 175. — **11.** $256.29. ART. **439.** — **1.** 30 cd. **2.** 48 cd. **3.** $5\frac{1}{3}$ ft. **4.** $693. **5.** 36 ft.

Page 176. — **6.** $426\frac{2}{3}$ bd. ft. **7.** $172.80. **8.** 2914 bd. ft. **9.** $468.48. **10.** 768 bd. ft. ART. **441.** — **1.** 6250.

Page 177. — **2.** 36. **3.** 77 bunches; $269.50. **4.** $258.40. **5.** $17.36. **6.** 9548. **7.** $18.78. **8.** $13.60.

Page 178. — **1.** $25\frac{5}{11}$ P. **2.** $50.75. **3.** $49.50. **4.** $77.25. **5.** $291.12. **6.** 2700. **7.** 31,360 bricks.

Page 179. — **8.** 19,110. ART. **445.** — **2.** 723.21+ bu. **3.** 819.63+ bu. **4.** 259.2 bu. **5.** 1132.2. bu.

Page 180. — **2.** 311.11 cu. ft. **3.** 1647.66 cu. ft. **4.** 80.26 cu. ft. **5.** 15.625 cu. ft. **6.** 550 cu. ft. **7.** 2.5 cu. ft.

Page 181. — **2.** 362.24 bu. **3.** 51.88 bu. **4.** 402.48 bu. **5.** 26.25 bu. ART. **451.** — **2.** 198.76+ cu. ft. **3.** 357.77 cu. ft.

Page 182. — **4.** 477.03 cu. ft. **5.** 80 cu. ft. **6.** 3.81 cu. ft. ART. **454.** — **2.** 2154.39 gal. **3.** 440.65 gal. **4.** 59.84 hhd.

Page 183. — **5.** 540.11 gal. **6.** 1,469,685.62 gal. ART. **456.** — **2.** 193.7+ cu. ft. **3.** 303 cu. ft. **4.** 228.02 bbl. **5.** 119.39 bbl. **6.** 80.28+ bbl.

Page 184. — **1.** $181.50. **2.** $21.06. **3.** 9.75 ch. **4.** $8.71. **5.** 100.92\frac{1}{2}$. **6.** $998. **7.** 240 ft. **8.** $40.18. **9.** $13.08. **10.** $9.83. **11.** 510.

Page 185.—**12.** 510. **13.** 255. **14.** 8572, about. **15.** $410.50. **16.** 4000 cu. ft. **17.** 8.35 ft. **18.** 12.8 ft. **19.** 1263 cu. ft. **20.** 1.6 ft.

Page 188.—**1.** 40 dm.; 55 dm. **2.** 350 cm. **3.** 45.6 Dm. **4.** 38.69 dm. **5.** 350 Dm. **6.** 52.43 m. **7.** $1.16. **8.** 340 cm. **9.** $2. **10.** $11.75. **11.** 9087 mm. **12.** 8700 cm. **13.** 14.6324 Km.; 14632.4 m. **14.** 36.849 Dm.; 36,849 cm. **15.** 8408.7 m. **16.** 91.554 m. **17.** 6.86 m. **18.** 22.2 m. **19.** $174.40. **20.** $2.

Page 190.—**1.** 386.49 Ha. **2.** 8,964,500 sq. cm. **3.** 2214 sq. m.; 22,140,000 sq. cm. **4.** 89,634,000,000 sq. mm.; 896.34 sq. Dm. **5.** $798. **6.** 536.84 Ha.; .38645 A. **7.** 896.45 sq. Dm. **8.** 419.24 ca. **9.** $3400. **10.** 22.7 m. **11.** 51,129 sq. yd. **12.** 9.03 A. **13.** 37 A.; 77.17 sq. rd. **14.** 3642.0363 A. **15.** 89.04 m.

Page 192.—**1.** 8500 cu. mm.; 4,500,000 cu. mm. **2.** 95.6 cu. m. **3.** 421,372 cu. dm.; 75.006 st. **4.** .13684 cu. Dm.; 136,840 cu. dm. **5.** 7.85 cu. m. **6.** $27.30. **7.** 55.31\frac{1}{4}$. **8.** $8.64.

Page 193.—**1.** 73.6 dl.; 7360 ml. **2.** 97.02 l. **3.** 53.86 l.; .5386 Hl. **4.** 72346.7 l.; 723,467 dl.; 72,346,700 ml. **5.** 8.364 l. **6.** 2.3328 l.; 8.1 ml. **7.** $8.88.

Page 194.—**8.** 9 Hl. 4 Dl. 5 l. **9.** $9.54. **10.** $6.076.

Page 195.—**1.** 2964.31 g.; 29.6431 Hg.; 2.96431 Kg. **2.** .07634 T.; 763.4 Hg.; 76,340 g. **3.** 7057.054 g. **4.** 36.288+ g. **5.** 2268.01+ g. **6.** 1181.6656 lb. **7.** 115.76 g. **8.** 23.855 Hg. **9.** $2.064. **10.** 500.

Page 196.—**1.** 196,850 in. **2.** 50.391 m. **3.** 36.67 m. **4.** 365.6 m. **5.** .392 m. **6.** 46.224 Km. **7.** 7264.38 A.; 72.6438 Ha. **8.** 80,000 ca. **9.** $73.60. **10.** 5906.25 Kg. **11.** 2099.5 Hl. **12.** 41.6052 m.; 5.6325 Km. **13.** $7.078. **14.** 25 l.

Page 197.—**1.** 400. **2.** 84.3. **3.** 2840. **4.** 25. **5.** 60$\frac{15}{44}$%. **6.** $1305 A; $1160 B. **7.** 6%. **8.** $10,488. **9.** 2%.

Page 198.—**10.** 400 bu. **11.** 25%. **12.** $3400. **13.** $6.75. **14.** 12%. **15.** 25%. **16.** 50%. **17.** 20%. **18.** 6 cwt. **19.** 900 bu. **20.** 1$\frac{103}{110}$ mi. **21.** 52$\frac{1}{2}$%.

Page 199.—**22.** 48 yr. **23.** $675. **24.** $.53$\frac{1}{2}$. **25.** 42$\frac{6}{7}$%. **26.** 300. **27.** $4 loss.

Page 200.—**1.** $9.45. **2.** $156.25. **3.** 145.12\frac{1}{2}$; 5659.87\frac{1}{2}$. **4.** $132.48 Com.; $2811.52 Pro. **5.** $3846. **6.** $4626. **7.** $8463. **8.** $466.

Page 201.—**9.** $45. **10.** $265.30. **11.** 5$\frac{3}{4}$%. **12.** 2$\frac{1}{2}$%. **13.** 3$\frac{1}{4}$%. **14.** 340 bbl.

Page 202.—**15.** 320 A.; $576. **16.** 2497 lb. **17.** $2370.20. **18.** $234; $360. **19.** $2800; $3500. **20.** 4$\frac{1}{2}$%. **21.** $63. **22.** 30%; 5%.

Page 203.—**23.** 19812.8 lb. **24.** 40 cd. **25.** 1633\frac{1}{3}$. Art. **495.**—**1.** $410.40.

Page 204.—**4.** 5$\frac{61}{100}$%; 2$\frac{1}{4}$%. **5.** No dif. **6.** 82$\frac{9}{10}$%. **7.** $105. **8.** 69¢. **9.** $.5814. **10.** $787.50. **11.** 300. **12.** $300.

Page 205.—**13.** 3331.45 yd. Art. **501.**—**1.** $54. **2.** $115.72. **3.** $511.50.

Page 206.—**4.** $146.33. **5.** $4,280,000. **6.** 8 mills. **7.** 5 mills. **8.** $85,000. **9.** $876,666$\frac{2}{3}$. **10.** $1,443,298.95.

Page 207.—**1.** $61.25. **2.** $30.75. **3.** $63. **4.** $45.50. **5.** 5 mills.

6. $4800. **7.** $35,000. **8.** $36,666⅔. **9.** $7.21. **10.** 25%. **11.** $44,557.70.

Page 208.—**2.** $125.62½. **3.** $154.53. **5.** $359.55. **6.** $645.38. **7.** $1546.81.

Page 209.—**9.** $28.53. **10.** $242.62. **11.** $199.07. **12.** $95.87½. **13.** $72.19. **14.** $38.05. **15.** $574.35. **16.** $748.69. **17.** $3917.51. **18.** $7559.32.

Page 210.—**3.** $38.88. **4.** $222.31. **5.** $1464.87.

Page 211.—**6.** $612.55. **8.** $216.62. **9.** $142.21. **10.** $2120.59. **12.** $27.78. **13.** $3.99. **14.** $262.26.

Page 212.—**16.** $14.50. **17.** $23.24. **18.** $97.20. **19.** $943.49. **20.** $423.55. **21.** $4212.05. **22.** $662.77. **23.** $8497. **24.** $8718.78. **25.** $10,849.25. **26.** $2124.10.

Page 213.—**1.** $481.88. **2.** $48.61. **3.** $3251.09. **4.** $449.24. **5.** $4321.05. **6.** $123.82. **7.** $43.60. **8.** $199.05. **9.** $9591.11. **10.** $3754.83.

Page 214.—**1.** $7.40. **2.** $18.24. **3.** $14.23. **4.** $35.18. **5.** $104.70. **6.** $3930.49. **7.** $133.88. **8.** $382.04. **9.** $1889.40. **10.** $8800.56. **11.** $538.33.

Page 215.—**1.** $385.72. **2.** $425.88. **3.** $3106.30. **4.** $4145.08. **5.** $5026.60. **6.** $4598.25. **7.** $63.89.

Page 216.—**8.** $888.52. **9.** $2714.81. **10.** $38.64. ART. **514.**—**2.** $162. **3.** $1434. **4.** $9027.81. **5.** $726.38. **6.** $8637.01.

Page 217.—**7.** $468.11. **8.** $7600.35. **9.** $380. **10.** $800. **11.** $1200. ART. **515.**—**2.** 9%. **3.** 10½%. **4.** 6%.

Page 218.—**5.** 6%. **6.** 5%. ART. **516.**—**2.** 1 yr. 2 mo. **3.** 1 yr. 9 mo. **4.** 1 yr. 11 mo. **5.** 5 yr. 10 mo. 15 da. **6.** 3 yr. 3 mo. **7.** 3 yr. 4 mo. 15 da. ART. **517.**—**1.** $528.43. **2.** $131.48.

Page 219.—**3.** $30.11. **4.** $9000. **5.** $493.566. **6.** $86.44. **7.** $2808.30. **8.** $12.72. **9.** $724.73. **10.** $467.82. **11.** $860. **12.** 8 yr. 11 mo. 4 da. **13.** 5$\frac{5}{9}$%. **14.** 10%. **15.** 12 yr. 8 mo. 21 da. **16.** $481.481. **17.** 22 yr. 2 mo. 20 da.

Page 220.—**18.** $17,274.577. **19.** $4360.237. **20.** 4 mo.

Page 221.—**2.** $353.96.

Page 222.—**3.** $3089.45. **4.** $469.16. **5.** $645.65. **6.** $981.99.

Page 223.—**7.** $895.49. **8.** $968.80. **9.** $757.21.

Page 225.—**2.** $7490.06. **3.** $5241.91. **4.** $10,974.20.

Page 226.—**5.** $7011.74. **6.** $8368.39. ART. **529.**—**2.** $111.68. **3.** $6215.08.

Page 227.—**4.** $127.98.

Page 228.—**1.** $297.80, Pro.

Page 229.—**2.** Due Dec. 7, 1898; Term of dis., 52 da.; Dis. $31.20; Pro., $3568.80. **3.** Due Aug. 8, 1898; Term of dis., 64 da.; Dis., $10.45; Pro., $969.55. **4.** Due June 30, 1898; Term of dis., 81 da.; Dis., $4.78; Pro., $381.72.

Page 230.—**6.** Due Nov. 20, 1898; Term of dis., 76 da.; Dis., $13.34; Pro., $776.66. **7.** Due Feb. 11, 1899; Term of dis., 144 da.; Dis., $9.27; Pro., $377.13.

Page 231.—**9.** Due April 21, 1899; Term of dis., 110 da.; Dis., $12.91; Pro., $691.235. **10.** $9.71. **11.** $751.93.

Page 232. — **12.** $50.72. **13.** $8221.40. **14.** $8925. **15.** $380.54. **16.** $486.67. **17.** $300.12. **18.** $875.08. **19.** $959.03. **20.** $677.10. **21.** $7620.66.

Page 233. — **23.** $1206.636. **24.** $811.359. **25.** $642.916. **26.** $1442.668. **27.** $6820.083. **28.** $842.409. **29.** $93.64. **30.** $401.429. **31.** $12,120.

Page 234. — **32.** $1597.35. Art. **541.** — **2.** $2400. **3.** $1376.14. **4.** $210.204. **5.** $41.81.

Page 235. — **6.** $3.374 int. more. **7.** At $$4\frac{1}{2}$ on 4 mo. is 44¢ more profitable. **8.** $54.059. **9.** Gain $30.297. **10.** $5333.094.

Page 237. — **2.** $106.968. **3.** $223.104. **4.** $625.077. **5.** $5568.526. **6.** $5277.507. **7.** $111.022. **8.** $54.68.

Page 239. — **9.** $246.435. **11.** $300.

Page 240. — **2.** $3678.60. **3.** $4227.868. **4.** $773.824. **5.** $920.927. **6.** $2585.869. **7.** $4.645 comp. int. greater. Art. **548.** — **1.** $863.199. **2.** $979.225.

Page 241. — **3.** $1336.406. **4.** $2087.144. **5.** $16\frac{2}{3}$%. **6.** $19.358 loss. **7.** $1398.987. **8.** $184.646. **9.** $209.26 gain. **10.** $2320. **11.** 60 yr.

Page 243. — **2.** $$2453\frac{1}{3}$. **3.** $$4663\frac{1}{3}$. **4.** $125. **5.** $5281.25. **6.** $450. **7.** 80 shares.

Page 244. — **8.** $4000. **9.** 72 shares. **10.** 120 shares. **11.** 79 shares; $31 surplus. **13.** $8600. **14.** $9000. **15.** $560,000; 500 shares.

Page 245. — **17.** $8400. **18.** $36,630. **20.** $522.50. **21.** $382.50. **23.** $5\frac{5}{14}$%. **24.** 5%.

Page 246. — **25.** $6\frac{12}{13}$%. **26.** $7\frac{1}{7}$%. **27.** $5\frac{5}{9}$%. **29.** $80. **30.** $$133\frac{1}{3}$. **31.** $125. **33.** 200 shares. **34.** $30,000; 300 shares.

Page 247. — **35.** $250. **36.** $180. **37.** $28,355. **38.** $7\frac{1}{2}$%. **39.** $4\frac{1}{2}$%. **40.** 5%; 6%. **41.** Increased $5; 133 shares; 30 surplus. **42.** No difference. **43.** $7\frac{1}{2}$%; $8\frac{8}{9}$%. **44.** $80.

Page 248. — **45.** Diminished $75. **46.** 20%. **47.** 27 shares; $77.25 surplus. **49.** $4390, N.Y.C.; $8980, D. and H. **50.** 7% at 90 is $40 more profitable. **51.** $16,000; $640.

Page 249. — **53.** $4\frac{12}{17}$%. **54.** $5\frac{15}{23}$%. **56.** $5\frac{15}{43}$%. **57.** $6\frac{78}{97}$%. **58.** 1st, $5\frac{20}{39}$% = 5.51 + %; 2d, $5\frac{545}{573}$% = 5.95%. **59.** 6% at 90 is $2\frac{7}{9}$% better.

Page 250. — **60.** *a*, $6\frac{18}{19}$%; *b*, $5\frac{5}{156}$%; *c*, $3\frac{31}{53}$%. **61.** $325 gain.

Page 252. — **2.** $803. **3.** $$758.43\frac{3}{4}$. **4.** $1481.25. **5.** $345.066. **6.** $954.

Page 253. — **8.** $320.24. **9.** $557.62. **10.** $2973.75. **12.** $1965.50. **13.** $997.50.

Page 254. — **15.** $2500. **16.** $3850. **17.** $2409.50. **19.** $2000.

Page 255. — **2.** $1830.56. **3.** $2291.157.

Page 256. — **5.** $2323.585. **7.** $871.70. **8.** $7513.38. **9.** $2663.40.

Page 257. — **1.** 585 bu., 1897; 450 bu., 1898. **2.** $321 A. 40 P. **3.** $6\frac{1}{4}$% loss. **4.** 30.22%. **5.** 54.3 + %. **6.** $810. **7.** $32.17.

Page 258. — **8.** $3007.024. **9.** $12\frac{1}{2}$ yr. **10.** 25 yr. **11.** $109.888. **12.** $202.122. **13.** $2870, flour; $2660, sugar. **14.** $19,000. **15.** $2574.71. **16.** $12\frac{128}{475}$%. **17.** $800; 6%. **18.** $9.21. **19.** $1486.428.

Page 259. — **20.** $36.178 gain. **21.** $558.362. **22.** $5122.29. **23.** 30%. **24.** Oct. 23. **25.** 4 yr. 8 mo. **26.** $47\frac{1}{5}$%. **27.** 84 shares. **28.** $2,368,421.

Page 261.—**1.** $\frac{1}{7}$; 17. **2.** $\frac{1}{8}$; $\frac{9}{16}$. **3.** 10; $\frac{41}{84}$. **4.** $\frac{1}{8}$; $\frac{1}{3}$. **5.** 6; $\frac{8}{75}$. **6.** $\frac{7}{12}$; $3\frac{3}{5}$; $21\frac{5}{7}$. **7.** 3. **8.** 212. **9.** 35. **10.** $4\frac{5}{6}$.

Page 262.—**1.** 7. **2.** 32. **3.** 3. **4.** 12. **5.** 90. **6.** 76 bu. **7.** 280 yd. **8.** $1\frac{5}{7}$. **9.** $69\frac{9}{14}$. **10.** 26.

Fage 264.—**3.** $168. **4.** $6\frac{2}{3}$ hr. **5.** $356. **6.** $341. **7.** 4000 lb. **8.** 400 mi. 6 rd. 11 ft. **9.** 91 bu. **10.** 9 da. **11.** $$47\frac{1}{4}$. **12.** $19\frac{1}{5}$ da. **13.** $44\frac{65}{72}$ ft.

Page 265.—**14.** 578.8 T. **15.** $41\frac{1}{4}$ bbl. **16.** $66\frac{2}{3}$ T. **17.** $379\frac{1}{11}$ bu. **18.** $11.255. **19.** 880. **20.** $$1743\frac{3}{31}$. **21.** $$2571\frac{3}{7}$. **22.** $37.44. **23.** $12\frac{1}{2}$ oz. **24.** 200 rd. **25.** 96 loaves. **26.** 138. **27.** 7920 times.

Page 266.—**28.** $411\frac{1}{4}$ yd. **29.** $74\frac{2}{3}$ yd. **30.** 160. **31.** 7%.

Page 267.—**3.** 192. **4.** $1141. **5.** $11.50. **6.** $31\frac{1}{4}$ lb. **7.** $44.10. **8.** $165.

Page 268.—**9.** $1216. **10.** $600. **11.** 38,400. **12.** $16\frac{13}{32}$ da.

Art. **597**—**2.** 55; 66; 77. **3.** 34; 51; 136; 153. **4.** 54; 72; 81.

Page 269.—**5.** $135; $324; $360. **6.** $1400; $1600; $1800. **7.** 189; 245. **8.** $1083; $1805; $2888. **9.** 288 bu. corn; 180 bu. rye. **10.** $985.60; $1056; $1161.60. **11.** 9180; 10,200; 12,240. **12.** $7528; $13,174; $16,938.

Page 271.—**2.** $375 A; $300 B. **3.** $90 A; $115 B; $148 C. **4.** $3825 A; $4050 B; $5400 C. **5.** $7500 A; $6500 B; $8400 C.

Art. **605.**—**2.** $1368 A; $760 B; $342 C.

Page 272.—**3.** $1080 S.; $600 J. **4.** $750 Samson; $300 Straw; $200 Riddle. **5.** $280 W.; $350 A.; $420 T. **6.** $5670 A.; $7560 R. **7.** $624 P.; $676 C.; $14 mo. A.

Page 273.—**2.** $5\frac{7}{10}$ mo. **3.** $5\frac{2}{7}$ mo. **4.** $46\frac{2}{3}$ da. = 47 da.

Page 274.—[illegible]. $3\frac{1}{3}$ mo. **8.** $1\frac{1}{2}$ mo.

Page 275.—**10.** $10\frac{1}{2}$ mo. Art. **609.**—**2.** April 11. **3.** Aug. 28. **4.** Oct. 18. **5.** $3\frac{2}{3}$ mo.

Page 276.—**1.** 800 bu. **2.** $6\frac{1}{4}$%. **3.** $16\frac{2}{3}$ yr. **4.** $11\frac{9}{10}$ yr. **5.** $4\frac{6}{11}$ ¢. **6.** $4660. **7.** $11,295.336. **8.** $5\frac{3}{5}$ hr. **9.** $3\frac{1}{3}$ mo.; July 3. **10.** 45; 51.

Page 277.—**11.** 715 bu. **12.** $1375.31. **13.** 19 mo. **14.** $2055.78. **15.** 18,824. **16.** $8\frac{8}{9}$%. **17.** 18 ¢. **18.** 15; 30; 90. **19.** $2.52; 20%. **20.** $10\frac{1}{8}$ mi.

Page 278.—**21.** $1505\frac{5}{11}$ bu.; $1254\frac{6}{11}$ bu. **22.** $18,000. **23.** $870; 6%. **24.** $136.17. **25.** 10 men. **26.** $46.57. **27.** $201.60. **28.** 88 ft. **29.** $\frac{20}{29}$. **30.** 172. **31.** 6000 bu. **32.** 25. **33.** 4056 bd. ft.

Page 279.—**34.** $7.84. **35.** $1375 in $1\frac{2}{3}$ is $50 better. **36.** $9.60. **37.** $168 A; $201.60 B; $192 C. **38.** $117,000. **39.** 36.16 bu. **40.** $3835.117. **41.** 9% loss. **42.** $512.263. **43.** 1128 bd. ft.

Page 280.—**44.** $33\frac{1}{3}$%; $66\frac{2}{3}$%. **45.** $$416\frac{2}{3}$. **46.** $33\frac{1}{3}$%; $66\frac{2}{3}$%. **47.** 1. **48.** $13\frac{3}{19}$ yd. **49.** $854.647. **50.** $563.176. **51.** 580.805. **52.** $\frac{7}{9}$.

Page 281.—**53.** 50%. **54.** 3200 bd. ft. **55.** $12\frac{1}{2}$%. **56.** $206\frac{2}{3}$ da. **57.** $16,277.804.

Page 282.—**1.** 6561; 2197. **2.** 16,807; 32,768.

Page 283.—**3.** 400; 4096; 144,000. **4.** $75\frac{1}{9}$; $29\frac{52}{81}$; $277\frac{7}{9}$. **5.** 1.898; $53\frac{7}{9}$. **6.** 41.64+; $\frac{343}{3375}$. **7.** .000000216; 903.904225. **8.** 7^7. **9.** 9^{14}. **10.** $(\frac{3}{4})^{14}$. **11.** 4th power. **12.** 6th power. **13.** 6th power. **14.** 16. **15.** 32. **16.** $\frac{1}{2}$. **17.** $\frac{1}{9}$. **18.** 5.

Page 288.—**3.** 499. **4.** 563. **5.** 591. **6.** 696. **7.** 758. **8.** 888.

Page 291.—1. 14. 2. 19. 3. 24. 4. 46. 5. 79. 6. 216. 7. 807. 8. 435. 9. 726. 10. 806. 11. .24. 12. .35. 13. .08. 14. .05108. 15. .413. 16. 7.3. 17. 34.0917. 18. 86.01. 19. $\frac{5}{9}$. 20. $\frac{14}{15}$. 21. $\frac{13}{19}$. 22. $\frac{21}{25}$. 23. $\frac{7}{8}$. 24. .6546+. 25. 2.738+. 26. 3.093+. 27. 2.217+. 28. $8\frac{3}{4}$. 29. 3.824+. 30. $\frac{13}{18}$. 31. 3.561. 32. 5.0074. 33. .02108.

Page 292.—1. 12 ft. 2. 80 rd. 3. 16.5 ft. 4. 742. 5. $96. 6. 220 yd. 7. 6 ft. sq. 8. $1.80. 9. 80 rd. long; 20 rd. wide. 10. 90 ft. long; 40 ft. wide.

Page 299.—1. 14. 2. 25. 3. 56. 4. 123. 5. 336. 6. 607. 7. 705. 8. 6.38. 9. 2.34. 10. 25.46. 11. 2.98+. 12. .6. 13. .07. 14. $\frac{4}{7}$. 15. $\frac{8}{13}$. **Art. 651.**—1. .4641+. 2. 1.81712. 3. .854+; $2\frac{2}{3}$; $16\frac{3}{4}$; .961+; .529+. 4. 23.9. 5. $20\frac{1}{4}$ sq. ft. 6. 4.63 ft. 7. 337.5 sq. ft.

Page 302, Art. 674.—2. 16 ft.

Page 303, Art. 676.—1. 504 sq. rd. 2. 840 sq. yd. 3. 205.8 sq. ch. 4. 80 rd. 5. 55 yd. 6. 150 sq. ft. 7. $5100. 8. 780.62 sq. rd.

Page 305.—1. 68. 2. 25. 3. 68. 4. 120 mi. 5. 30 ft. 6. 114 ft. 7. 33 ft. 8. 271.75 ft. 9. 106.12 ft. 10. 43.86 ft.

Page 309.—1. 750 sq. rd. 2. 700 sq. yd. 3. $18\frac{3}{4}$ A. 4. 40 sq. ft. 5. 2100 sq. ft.

Page 310.—6. 517.42 sq. rd. 7. 2592 sq. ft. 8. 960 sq. ft. 9. 972 sq. yd.

Page 312.—1. 94.248 ft. 2. 10.82+ rd. 3. 1385.4456 sq. ft. 4. 36.78 sq. yd. 5. 19.54+ ft.; 61.39+ ft. 6. 14.27+ rd.; 44.84+ rd. 7. 14.14+ ft. 8. 22.62+ yd. 9. 32 sq. ft. 10. 7.07+ ft.

Page 315.—1. 81 sq. ft. 2. 25 ft. 3. 25 times. 4. 69.28 rd. 5. 80 min. 6. 5.125 sq. ft. 7. 4 ft. 8. 24 ft. 9. 150 rd. long; 100 rd. wide.

Page 317.—2. 160 sq. ft. 3. $112\frac{1}{2}$ sq. ft.

Page 318.—4. 140 sq. ft. 5. 103 sq. ft. **Art. 720.**—1. 201.06 + cu. ft. 2. 207.6 cu. ft. 3. 848.23+ cu. ft. 4. 34.91 cu. ft. 5. 8.94+ in.

Page 319.—1. 162 sq. ft. 2. 468 sq. in. 3. 8000 sq. ft. 4. 169.64 + sq. ft.

Page 320.—1. 96 cu. ft. 2. 95.49 + cu. ft. 3. 215.98 + cu. ft. 4. 279.55 + sq. ft. 5. 41,310.4 cu. ft. 6. 295.57 cu. ft. **Art. 731.**—1. 207 sq. ft.

Page 321.—2. 2880 sq. ft. **Art. 733.**—2. 697.43 + cu. ft. 3. 137.04 cu. ft.

Page 322.—1. 34.9 + sq. ft. 2. 452.39 sq. in. **Art. 739.**—1. 113.09 + cu. ft. 2. 179.59 + cu. ft. 3. 12.56 + ft.

Page 323.—1. 8.

Page 324.—2. 343 lb. 3. 6 lb. 4. 2 in. 5. $250\frac{14}{25}$ lb. 6. $6\frac{2}{3}$ ft. 7. 6.34 + ft. 8. 12 ft. 9. 27. 10. $1\frac{2}{25}$ T. **Art. 742.**—1. 216 sq. in. 2. 136 sq. ft. 3. 848.23 + cu. ft. 4. 100 ft.

Page 325.—5. 30.59 + ft. 6. 140 ft. 7. 16 ft. 8. 55 A. 88 sq. rd. 26 sq. yd. 8 sq. ft. 9. 10 ft. 10. 14.14 + in. 11. 672.09 gal. 12. 128 sq. ft. 13. 11.31 + in. 14. $157.08. 15. 18 in. 16. 32.56 + gal. 17. $\frac{27}{32}$ hr.

Page 326.—18. 80 rd. long; 60 rd. wide. 19. 16 in. 20. 209.21 + cu. ft. 21. 237.37 + sq. ft. 22. $106.94 +. 23. $3.09 +. 24. 38 ft. 25. 16. 26. 52.201 ft. 27. $209.86. 28. 3. 29. 10.19 + ft. 30. 1944 sq. ft

Page 327.—**2.** 24 bu., W.; 12 bu., J.

Page 328.—**4.** 60¢, R; 20¢, E. **5.** $200, H; $50, C. **6.** 90; 18. **8.** $3, H; $6, S; $24, C. **9.** $$4222\frac{2}{9}$, 1st; $$12,666\frac{2}{3}$, 2d; $$21,111\frac{1}{9}$, 3d. **10.** 9. **11.** 9. **12.** 21; 63.

Page 329.—**13.** 24; 96. **14.** 128 bu., O; 512 bu., W. **15.** 13. **Art. 746.**—**1.** 24. **2.** 36. **3.** 28. **4.** 23. **5.** 7. **6.** 27. **7.** 24. **8.** 10. **9.** 21. **10.** 12. **11.** 8. **12.** 32. **Art. 747.**—**1.** 30. **2.** 14. **3.** 8.

Page 330.—**4.** 10. **5.** 12; 36; 72. **6.** 8; 24; 64. **7.** 8 qt.; 16 qt.; 48 qt. **8.** 17; 51. **9.** 32 rd.; 64 rd. **10.** 9 bu.; 18 bu.; 36 bu.

Page 331.—**2.** $300. **3.** 180; 270. **4.** 12 yr. **5.** $6000. **6.** 45 yr. **7.** $2400. **9.** 480.

Page 332.—**4.** 20. **5.** 24. **6.** 30. **7.** 36. **8.** 44. **9.** 45. **10.** 21. **11.** 15. **12.** 30. **Art. 751.**—**4.** 30. **5.** 36. **6.** 60. **7.** 96. **8.** 72. **9.** 400. **10.** 100. **11.** 60. **12.** 36.

Page 333.—**4.** 16 yr.

Page 334.—**5.** $10, H.; $16, C. **6.** 35 mi. **7.** 10, M.; 6, S. **8.** 60. **9.** $20, M.; $32, E. **10.** $140. **Art. 753.**—**1.** 29. **2.** 52. **3.** 47. **4.** 39. **5.** 106. **6.** 85. **7.** 44. **8.** 80. **9.** 105. **10.** 120. **11.** 24. **12.** 22. **13.** 35. **14.** 82. **15.** 42. **16.** 24.

Page 335.—**1.** 16. **2.** 6. **3.** 77. **4.** 18. **5.** 56. **6.** 39. **7.** 9. **8.** 202. **9.** 25. **10.** 3. **11.** 6. **12.** 14. **13.** 29. **14.** 4. **15.** 9. **Art. 757.**—**2.** 5. **3.** 7. **4.** 6. **5.** 3. **6.** 3. **7.** 4. **8.** 1. **9.** 8.

Page 336.—**10.** 2. **11.** 4. **12.** 4. **13.** 9.

Page 337.—**4.** $80. **6.** 8. **7.** $376; $532; $782. **8.** $8000.

Page 338.—**1.** $3600. **2.** $325. **3.** 7%. **4.** $2\frac{1}{2}$%. **5.** $60 per share. **6.** $\frac{181}{400}$. **7.** 12¢ loss. **8.** 11 min. 56 sec. past 3 P.M. **9.** 84° 24′ west. **10.** 155.97+ sq. yd.

Page 339.—**11.** $265.68. **12.** $537.279. **13.** $76,000. **14.** $820.869, P.W.; $123.13 Dis. **15.** $1904 in 9 mo. **16.** $1456.796. **17.** $1925.025. **18.** $4298.90. **19.** $3017.20. **20.** $729.49. **21.** 3 yr. 4 mo. **22.** $1600.667.

Page 340.—**23.** $6391.853. **24.** $3890.75. **25.** $17\frac{24}{31}$ da. **26.** $1680, A; $2016, B; $5544, C. **27.** $9774.968. **28.** $15,000, A; $12,500, B; $10,000, C. **29.** 1650 bd. ft. **30.** $$2531\frac{1}{4}$, A; $4050, B; $$3543\frac{3}{4}$, C. **31.** $200 loss. **32.** $112. **33.** 521.6 A.

Page 341.—**34.** 60.345 ft. **35.** 8%. **36.** $46\frac{2}{3}$%. **37.** $1.32. **38.** 25 bd. ft. **39.** About 12.86 rd. around the square. **40.** 40 ft. **41.** $73.75. **42.** $5.50 gain. **43.** $4634.994. **44.** 3 mo.

Page 342.—**45.** $5768. **46.** $80 per share. **47.** $33\frac{1}{3}$%. **48.** 54.3%. **49.** $583\frac{1}{3}$ cu. ft. **50.** 43.86 ft.

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